Date: ____________

Name:- Muhammad Daniyal

ID No:- 17011

Class timming:- wednesday
8:00am to 11:00am

Final assignment:-
Discrete
Mathematics

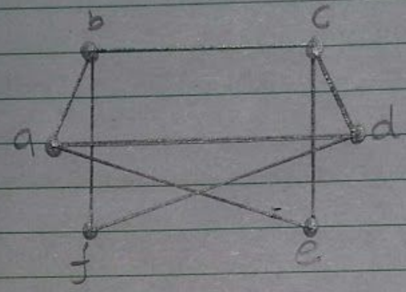
Instructor:-
Saifullah jan.

Date: ____________

"Question no 1"

Determine whether the graph is bipartite.

(i)



Solution:-

A bipartite graph is a Simple graph whose vertices Canbe partitioned into two Sets V_1 and V_2 Such that there are no edges among the vertices of V_1 and no edge among the vertices of V_2, while there Canbe edges between a vertex of V_1 and a vertex of V_2.

A Simple graph is bipartite if and only if it is possible to assign each vertex - which we Can show with red and blue Color.

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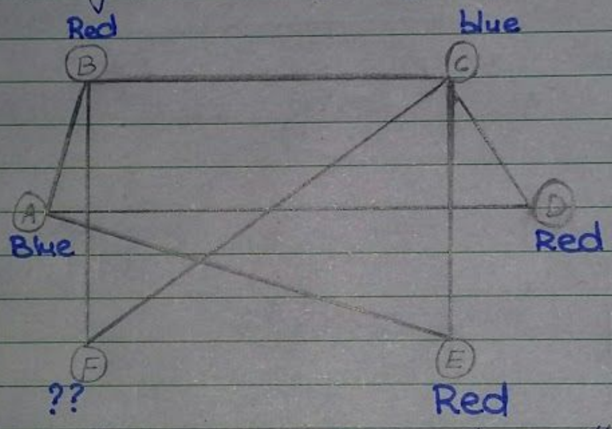
Now:-

If two vertices are Connected, then they should not have same Color.

Let us assign red or blue, to each other.



we start by assigning "Blue" to a.

Since b is Connected to the blue a, we assign "Red" to b.

Since C is Connected to the red b, we assign "blue" to C.

we then note that f is Connected to the red b and the blue C, which means that we Can not assign a Color to f Such that

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it differs from the Colors of the Conneted vertices (as there are only two Colours: red and blue)

Thus it is not possible to assign red or blue to each vertex such that Connected vertices do not have the Same Color and thus the graph is not bipartite.

"So the graph is no bipartite"

Answer.

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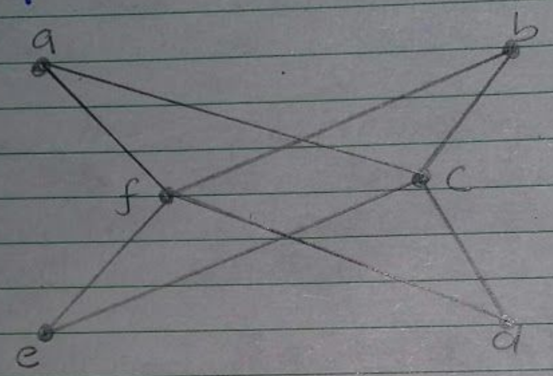
"Question no 1"

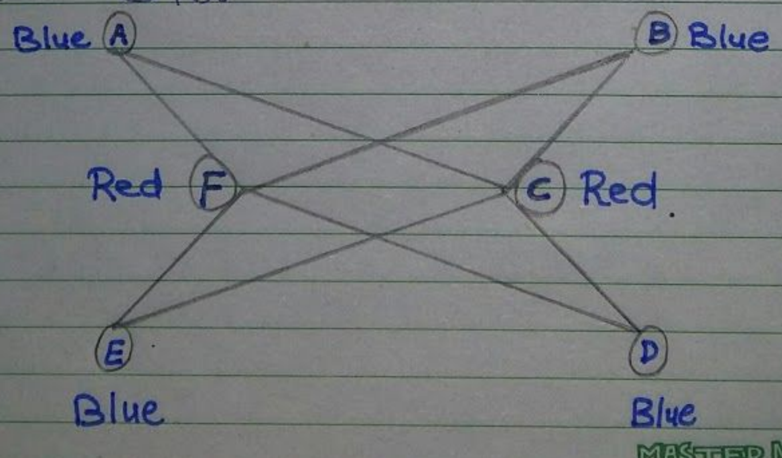
Part II



Solution:-

Let us assign red or blue to each vertex.

