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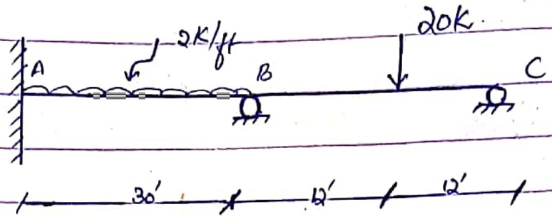
Q No 1: Analyze the given beam show in Fig-1 by flexibility method. EI is constant.



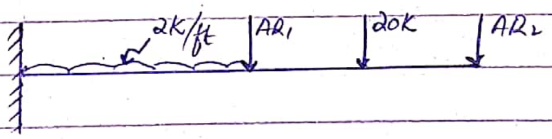
Sol:-

$= R - 3n = 5 - 3(1)$

Structural Indeterminancy $= 2°$

Step No # 1

Select Redundant Actions.



$$\begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} , \quad \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = [DRL] + [F] \times [AR]$$

Step No # 2

Compute the values of [DRL]

(P.T.O)

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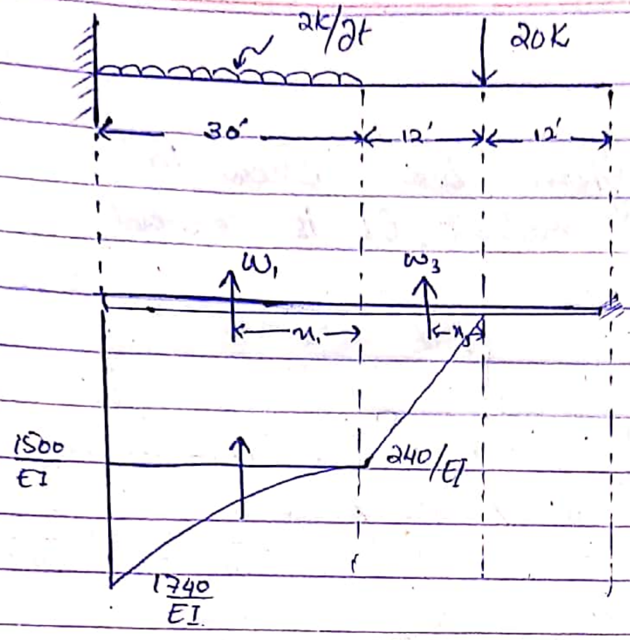

$w_1 = 1500 \times 30 = 4500.$

$w_2 = \frac{1}{3} \times 30 \times 240 = 2400$

$w_3 = \frac{1}{2} \times 12 \times 240 = 1440$

$x_1 = \frac{b}{2} = \frac{30}{2} = 15'$

$x_2 = \frac{3}{n+2} \times L = \frac{3}{2+2} \times 30 = 22.5'$

$x_3 = \frac{2}{3} \times L = \frac{2}{\cancel{3}} \times \cancel{12}^{4} = 8'$

Now Finding DRL.

$DRL = w_1 \times (x_1 + 24) + w_2 \times (x_2 + 24) + w_3 \times (x_3 + 12)$

$= 45000 (15 + 24) + 2400 (22.5 + 24) + 1440 (8 + 12)$

$=$ ~~45000~~

$175000 + 111600 + 28800$

$DRL = 1895400/EI$

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$DRL_1 = w_1(x_1) + w_2(x_2)$
$= 45000(15) + 2400(22.5)$
$= 675000 + 54000$
$= 729000$

So,

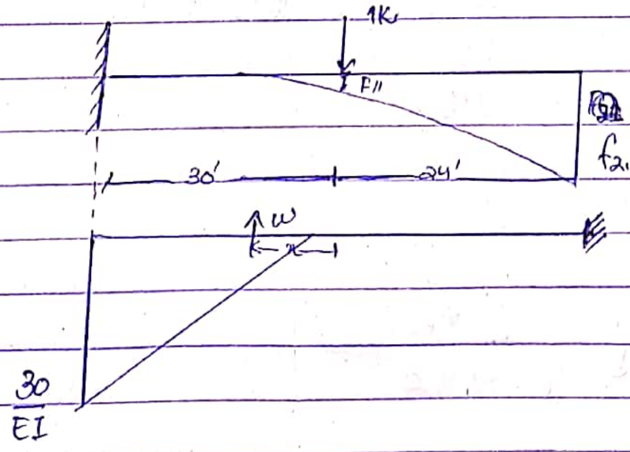
$$DRL = \frac{1}{EI}\begin{pmatrix}729000\\1895400\end{pmatrix}$$

