Mid Assignment - Sping 2020 Course Differential Equation

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Q1 a) Define differential equation along with 2 examples

Ans Differential equation: Differential equation is an equation that relates are one more functions and their derivatives. In applications the functions generally represent physical quantities. The derivatives represents their rates of charge and the differential equation defines a relationship between the two.

Exp (1) $\frac{dy}{dx} + xy = e^{2x}$ (2) $Uxx + Uyy + 0$

As both of the above example contains the derivative so these are differential equation.

(b) Define a separable Differential equation (DE)?

Ans Separable (DE): A separable differential equation is one that can be broken into a set of separate equations of lower dimensionality by a method of separation of variasles

Solve the following initial value problem (IVP) using separable DE and Find the interval of validity of the solution

(a) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$

Sol $\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$

$= \int y^{-3} dy = \int \frac{x}{\sqrt{1+x}} dx$

$\Rightarrow 1+x^2 = u$

$2x dx = du$

$\Rightarrow x du = \frac{du}{2}$

$\Rightarrow \int y^{-3} dy = \int \frac{1}{\sqrt{u}} \frac{du}{2}$

$\Rightarrow \frac{y^{-3+1}}{-3+1} = \frac{1}{2} \frac{u^{-1/2+1}}{-1/2+1} + c$

$\Rightarrow \frac{y^{-2}}{-2} = \frac{2}{2} \sqrt{u} + c$

$\Rightarrow \frac{1}{-2y^2} = \sqrt{u} + c$

$\Rightarrow \frac{1}{-2y^2} = \sqrt{1+x} + c$

$\Rightarrow y(0) = -1$

$\frac{1}{-2(-1)^2} = \sqrt{1} + c$

$= \frac{1}{-2} = 1 + c$

$\Rightarrow -1 \; \frac{-1}{2} = c$

$\Rightarrow c = \frac{-2-1}{2}$

$\Rightarrow c = \frac{-3}{2}$

$\Rightarrow \frac{y^{-2}}{2} = \sqrt{1+x^2} - 3/2$ Ans

(b) $y' = e^{-y}(2x-4) \quad y(5) = 0$

Sol $\frac{dy}{dx} = e^{-y}(2x-4)$

$\Rightarrow \int \frac{dy}{e^{-y}} = \int (2x-4)\,dx$

$\Rightarrow \int e^{y}dy = \frac{2x^2}{2} - 4x + c$

$\Rightarrow e^{y} = x^2 - 4x + c$

$\Rightarrow y = \ln(x^2 - 4x) + c$

$\Rightarrow y = \ln((5)^2 - 4(5)) + c$

$\Rightarrow 0 = \ln(25-20) + c$

$\Rightarrow 0 = \ln 5 + c$

$\Rightarrow c = -\ln 5$

$\Rightarrow y = \ln(x^2 - 4x) - \ln 5$

Q2 a Solve the following IVP using Linear Differential method

(i) Explain the steps for solving Linear Differential eq.

Following are the steps for solving Linear DE.

(1) Substitute $y = uv$ and

$$\frac{dy}{dx} = \frac{u\,dv}{dx} + v\frac{du}{dx}$$

into $\frac{dy}{dx} + P(x)y = Q(x)$

(2) Factor the parts involving v.

3 Analving v term equal to zero (this given D.E in u and x which can be solved in the next step).

4 Solve using separation of variable to find u.

5 Substitute u back into the equation we get at step 2.

6 Solve that to find v.

7 Finally substitute u and v into y = uv to get solution.

2] $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1 \quad y\left[\frac{\pi}{4}\right] = 3\sqrt{2} \quad 0 \le x < \frac{\pi}{2}$

Sol ÷ing by $\cos(x)$

$$y' + \frac{\sin(x)y}{\cos x} = \frac{2\cos^3(x)\sin(x) - 1}{\cos(x)}$$

$$y' + \tan(x)y = 2\frac{\cos^3(x)\cdot\sin(x) - 1}{\cos(x)} \quad -(1)$$

It has the form $y' + P(x)y = Q(x)$

where $P(x) = \tan x$ & $Q(x) = \frac{2\cos^3(x)\sin(x) - 1}{\cos(x)}$

Integrating factor $= e^{\int P(x)\,dx} = e^{\int \tan x\,dx} = e^{\ln[\sec x]}$

=> Integrating factor = $\sec x$.

Now multiplying (1) by I.f or $\sec(x)$.

