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**Subject:** Discrete Structure

**Q.1**

 **a) Explain the concept of Biconditional statement.**

**ANSWER:**

A biconditional statement is a combination of a [conditional statement](https://www.varsitytutors.com/hotmath/hotmath_help/topics/conditional-statements.html)and its converse written in the if and only if   form.

Two line segments are congruent if and only if  they are of equal length.

It is a combination of two conditional statements, “if two line segments are congruent then they are of equal length” and “if two line segments are of equal length then they are congruent”.

A biconditional is true if and only if both the conditionals are true.

Bi-conditionals are represented by the symbol ↔↔ or ⇔⇔ .

p↔qp↔q means that p→qp→q and q→pq→p . That is, p↔q=(p→q)∧(q→p)p↔q=(p→q)∧(q→p) .

**Example:**

Write the two conditional statements associated with the bi-conditional statement below.

A rectangle is a square if and only if the adjacent sides are congruent.

The associated conditional statements are:

a) If the adjacent sides of a rectangle are congruent then it is a square.

b) If a rectangle is a square then the adjacent sides are congruent.

 **b) Let p, q, and r represent the following statements:**

**p: Sam had pizza last night.**

**q: Chris finished her homework.**

**r: Pat watched the news this morning**

 **Give a formula (using appropriate symbols) for each of these statements.**

1. **Sam had pizza last night if and only if Chris finished her homework.**
2. **Pat watched the news this morning if Sam did not have pizza last night.**
3. **Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.**
4. **In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework**

**ANSWER:**

a) Sam had pizza last night if and only if Chris finished her homework.

p⇔q

b) Pat watched the news this morning if Sam did not have pizza last night.

r⇔¬p

c) Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

r⇔(q∧¬p)

d) In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework.

r⇔(p∧q)

e) q ⇔ r

Chris finished his homework if and only if Pat watched the news this morning

f) p ⇔ (q ∧ r)

Sam had pizza last night if and only if Chris finished his homework and Pat watched the news this morning

g) (¬p) ⇔ (q ∨ r)

Sam didn't have pizza last night if and only if Chris finished his homework or Pat watched the news this morning

h) r ⇔ (p ∨ q)

Pat watched the news this morning if Sam had pizza last night or Chris finished his homework.

**Q.2**

**a) Explain the concept of Venn diagram with examples.**

**b) Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram.**

**c) Draw and label a Venn diagram to represent the set**

***R*= {Monday, Tuesday, Wednesday}.**

**d) Given the set Q = {x : 2x – 3 < 11, x is a positive integer }. Draw and label a Venn diagram to represent the set Q.**

**ANSWER:**

**a) Explain the concept of Venn diagram with examples.**

A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something in common. Usually, Venn diagrams are used to depict set intersections (denoted by an upside-down letter U). This type of diagram is used in scientific and engineering presentations, in theoretical mathematics, in computer applications, and in statistics. The drawing is an example of a Venn diagram that shows the relationship among three overlapping sets X, Y, and Z. The intersection relation is defined as the equivalent of the logic AND. An element is a member of the intersection of two sets if and only if that element is a member of both sets. Venn diagrams are generally drawn within a large rectangle that denotes the universe, the set of all elements under consideration. In this example, points that belong to none of the sets X, Y, or Z are gray. Points belonging only to set X are cyan in color; points belonging only to set Y are magenta; points belonging only to set Z are yellow. Points belonging to X and Y but not to Z are blue; points belonging to Y and Z but not to X are red; points belonging to X and Z but not to Y are green. Points contained in all three sets are black. Here is a practical example of how a Venn diagram can illustrate a situation. Let the universe be the set of all computers in the world. Let X represents the set of all notebook computers in the world. Let Y represent the set of all computers in the world that are connected to the Internet. Let Z represent the set of all computers in the world that have anti-virus software installed. If you have a notebook computer and surf the net, but you are not worried about viruses, your computer is probably represented by a point in the blue region. If you get concerned about computer viruses and install an anti-virus program, the point representing your computer will move into the black area.