Step #3

Flexibility Matrix

$$[F]_{2\times2} = \begin{bmatrix}f_{11} & f_{12}\\f_{21} & f_{22}\end{bmatrix}$$

a) Applied unit load on R_1



$x = \frac{2}{3} \times 30$

$= 20$

$w = \frac{1}{2}\left(\frac{30}{EI} \times 30\right)$

$= 450/EI$

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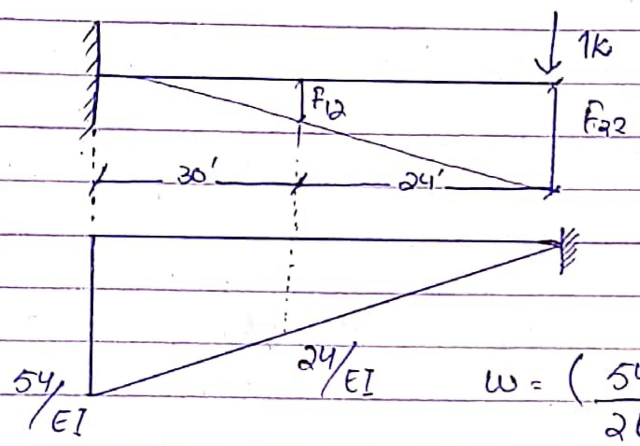
So,

$$F_{11} = \frac{450}{EI}(20) = 9000/EI$$

$$F_{21} = \frac{450}{EI}(20+24) = 19800/EI$$

Now apply unit load on AB_2



$$w = \left(\frac{54+24}{2EI}\right) \times 30$$

$$= 1170/EI.$$

The distance

$$x = \frac{c}{3}\left[\frac{b+2(a)}{a+b}\right]$$

$$:= \frac{30}{3}\left[\frac{24+2(54)}{54+24}\right] = 16.92'$$

$$\Rightarrow F_{12} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$\Rightarrow F_{22} = \frac{1170}{EI} \times (16.92+24) = \frac{47876.4}{EI}$$

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Hence.

$$F_{2\times2} \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix} \frac{1}{EI}$$

Step # 4

Compute the value of AR

$$[DRS] = [DRL] + [F] \times [AR]$$

$$[AR] = [DRS - DRL] \times [F]^{-1}$$

$$[F]^{-1} = \frac{1}{|F|} \times Adj\ F.$$

$$= \frac{1}{\begin{vmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{vmatrix}} \times Adj \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 478704 \end{bmatrix}$$

$$[F] = (9000 \times 47876.4 - 19796.4 \times 19800)$$
$$(430887600 - 391968720)$$

$$\Rightarrow |F| = 38918880.$$

$$\Rightarrow Adj\ A = \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 - 729000 \\ 0 - 1895400 \end{bmatrix} \frac{1}{EI} \times \frac{1}{38918880} \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$= \begin{bmatrix} -729000 \\ -1895400 \end{bmatrix} \frac{1}{EI} \times \frac{\begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}}{38918880}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 66.193 \\ -67.505 \end{bmatrix}$$

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Q No # 2

Differentiate between force method & displacement method & suggest which method is more suitable for structure analysis of matrix approach.

Ans:

Force Method	Displacement Method
$D_s < D_k$	$D_s > D_k$
Forces are redundent or unknown	Displacements are redundant or unknown
Starts with equilibrium of forces	Starts with compatible deformation
Forces found by compatibility equation of displacements.	Displacements found by equillibrium equilibrium equation of force.
No. of redundant $= D_s$	No. of reduntants $= D_k$
Not suitable for computer	Not suitable for truss.

In structure analysis there are two classes of variable that are obtained through the ~~ang~~ analysis process.

The first class of variable are force (here it can be forces & couple). So the force

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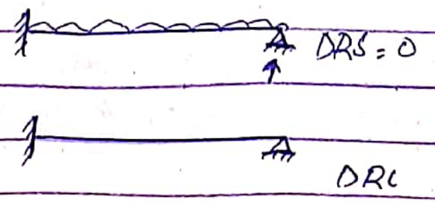
method uses the unknown forces as variables to ~~Constant~~. construct equations based on equilibrium and compatability then solves the equation to arrive ~~of~~ ~~sto~~ at final moments and reactions. So here the forces are variables.