If two vertices are connected, then they should not have the same color.



MASTER NOTES

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we then note that it is possible to assign red or blue to each, vertex Such that Connected vertices do not have the Same Color and thus the graph is bipartite

Moreover, the partitioning of the vertices are the set with the blue vertices and the set with the red vertices.

$$V_1 = \{a, b, d, e\}$$

$$V_2 = \{c, f\}$$

Result:-

The Graph is bipartite.

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Let us add all elements in the Two matrices, which will represent the number of Connections of a vertex to another vertex (which is double the number of edges.

A Contains 8 ones and thus the Simple graph Corresponding to A Contains 8 Connections.

B Contains 10 ones and thus the Simple graph Corresponding to A Contains 10 Connections.

Since the number of Connections of the two graphs are not the Same, the number of edges in the graphs are not the Same and then the graphs are not isomorphic.

Result:-

This is not isomorphic

"Question no 2"
"part I"

Determine whether the given pair of graph is isomorphic.

Solution:-

Let us first determine the set of vertices and set of edges of the left graph.

$V_1 = \{V_1, V_2, V_3, V_4, V_5, V_6\}$

$E_1 = \{(V_1, V_2), (V_1 V_4), (V_1, V_6), (V_2, V_3) \cdot (V_2, V_6)$
$(V_3, V_4), (V_3, V_5), (V_4, V_5), (V_5, V_6)\}$

Let us first determine the set of vertices and set of edges of the right paragraph:

$V_2 = \{V_1, V_2, V_3, V_4, V_5, V_6\}$

$E_2 = \{(V_5, V_2), (V_5, V_6), (V_5, V_1) (V_2, V_3)$
$(V_2, V_1) (V_3, V_6) (V_3, V_4) (V_6, V_4) (V_4, V_1)\}$

By Comparing the Two sets of edges, we can define the following one-to-one and onto function f from V_1 to V_2.

we Could also use the degrees of the vertices, because the vertex and their image need to have the same degree.

$$f(v_1) = v_5$$
$$f(v_2) = v_2$$
$$f(v_3) = v_3$$
$$f(v_4) = v_6$$
$$f(v_5) = v_4$$
$$f(v_6) = v_1$$

f is then a function that makes the two graphs isomorphic, Since

v_1 and v_2 are adjacent while $f(v_1) = v_5$ and $f(v_2) = v_2$ are adjacent

v_1 and v_4 are adjacent, while $f(v_1) = v_5$ and $f(v_4) = v_6$ are adjacent

v_1 and v_6 are adjacent while $f(v_1) = v_5$ and $f(v_6) = v_1$ are adjacent

v_2 and v_3 are adjacent $f(v_2) = v_2$ and $f(v_3) = v_3$ are adjacent

v_2 and v_6 are adjacent $f(v_2) = v_2$ and $f(v_6) = v_1$ are adjacent

v_3 and v_4 are adjacent $f(v_3) = v_3$ and $f(v_4) = v_6$ are adjacent.

v_5 and v_6 are adjacen $f(v_5) = v_4$ and $f(v_6)$ are adjacent

Result:-

Isomorphic.

"Question no 3"

"Part II"

Are the Simple graphs with the following adjancy matrices isomorphic?

