DEPARTMENT OF

ELECTRICAL ENGINEERING

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COURSE TITLE:-

CALCULAS AND ANALYTIC GEOMETRY

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FINAL PAPER

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①

QNO1:

Part A:

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

Solution:

Let $1-\theta^2 = u$

Then

$$1-\theta^2 = u$$

$$d/d\theta\,(1-\theta^2 = du/d\theta$$

$$0-2\theta = du/d\theta$$

$$-2\theta = du/d\theta$$

$$\theta\, d\theta = -1/2\, du$$

Put $-\frac{1}{2}\,du = \theta\, du$

& $(1-\theta^2) = u$

Q1 =>

$$\int -\sqrt[4]{\frac{u}{2}}\, du$$

$$-1/2 \int \sqrt[4]{u}\, du$$

$$-1/2 \int u^{1/4}\, du$$

(2)

$$-\frac{1}{2}\ \frac{u^{\frac{1}{4}+1}}{\frac{1}{4}+1}+c$$

$$-\frac{1}{2}\ \frac{(u)^{5/4}}{5/4}+c$$

put $u = 1-\theta^2$

$$-\frac{1}{2}\ \frac{(1-\theta^2)^{5/4}}{5/4}+c$$

$$-\frac{1}{\cancel{2}}\ \frac{\cancel{4}^{2}(1-\theta^2)^{5/4}}{5}+c$$

$$-\frac{2}{5}\ \frac{(1-\theta^2)^{5/4}}{5}+c$$

$$= \boxed{\frac{-2}{5}\ \frac{(-\theta^2+1)^{5/4}}{5}+c} \rightarrow \text{Ans.}$$

X ———— X ———— X ——

③

Q No1 b:-

$$\int_0^1 x^{3}(1+x^4)^3\,dx$$

by substitution method.

Solution:

$$\int_0^1 x^4 (1+x^4)^3\,dx \quad \text{—①}$$

by substitution:-

Let

$$1+x^4 = u$$

$$d/dx\,(1+x^4) = du/dx$$

$$0+4x^3 = du/dx$$

$$x^3\,dx = 1/4 \cdot du \quad \text{put in ①}$$

$$\int_0^1 (u)^3 \cdot 1/4\,du$$

$$1/4\left(\int_0^1 u^3\,du\right)$$

$$1/4 \int_0^1 \frac{u^{3+1}}{3+1}$$

(4)

$\frac{1}{4}\int_0^1 \frac{u^4}{4} \rightarrow ii$

~~App~~ put $u = 1+x^4$

$\frac{1}{4}\int_0^1 \frac{1+x^4}{4}$

Apply limits.

$\frac{1}{4}\left(\frac{1+(1)^4}{4} - \frac{1+0}{4}\Big|_0^1\right)$

$\frac{1}{4}\cdot\left(\frac{2}{4} - \frac{1}{4}\right)$

$\frac{1}{4}\left(\frac{2-1}{4}\right)$

$\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$

$$\boxed{\int_0^1 x^3(1+x^4)^3 du = \frac{1}{16}}$$

$\Rightarrow$ ANS.

Q No 2 A:-

Find the centre & radius of sphere

$x^2 + y^2 + z^2 + 3x - 4z + 1$

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

$$(x^2 + 3x) + y^2 + (z^2 - 4z) + 1 = 0$$

$$(x^2 + 3x) + (y^2 + 0) + (z^2 - 4z) + 1 = 0$$

Add $(3/2)^2$ & $(-4/2)^2$ both side

$$\left(x^2 + 3x + (3/2)^2\right) + (y^2 + 0)$$

$$z^2 - 4z + (-4/2)^2 + 1 = (3/2)^2 + (-4/2)^2$$

$$(x + 3/2)^2 + (y + 0)^2 (z - 2)^2 = -1 + (3/2)^2 + (-4/2)^2$$

$$(x + 3/2)^2 + (y + 0)^0 (z - 2)^2 = -1 + 25/4$$

$$(x + 3/2)^2 + (y + 0)^0 (z - 2)^2 = \frac{-4 + 25}{4}$$

$$(x + 3/2)^2 + (y + 0)^2 (z - 2)^2 = \frac{21}{4}$$

so

(x_0, y_0, z_0) = center
$= (-3/2, 0, 2)$

$$\boxed{\text{center} = -3/2, 0, 2}$$

$$\boxed{\text{Radius} = a = \sqrt{21/4}}$$

x ——— x ——— x ——— x — x

Q no 2 b:-

Find the region b/w the curve $y = \sqrt{x}$ $0 \le x \le 4$ & the x-axis is revolved about the x-axis to generate solid. Aply integration find volume of solid.