The second class of variables are displacement method (also called stiffness method), uses ~~to~~ the unknown displacements as unknown to arrive ~~of~~ at final moments and reaction

FORCE METHOD

This method is also known as flexibility or compalibity method In this method the degree of static indetermancy of the determined & the vendindants are identified. A co-ordinate is assigned to each redudand. thus AR_1, AR_2 ---- AR_n are the redudant at co-ordinate 1, 2 --- n if all the redundants are removed the resulting structure know as the released state, is statically determinate from the principle of super position, the net displacement at any point in a statically determinate strc, is the sum of the displacement in the basic determinate strc due to applied load and the redudants, thus condution, known as compalibility condition, know as compalibility condition may be expressed by the equations for "n" vedudant action



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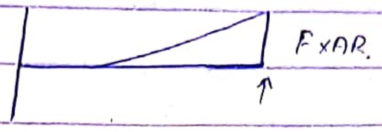

$$DRS = DRL + F \times AR.$$

$$DRS_1 = DRL_1 + F_{11}AR_1 + F_{12}\,AR_2 + \;------\; F_{1n}\,AR_n$$

$$DRS_2 = DRL_2 + F_{21}\,AR_1 + F_{22}\,AR_2 + \;-----\; F_{2n}\,AR_n$$

$$DRS_n = DRL_n + F_{n1}\,AR_1 + F_{n2}\,AR_2 + \;-----\; F_{nn}\,AR_n$$

Writing these equation in matrix from

$$\begin{bmatrix} DRS_1 \\ DRS_2 \\ DRS_n \end{bmatrix} = \begin{bmatrix} DRL_1 \\ DRL_2 \\ DRL_n \end{bmatrix} + \begin{bmatrix} F_{11} & F_{12} & ---- & F_{1n} \\ F_{12} & F_{22} & ------ & F_{2n} \\ F_{n1} & F_{n2} & ----- & F_{nn} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \\ AR_n \end{bmatrix}$$

$$[DRS]_{n\times 1} = (DRL)_{n\times 1} + (F)_{n\times n}\;(AR)_{n\times 1}$$

$$(F)(AR) = (DRS) - (DRL)$$

$$AR = (F)^{-1}\,(DRS - DRL)$$

n = Degree of indeterminacy where, DRS = Support settlement / Rotation corresponding to the redundant action

DRL = Displacement (Rotation / translation) corresponding to the recbidiant action in a released stre (Basic dot str) due to applied load

AR = The redudant action.

F = flexibility co-officient i-e displacement caused by unit action.

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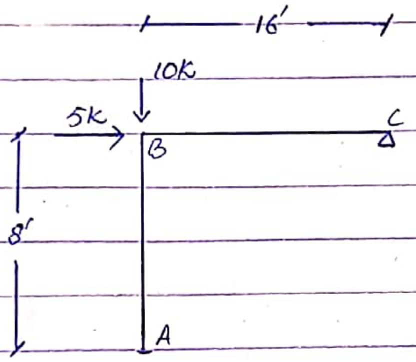
Q No 3.
Analyze the rigid-joint frame shown in fig (2) by flexibility method. Assume EI is constant for all members.

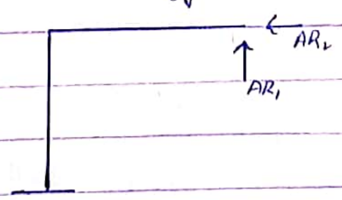
Sol:



total statical indeterminacy
=> $R - 3 = 5 - 3 = 2^{\circ}$

Step # 01

Identify redundant Action.



$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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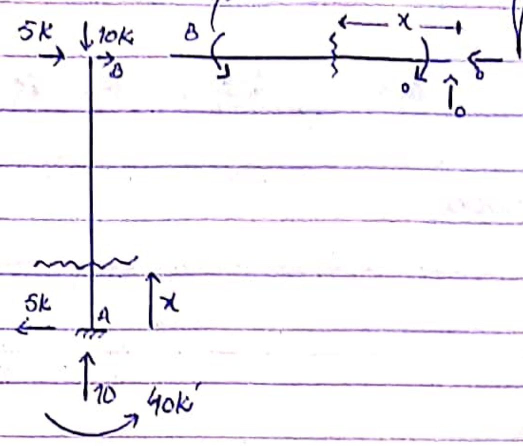
Step # 2

Compute value of [DRL]



Step # 3

[F] or [AMR]

a)