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Solution:-

$$A = \begin{bmatrix} 0 & \underline{1} & 0 & \underline{1} \\ \underline{1} & 0 & 0 & \underline{1} \\ 0 & 0 & 0 & \underline{1} \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \underline{1} & \underline{1} & \underline{1} \\ \underline{1} & 0 & 0 & \underline{1} \\ \underline{1} & 0 & 0 & \underline{1} \\ \underline{1} & 1 & 1 & 0 \end{bmatrix}$$

"Question no 3"

"part II"

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solution:-

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Let us add all elements in the two matrices, which will represent the number of connections of a vertex to another vertex (which is double the number of edges)

A contains 8 ones and thus the simple graph corresponding to A contains 8 connections.

B contains 6 ones and thus the simple graph corresponding to A contains 6 connections.

Since the number of Connections of the two graphes are not the Same, the number of edges in the graphs are not the Same and then the graph are not isomorphic

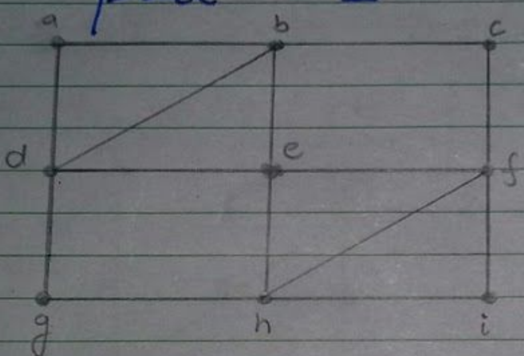
Result:-

NOT Isomorphic

"Question no 4"

Determine whether the given graph has an euler Circuit.

"part I"



Solution:-

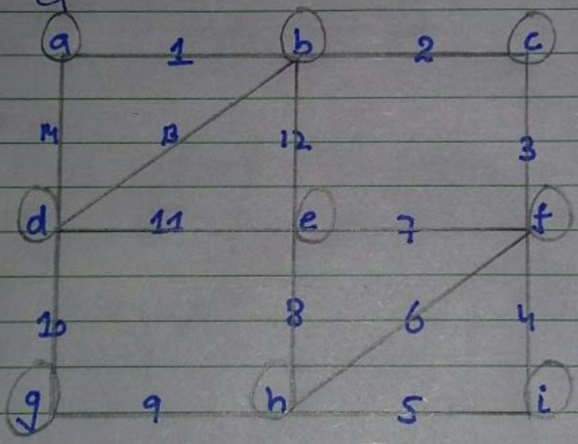
Let us first determine the degree of every vertex in the given graph.

deg (a) = 2
deg (b) = 4
deg (c) = 2
deg (d) = 4
deg (e) = 4
deg (f) = 4
deg (g) = 2
deg (h) = 4
deg (i) = 2

A graph has an Euler Circuit if and only if each of the vertices has an even degree. Since all degrees are even, there exists an Euler Circuit.

A possible Euler theorem is:

a, b, c, f, i, h, f, e, h, g, d, e, b, d, a



Result:-
Euler Circuit exists.

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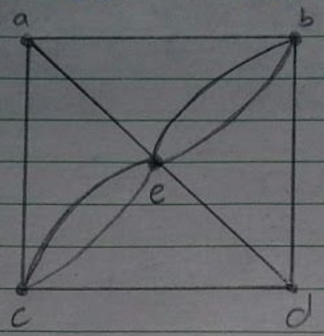
"Question no 4"

"Part II"



Solution:-

Let us first determine the degree of every vertex in the given graph.

deg (a) = 3
deg (b) = 4
deg (c) = 4
deg (d) = 3
deg (e) = 6

A graph has an Euler Circuit if and only if each of the vertices has an even degree. Since some are odd, there is no Euler Circuit.

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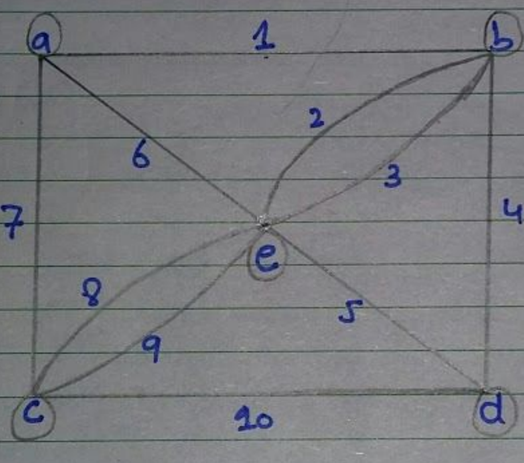
A graph has an Euler path if and only if there are exactly two vertices who have no odd degrees. we note that vertices a and b have an odd degree and thus an Euler path exists.