P.T.O

7)

Solution:

Given That:

$$y = \sqrt{x}$$

$$0 \leq x \leq 4 \Rightarrow a \leq x \leq b$$

Find : volume of solid $(V) = ?$

As we know that

$$V = \int_a^b \pi y^2 \, dx$$

here, $a = 0$, $b = 4$ & $y^2 = (\sqrt{x})^2$

$$\Rightarrow V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \left. \frac{x^2}{2} \right|_0^4$$

$$V = \pi \left(\frac{(4)^2}{2} - \frac{0^2}{2} \right)$$

$$V = \pi \frac{16}{2} - 0$$

$$V = \frac{16}{2} \pi \Rightarrow V = 8\pi$$

$$\boxed{V = 8\pi \, u^3} \rightarrow \text{ANS}$$

Q No 3:-

$A = 2i - 4j + \sqrt{5}k$

$B = -2i + 4j - \sqrt{5}k$

~~the~~ Find vector projection A on b:

Solution:-

We know that,

$$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A \rightarrow \text{①}$$

Now we find B·A

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$= -2 \times 2 (i \cdot i) + (4 \times -4)(j \cdot j) + (-\sqrt{5} \times \sqrt{5})(k \cdot k)$$

We know that

$i \cdot i = 1$, $k \cdot k = 1$ & $j \cdot j = 1$

simply the dot product of same unit vector = 1

& different unit vector = 0

$$B \cdot A = -4(1) + (-16)(1) + -\sqrt{25}.$$

PTO

(9)

$B \cdot A = -4 - 16 - 5$

$\boxed{B \cdot A = -25} \longrightarrow (1)$

Now $A \cdot A$

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= 2 \times 2 (i \cdot i) + (-4 \times -4) j \cdot j + (\sqrt{5} \times \sqrt{5})(k \cdot k)$

$= 4 + 16 + \sqrt{25}$

$A \cdot A = 20 + 2$

$\boxed{A \cdot A = 25} \longrightarrow (ii)$

Now put equation (1) and (2) in (*)

$* \Rightarrow$

$$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$$

$$= \frac{-\cancel{25}^{1}}{\cancel{25}} (2i - 4j + \sqrt{5}k)$$

$\boxed{Proj_A B = -2i + 4j - \sqrt{5}K} \rightarrow$ An

QNo 4:-

Find the area of the region b/w the graph & the x-axis.

Solution:

Given that

$F(x) = y = -x^2+5x-4$

$[0, 2]$

We know that Area under the graph

$$A = \int_a^b F(x)\, dx$$

Here $[a, b] = [0, 2]$

$\Rightarrow a = 0, \; b = 2$

& $Fx = -x^2+5x-4$

$$A = \int_0^2 (-x^2+5x-4)\, dx$$

by Applying integration

(11)

$$A = \frac{-x^3}{3} + \frac{5x^2}{2} - 4x \Big|_0^2$$

Now Apply limmit

$$A = \left[\frac{-(2)^3}{3} + \frac{5(2)^2}{2} - 4(2)\right] - \left[\frac{(-0^3)}{3} + \frac{5(0)}{2} - 4(0)\right]$$

$$= \left[\frac{-8}{3} + \frac{20}{2} - 8\right] - \left[0 + 0 - 0\right]$$

$$A = \left[\frac{-8}{3} + \frac{20}{2} - \frac{8}{1}\right] - 0$$

Taking lcm-

$$A = \frac{(2 \times 8) + (3 \times 20) - (6 \times 8)}{6}$$

$$A = \frac{-16 + 60 - 48}{6} = \frac{-4^2}{6^3}$$

$$A = -2/3 \Rightarrow A = -0.66$$

we know that the can not be neagative so we remove - sing $\Rightarrow$ $\boxed{A = 0.66 m^2}$ → ANS

Q NO 5 A:-

Find Angle b/w

$A = 1 - 2j - 2K$

$B = 6i + 3j + 2K$

Solution:

As

$$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$$

$$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{|A|\,|B|}$$

$$\theta = \cos^{-1} \frac{A \cdot B}{|A|\,|B|} \rightarrow A$$

Now

$$A \cdot B = (i - 2j - 2k) \cdot (6i + 3j + 2k)$$

$$= 6 \times 1 (i \cdot i) + (-2 \times 3)(j \cdot j) + (2 \times -2)(k \cdot k)$$

$$A \cdot B = \cancel{6}(1) - \cancel{6}(1) - 4(1)$$

$$\boxed{A \cdot B = -4} \rightarrow (i)$$

(13)

Now

$|A| = |i - 2j - 2k|$

$= \sqrt{(1)^2 + (-2)^2 + (-2)^2}$

$= \sqrt{1+4+4} = \sqrt{9}$

$\boxed{|A| = 3} \longrightarrow (ii)$

$|B| = |6i + 3j + 2k|$

$= \sqrt{(6)^2 + (3)^2 + (2)^2}$

$\sqrt{36+9+4} = \sqrt{49}$

$\boxed{|B| = 7} \longrightarrow (iii)$

put value (A·B)

|A| & |B| in *

we get

$\theta = \left(\frac{A \cdot B}{|A||B|}\right) \cos^{-1}$

$= \cos^{-1}\left(\frac{-4}{7 \times 3}\right)$

$\theta =$ P-T-O

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$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\boxed{\theta = 100.9^\circ} \rightarrow \text{Ans}$$

x —— x —— x —— x

QNO5 b) :-

Change into spherical coordinate equation for the sphere

$$x^2+y^2+(z-1)^2=1$$

Solution:

$$(\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2$$

$$+(\rho \cos\phi - 1)^2 = 1$$

$$\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\theta \sin^2\phi$$

$$+\rho \cos^2\phi + 1 - 2\rho \cos\phi = 1$$

(15)

$\rho \sin^2\phi \, (\cos\theta^2 + \sin^2\theta)$

$+ \rho \cos + 1 - 2 + \cos\phi = 1$

$\rho^2 (\sin\phi^2 + \cos^2\phi) - 2\rho\cos\phi = 0$

$\rho^2 = 2\rho\cos\phi$

$$\boxed{\rho = 2\cos\phi}$$

x ———— x ———— x

End-

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