**b) Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram.**

**Solution:** List out the elements of P.

P = {16, 18, 20, 22, 24} ← ‘between’ does not include 15 and 25

Draw a circle or oval. Label it P. Put the elements in P



**c) Draw and label a Venn diagram to represent the set**

***R*= {Monday, Tuesday, Wednesday}.**

**Solution:**

Draw a circle or oval. Label it R. Put the elements in R



**d) Given the set Q = {x : 2x – 3 < 11, x is a positive integer }. Draw and label a Venn diagram to represent the set Q.**

**Solution:**

Since an equation is given, we need to first solve for x.

2x – 3 < 11 ⇒ 2x < 14 ⇒ x < 7

So, Q = {1, 2, 3, 4, 5, 6}

**Q.3**

 **Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments.**

**ANSWER:**

**ARGUMENT WITH PROPER EXAMPLE**

**Definition:** An argument consists of a sequence of statements called premises and a statement called a conclusion. An argument is valid if the conclusion is true whenever the premises are all true.

**Example**: My program won’t compile or it produces a division by 0 errors. My program does not produce a division by 0 errors. Therefore my program will not compile.

**Now:** Rewrite this argument in its general form by defining appropriate propositional variables.

This is one example of an argument form that is called **disjunctive syllogism.**

**Example:** Another disjunctive syllogism example

Ladybugs are purple or green.

Ladybugs are not green.

Therefore ladybugs are purple.

Let P be: Ladybugs are purple.

Let Q be: Ladybugs are green.

**Rewritten argument:**

P or Q

Not Q

Therefore P

This argument is valid, but it isn’t very meaningful since P and Q are not true

**DIFFERENTIATE BETWEEN VALID & INVALID ARGUMENTS**

An argument form is valid if whenever true statements are substituted in for the statement variables the conclusions is always true. To say an argument is invalid means that it is not valid.

The main point regarding a valid argument is that it follows from the logical form itself and has nothing to do with the content. When a conclusion is reached using a valid argument, we say the conclusion is inferred or deduced from the premises. Before we consider examples, we shall brieﬂy examine how one can tell if a given argument form is valid or invalid.

Result 1.3. To test whether or not an argument is valid, we do the following: (i) Identify the premises and the conclusion (ii) Construct a truth table showing the truth values of the premises and the conclusion (iii) Look for all the rows where the premises are all true - we call such rows critical rows. If the conclusion is false in a critical row, then the argument is invalid. Otherwise, the argument is valid (since the conclusion is always true when the premises are true).

**Example 1.4.** Determine whether the following arguments are valid.

(i)

p → q q → r

◦ ◦ ◦ p ∨ q → r

Constructing a truth table, we have: p q r p → q q → r p ∨ q p ∨ q → r → T T T T T T T T T F T F T F T F T F T T T T F F F T T F → F T T T T T T F T F T F T F → F F T T T F T → F F F T T F T To help, we mark the critical rows. Notice that all critical rows have a true conclusion and thus the argument is valid.

(ii)

p ∨ q p →∼ q p → r

◦ ◦ ◦ r

Constructing a truth table, we have: p q r p ∨ q p →∼ q p → r r T T T T F T T T T F T F F F → T F T T T T T T F F T T F F → F T T T T T T → F T F T T T F F F T F T T T F F F F T T F To help, we mark the critical rows. Notice that the 6th row is a critical row with a false conclusion, so it follows that the the argument is invalid.

Warning. In logic, the words “true” and “valid” have very diﬀerent meanings - truth is talking about the statements making up an argument and validity is talking about whether the conclusion follows from the premises. Note that a perfectly valid argument may have a false conclusion depending upon the truth value of the premises. Likewise, an invalid argument may have a true conclusion depending upon the truth value of the premises.

**Example 1.5.** As we noted above, the argument

p → q q → r

◦ ◦ ◦ p ∨ q → r

is a perfectly valid argument. Let p :=“I sleep a lot”, q :=“I don’t do any homework” and r :=“I will do well in this class”. Then this translates to: “If I sleep a lot, then I don’t do any homework. If I don’t do any homework, then I will do well in the class. Therefore, if I sleep a lot or don’t do any homework, I will do well in the class”. As noted above, this is a perfectly valid argument, but clearly not a true conclusion! This is because though the ﬁrst hypothesis is true, the second hypothesis is false (and hence the conclusion is false - see the truth table).