A possible Euler path is:

a, b, e, b, d, e, a, c, e, c, d



Result:- No Euler Circuit exists

Euler path exists.

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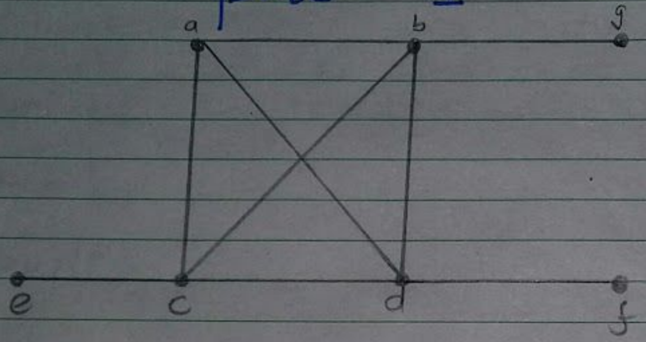
"Question no 5"

"Part I"



Solution:-

Let us first determine the degree of every vertex in the given graph.

deg (a) = 3
deg (b) = 3
deg (c) = 3
deg (d) = 2
deg (e) = 4
deg (f) = 1

we note that Dirac's theorem is not satisfied (Since Some degrees are less than $n/2 = 6/2 = 3$), but this does not necessarily mean that no hamilton Circuit exists.

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However, we do note that there is only one edge ({c,f}) Connecting to f and thus any Circuit that Contains f needs to pass through C twice which means that no Circuit can be a Hamilton Circuit (as we pass through a vertex more than once).

Result:-

Hamilton Circuit does not exist, because f has only 1 edge Connecting to it.

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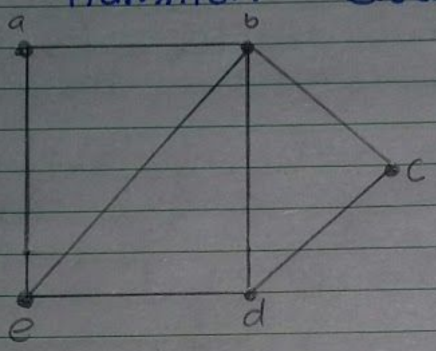
"Question no 5"
"Part II"

Determine whether the given graph has a Hamilton Circuit.



Solution:-

Let us first determine the degree of every vertex in the given graph.

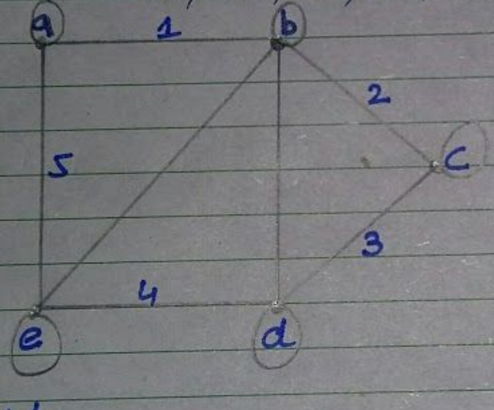
deg(a) = 2
deg(b) = 4
deg(c) = 2
deg(d) = 3
deg(e) = 3

we then note that Dirac's theorem is not satisfied (since some degrees are less than $n/2 = 5/2 = 2.5$) but this does not necessarily mean that no hamilton Circuit exists.

However, we do note that the given graph Contains the Cycle C_5 and the cycle C_5 within the given graph forms a Hamilton Circuit (as the Circuit will pass through all vertices exactly once.)

A possible hamilton Circuit is thus the path of e.g. C_5

a, b, c, d, e, a



Result:-

Hamilton theorem exists.