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Q NO 1

A man throws two fair dice, what is Conditional probability that the sum of the two dice will be 7 given that

- 1) The sum is even
- 2) The sum is greater than 8
- 3) The two dice had the same outcome.

Ans.:

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (3,8) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (4,7), (4,8) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (5,7), (5,8) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8) \\ (7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8) \\ (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8) \end{array} \right\}$$

Let :

$$A = \left\{ \text{The sum is 7} \right\}$$



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$B = \{ \text{The sum is even} \}$   
 $C = \{ \text{The sum is greater than 8} \}$   
 $D = \{ \text{The two dice had the same outcome} \}$

NOW

$A = \{ (1,6), (2,5), (3,4), (5,2), (6,1), (4,3) \}$

$B = \{ (1,1), (1,3), (1,5), (1,7), (2,2), (2,4), (2,6), (2,8), (3,1), (3,3), (3,5), (3,7), (4,2), (4,4), (4,6), (4,8), (5,1), (5,3), (5,5), (5,7), (6,2), (6,4), (6,6), (6,8), (7,1), (7,3), (7,5), (7,7), (8,2), (8,4), (8,6), (8,8) \}$

$C = \{ (1,8), (2,7), (2,8), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8), (5,4), (5,5), (5,6), (5,7), (5,8), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8) \}$

$D = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8) \}$



$$A \cap B = \{ \} \text{ OR } \phi$$

$$A \cap C = \{ \}$$

$$A \cap D = \{ \}$$

$$P(A) = 6/64, \quad P(B) = 32/64$$

$$P(C) = 36/64, \quad P(D) = 8/64$$

$$P(A \cap B) = 0, \quad P(A \cap C) = 0, \quad P(A \cap D) = 0$$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0 \times 32/64$$

$$P(A/B) = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times 36/64$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times 8/64$$

$$P(A/D) = 0$$



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Q NO 2

show that in a single throw of two dice the probability of throwing more than 7 is equal to that of throwing less than 7, and hence find the probability of throwing exactly 7 state clearly your assumption you are making.

Ans:

when we are rolling two dice there are 36 different combinations counting those up there are 15 possibilities are less than 7

(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1) The probability of getting less than a 7 is

$$\frac{15}{36} = \frac{5}{12}$$

These are 6 possible combinations of getting a 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) which gives a

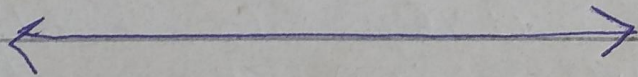
Probability of

$$\frac{6}{36} = \frac{1}{6}$$



<5>

This mean that 21 possibilities  
account for getting less than  
are equal to 7 this is  
the same as the possibility of  
getting less than 7 so the  
probability must be  $\frac{5}{12}$  as  
well in calculating this we  
must assume that each combination  
is equally likely to roll as any  
other one therefore the dice  
are fair or also the  
calculation don't work.





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Q NO 3

A and A play a game in which A's probability of winning is  $\frac{2}{3}$ . In a series of 8 games, what is the probability that A will win.

1. Exactly 4 games
2. at least 4 games
3. from 3 to 6 games

Ans:

Given that  $p = \frac{2}{3}$   $n = 8$

$$q = 1 - p$$
$$= 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let "x" denotes the number of games won by A then

$$(i) P(X=4)$$

$$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$= 0.1707$$



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$$(ii) P(x \geq 4)$$

$$= 1 - P(x < 4)$$

$$= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[ \binom{8}{0} \left(\frac{1}{3}\right)^8 + 8 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + \right.$$

$$\left. 56 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right]$$

$$= 1 - \frac{1}{6561} [1 + 16 + 112 + 448]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

$$(iii) P(3 \leq x \leq 6) = \sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8} [56 + 146 + 224 + 224] \Rightarrow \frac{5152}{6561}$$

$$= 0.7852$$



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Q NO 4

Let  $C_1, C_2, \dots, C_M$  be a partition of the sample space  $S$  of  $B$  be two events. Suppose we know that

- $A$  &  $B$  are conditionally independent given  $C_i$  for all  $i \in \{1, 2, \dots, M\}$
- $B$  is independent of all  $C_i$

Ans:

Proof:

Since the  $C_i$ 's form a partition of the sample space we can apply the law of total probability for  $A \cap B$ .

$$P(A \cap B) = \sum_{i=1}^M P(A \cap B | C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^M P(A | C_i) P(B | C_i) P(C_i)$$

(  $A$  and  $B$  are conditionally independent )



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$$P(A \cap B) = \sum_{i=1}^n P(A/c_i) P(B) P(c_i)$$

$\therefore$  (B is independent of all  $c_i$ 's)

$$P(A \cap B) = P(B) P(A)$$

$$P(A \cap B) = P(B) P(A)$$

(Law of total probability)

Hence A & B are independent



0



Q5  
Solve the binomial distribution and find its mean and variance.

ans  
Mean and variance of binomial Random variable.

The probability function for a binomial random variable is  
 $b(x, n, p) = \binom{n}{x} p^x (1-p)^{n-x}$

This is the probability of having  $x$  successes in a series of  $n$  independent trials when the probability of success is any one of the trials is  $p$ . If  $x$  is a random variable with the probability distribution

$$\begin{aligned}
 F(x) &= \sum_{z=0}^x \binom{n}{z} p^z (1-p)^{n-z} \\
 &= \sum_{z=0}^x \binom{n}{z} \frac{n!}{z!(n-z)!} p^z (1-p)^{n-z} \\
 &= \sum_{z=0}^x \frac{n!}{z!(n-z)!} p^z (1-p)^{n-z}
 \end{aligned}$$

Since  $x=0$  term vanishes let  $y=x$   
by  $m=n-1$  Subbing  $n=y+1$  by  
 $m=m+1$  into the last  
sum



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$$\begin{aligned} E(X) &= \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y} \\ &= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\ &= np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \end{aligned}$$

By Binomial theorem

$$= (a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

set  $a=p$  and  $b=1-p$

$$= \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

$$= (a+b)^m$$

$$= (p+1-p)^m$$

$$= 1$$

So that

$$\boxed{E(X) = np}$$

Similar but this



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$$y = n - 2, \quad m = n - 2$$

$$E = (x(x-1)), \quad \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1) p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1) p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} (p^y (1-p)^{m-y})$$

$$= n(n-1) p^2 (p + (1-p))^m$$

$$= n(n-1) p^2$$

So the variance of size

$$= (x^2) - E(x)^2 = E(x(x-1)) + E(x)$$

$$E(x^2) = (n(n-1) p^2) + (np)$$

$$= np(1-p)$$



Q NO 6

Differentiate b/w Binominal frequency distribution of Bi-nominal distribution with the help of formulas?

## Bi-nominal Distribution

A binominal distribution can be through of as simply the probability of a success or failure outcome come in an experiment or survey that it repeated multiple time.

## Bi-nominal frequency distribution

if the bi-nominal probability distribution is multiplied by  $N$  the number of experiment or sets are resulting distribution is known as the binominal frequency distribution

$$N \binom{n}{x} p^x q^{n-x}$$



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Q NO 7

Coefficient of Variation:

For Data Set A

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$\boxed{CV = 6.7}$$

For Data Set B

$$CV = \frac{11}{60} \times 100$$

$$\boxed{CV = 18.3}$$

For Data Set C

$$CV = \frac{5}{50} \times 100$$

$$\boxed{CV = 10}$$

For Data Set D

$$CV = \frac{15}{25} \times 100$$

$$\boxed{CV = 60}$$