We ﬁnish with one more example of translating an argument into logical form and then testing validity.

**Example 1.6.** Determine the validity of the following argument: “Robbery was the motive for the crime only if the victim had money in his pockets. But robbery or vengeance was the motive for the crime. Therefore, vengeance must have been the motive for the crime.” Let p :=“robbery was the motive for the crime”, q :=“the victim had money in his pockets”, and r :=“vengeance was the motive for the crime”. Then the argument translates as follows:

p → q p ∨ r

The truth table is: p q r p → q p ∨ r r → T T T T T T → T T F T T F T F T F T T T F F F T F → F T T T T T F T F T F F → F F T T T T F F F T F F This is clearly not a valid argument - as stated above, if the victim had money in their pockets, and the motivation of the crime was robbery but not vengeance, this satisﬁes all hypothesis, but not the conclusion as suggested by the truth table.

**Q.4**

**a) Explain the concept of Union, also explain membership table for union by giving proper example of truth table.**

**b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table**

**ANSWER:**

**Union:**

In set of theory, the union denoted by U of a collection of sets is the set of all elements in the collection it is one of the fundamental operations through which sets can be combined and related to each other

Let A & B are subsets of universal sets U

The union of A & B is the set of all elements in U that belong to A or to B or both

It is denoted by A ∪ B

A U B= {x ∈U | x ∈A or x ∈ B}

 Union is commutative: A ∪ B = B ∪ A

A ⊆ A ∪ B and B ⊆ A ∪ B

**Example:** Let U = {10,11,12,13,14,15,16,17 } A = {10, 12, 14, 17}, B = {13,14, 15,16} Then A ∪ B = {10,11,12, 13, 14, 15, 16,17}



**Membership Table for Union:**

* The Membership table for the union of sets A and B is given below
* The truth table for disjunction of two statements P and Q is given below
* In the membership table of Union replace, 1 by T and 0 by F then the table is same as of disjunction
* So membership table for Union is similar to the truth table for disjunction

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **A ∪ B** |
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P v Q** |
| T | F | T |
| T | F | T |
| F | T | T |
| F | F | F |

**b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table**

**Intersection:**

The intersection of two sets A & B denoted by A ∩ B is the set containing all elements of A that also belongs to B or all elements of B that also belong to A

Intersection is written using the sign ∩

A ∩ B = {x ∈U | x ∈A and x ∈ B}

Intersection is commutative: A ∩ B = B ∩ A

A ∩ B ⊆ A and A ∩ B ⊆ B

If A and B are disjoint, then A ∩ B = φ

**Example:** Let U = {a, b, c, d, e, f, g}

A = {a, c, e, g}, B = {d, e, f, g}

Then A ∩ B = {e, g}

**Membership Table For Intersection:**

* The Membership table for intersection of sets A and B is given
* below
* The truth table for conjunction of two statements P and Q is given
* below
* In the membership table of Intersection, replace 1 by T and 0 by F
* then the table is same as of conjunction
* So membership table for Intersection is similar to the truth table
* for conjunction (∧)

|  |  |  |
| --- | --- | --- |
| **A** | **B** | **A ∩ B** |
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P ∧ Q** |
| T | F | T |
| T | F | T |
| F | T | T |
| F | F | F |

**Q.5**

**a) Lets p, q, r represent the following statements:**

*P: it is hot today.*

*Q: it is sunny*

*R: it is raining*

**Express in words the statements using Bicondtional statement represented by the following formulas:**

1. *q ↔ p*
2. *p ↔ ( q ˄ r )*
3. *p ↔ ( q ˅ r)*
4. *r ↔ ( p ˅ q)*

**ANSWER:**

P means “it is hot”

Q means “it is sunny”

R means “it is raining”

R ↔ (p ˅ q)

“If it is hot & sunny, then it is raining”

q ↔ p

“If it is sunny, then it is hot”

\_ P ˄ \_q

\_ (p ˄ q)

(P ˄ q) ˄ R

P ˄ q ˄ r

(((\_p) ˄ q) ↔ ((\_r) ˅p)

(\_ (p ˅ q) ↔ q) ↔ R

((\_p) ˅ (\_q)) ↔ (\_r).