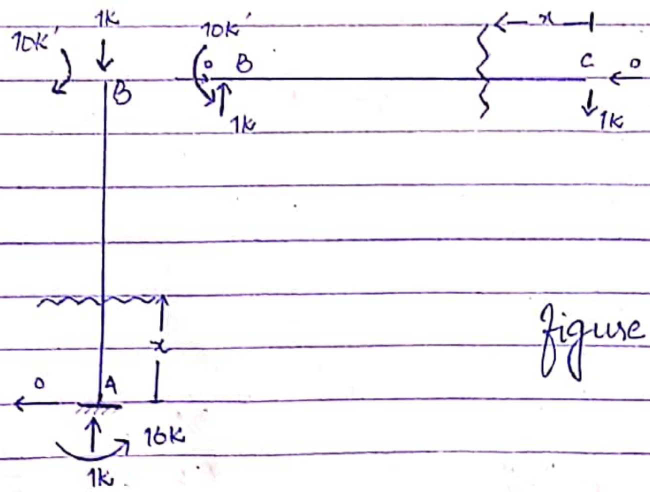


figure: AMR - value (m, values)

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b)

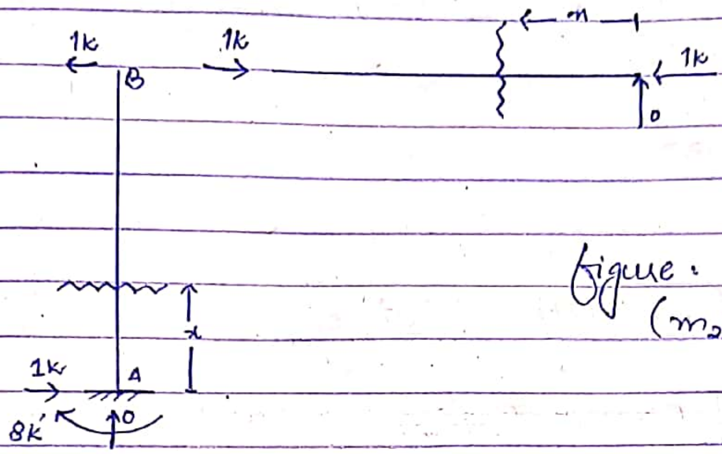

figure: AMR values (m_2 values)

Member	AB	BC
Origin	A	C
Limits	0 - 8	0 - 16
I	I	2I
M	5x - 40	0
m_1	- 16	x
m_2	8 - x	0

⇒ finding values of DRL

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot m_1}{EI} + \int_0^{16} \frac{M_{BC} \cdot m_2}{EI}$$

$$= \int_0^8 \frac{(5x-40)(-16)\,dx}{EI} + \int_0^{16} \frac{0}{E(2I)} x\,dx$$

$$DRL_1 = \frac{2560}{EI}$$

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$$DRL_2 = \int_0^8 \frac{(5x-40)(8-x)}{EI}dx + \int_0^{16} \frac{0.0dx}{E(2I)}$$

$$DRL_2 = \frac{-853.33}{EI}$$

⇒ Compute flexibility Matrix.

$$F_{2\times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

ϕ :- F_{11} $\int_0^8 \frac{m_{1(AB)}^2}{EI} + \int_0^{16} \frac{m^2_{(BC)}}{EI} = \int_0^8 \frac{(-16)^2 dx}{EI} + \int_0^{16} \frac{x^2}{E(2I)}$

$$F_{11} = \frac{2730.67}{EI}$$

ϕ :- $F_{12} = F_{21} = \int_0^8 m_{1(AB)} \cdot m_{2(AB)} + \int_0^{16} m_{1(BC)} \cdot m_{2(BC)}$

$$= \int_0^8 \frac{(-16)(8-x)dx}{EI} + \int_0^{16} \frac{(x)(0)}{2EI}dx$$

ϕ :- $F_{12} = F_{21} = \frac{-512}{EI}$

ϕ :- $F_{22} = \int_0^8 (m_2)^2_{(AB)} dx + \int_0^{16} (m_2)^2_{BC} dx$

$$= \int_0^8 \frac{(8-x)^2 dx}{EI} + \int_0^{16} \frac{0^2}{2EI} dx.$$

$F_{22} = 170.67$

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As we know

★ $[DRS] = [DRL] + [AR] \times [F]$

$$\Rightarrow [AR] = \frac{[DRS] - [DRL]}{[F]}$$

★ $[AR] = [F]^{-1} \times [DRS - DRL]$

$$= \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 - 2560 \\ 0 + 853.33 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.00005 \\ 4.997 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$