(1) => $\sec x\, y' + \sec x \tan(x) y = 2\frac{\cos^3 6(x)\sin(x) - 1}{\cos(x)}$

$$\Rightarrow \frac{d}{dx}[y \sec x] = 2\frac{\cos^3 x \sin x - 1}{\cos 2x}$$

$$\int d[y \sec x] = \int 2\frac{\cos^3(x)\sin(x) - 1}{\cos^2 x}\,dx$$

By solving this Integration we get.

$y \sec x = \frac{-1}{\tan^2 x + 1} - \tan x + c$

$y = \cos x \left[\frac{-1}{\tan^2 x + 1} - \tan x + c \right]$

Now $y(\pi/4) = 3\sqrt{2}$

$\Rightarrow y(\pi/4) = \cos \pi/4 \left[\frac{-1}{\tan^2(\pi/4) + 1} - \tan(\pi/4) + c \right]$

$\Rightarrow 3\sqrt{2} = \frac{1}{\sqrt{2}} \left[-\frac{1}{2} - 1 + c \right]$

$\Rightarrow 3 \times 2 = \frac{-3}{2} + c$

$= c = -4$

$y = \cos x \left[-\frac{1}{\tan^2 x + 1} - \tan x - 4 \right], 0 \leq x \leq \pi/2$

(3) $x' + 2x = \sin t$

Sol $x' + 2x = \sin t$ — (1)

This DE has the form $x' + P(t)x = Q(t)$

where $P(t) = 2$ and $Q(t) = \sin t$.

Integrating Factor $= e^{\int P(t)\, dt}$.

$I.F = e^{\int 2 dt} = e^{2t}$

Multiplying (1) by e^{2t} we get

$e^{2t} x' + 2x e^{2t} = e^{2t} \sin t$.

$\Rightarrow \frac{d}{dt} [x e^{2t}] = e^{2t} \sin t$

$\Rightarrow \int \cancel{d}[xe^{2t}] = \int e^{2t} \sin t\,dt$

$\Rightarrow xe^{2t} = \int e^{2t} \sin t\,dt$

Solving by Integration by parts method.

$\Rightarrow xe^{2t} = \left(\frac{2\sin t - \cos t}{5}\right)e^{2t} + c$

$x = e^{-2t}\left[c + \frac{2\sin t - \cos t}{5})e^{2t} - e^{2t}\frac{\cos t}{5}\right]$

Q3 Solve the following IVP for the exact equation and find the interval of validity for the solution

(1) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, \quad y(0) = -3$

Sol $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$ — (i)

$\Rightarrow (2y + x^2 + 1)\frac{dy}{dx} = -2xy + 9x^2$

$\Rightarrow (2y + x^2 + 1)\,dy = (9x^2 - 2xy)\,dx$

$\Rightarrow (9x^2 - 2xy)\,dx - (2y + x^2 + 1)\,dy = 0$

$\Rightarrow (9x^2 - 2xy)\,dx + (-2y - x^2 - 1)\,dy = 0$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$\frac{\partial M}{\partial y} = -2x$

$\frac{\partial N}{\partial x} = -2x$

So. Exact Equation solution exists

$\int M dx + \int$ (term of N, (free of x)) dy

y-axis

$$\int (9x^2 - 2xy)\,dx + \int (-2y-1)\,dy = C$$

$$\frac{9x^3}{3} - \frac{\cancel{2}x^2y}{\cancel{2}} - \frac{\cancel{2}y^2}{\cancel{2}} - y = C$$

$$\frac{9x^3}{3} - x^2y - y^2 - y = C$$

$$y(0) = -3$$

$$(-3)^2 - (-3) = c$$

$$-9+3 = C$$

$$\boxed{C = -6}$$

2 $\frac{2+y}{t^2+1} - 2t - (2-\ln(t^2+1))y' = 0 \quad y(5) = 0$

Sol $\frac{2t y}{t^2+1} - 2t - (2-\ln(t^2+1))\frac{dy}{dx} = 0$

$\left(\frac{2+y}{t^2+1} - 2t\right)dx - (2-\ln(t^2+1))dy = 0$

$M(t,y) = \frac{2+y}{t^2+1} - 2t$

$N(t,y) = \ln(t^2+1) - 2t$

$\frac{\partial M}{\partial y} = \frac{2t}{t^2+1}$

$\frac{\partial N}{\partial t} = \frac{2t}{t^2+1}$

So Exact Equation. Solution exists

$\int M dx + \int$ t term of N (free of x) dy

y-zeros

$\int\left(\frac{2+y}{t^2+1} - 2t\right)dt + \int -2dy = c$

$y\ln(t^2+1) - t^2 - 2y c$

$y(5) = 0$

$-(5)^2 = c$

$\boxed{c = -25}$