Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th

Instructor: ______ **Total Marks:** 30

Student Details

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	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
Q1.	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
	(a)		Marks 5

Q2.		Determine the response of the system to the following input signal with given impulse response $x[n] = \{\, 2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	CLO 2
	(b)	Compute the convolution y(n) of the following signal $x(n) = \begin{cases} \alpha^{n+1}, & -3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$ $h(n) = \begin{cases} 2^{n}, & 0 \leq n \leq 4 \\ 0, & elsewhere \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{3})^{n}, & n \geq 0 \\ (\frac{1}{4})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \geq 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Q No (1) (a)

$x_0(t) = 3\cos 100\pi t + 4\sin \overset{200}{\pi} t$

Sol: The frequency present in the signal above are -

$\Rightarrow f_1 = 50\,Hz \qquad f_2 = 100\,Hz$

So the minimum frequency is = 50 Hz

$\Rightarrow f_s \Rightarrow 2 f_{max}$

f_2 is max $\Rightarrow f_2 = 100\,Hz$

$\Rightarrow f_s = 2\times 100 = 200\,Hz$

This required to avoid aliasing

(ii) we have

$F_s = 100\,Hz$

So f_1 becomes

$f_1' = f_1/F_s = \overset{50}{\cancel{100}}/\overset{100}{\cancel{200}} = 0.5\,Hz$

f_2 becomes

$f_2' = f_2/F_s = 100/100 = 1\,Hz$

So $\omega_1' = 2\pi f_1' \qquad \omega_2' = 2\pi f_2'$

$\omega_1' = 2\pi \times 0.5 \qquad \omega_2' = 2\pi \times 1$

$\omega_1' = \pi \qquad \omega_2' = 2\pi$

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The signal becomes

$$\boxed{x[n] = 3\cos \pi n + 4\sin 2\pi n}$$

iii) Since only the frequency components at 50 Hz at 100 Hz are Present in the Sampled Signal. The analog Signal we can recover is

$$Y_a(t) = 3\cos \pi n + 4\sin 2\pi n$$

which aibvously different from the orignal analog Signal was caused by the aliasing effect due to the low Sampling rate used.

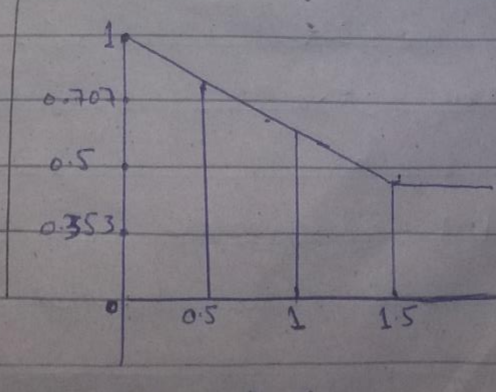
QNo (1)(b)

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$F_s = 2HZ \Rightarrow F_s = \frac{1}{t} = T = \frac{1}{F_s} = \frac{1}{2} = 0.5\ Sec$$

i) Draw The Sampled Signal

X_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



T = 0.5 Sec

J

ii) Quantization level.

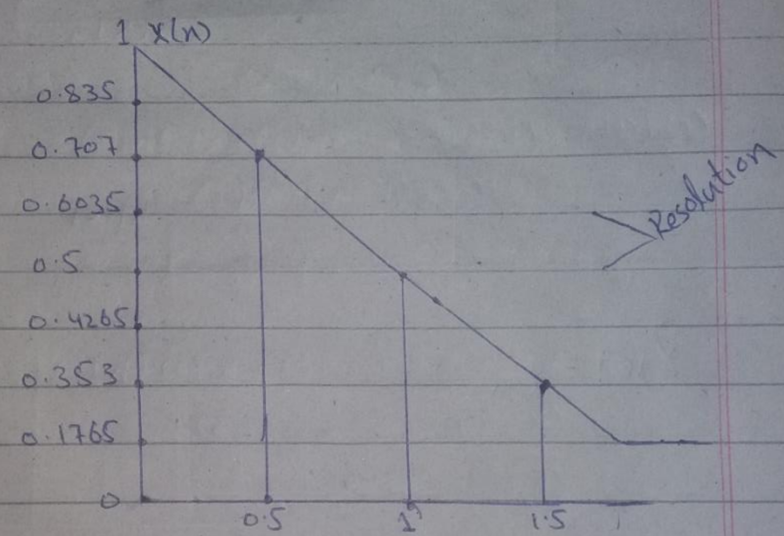
$$L = 2^n$$

$$n = bits = 3$$

$$L = 2^3 = 8 \text{ levels}$$

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$



iii)

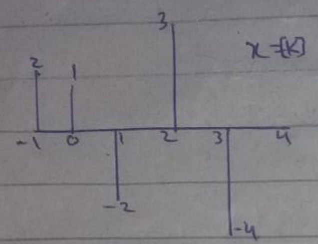
	Discrete time Signal	Truncation	Rounding	error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

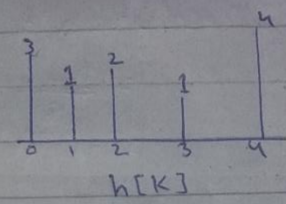
Q No 2 (a)

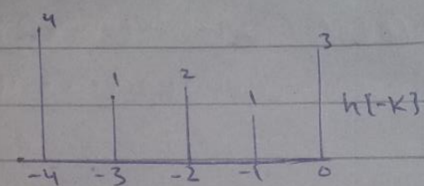
$x(n) = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$; $h(n) = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$

Sol :-

$$y[n] = \sum_{K=0}^{n} x[K]\, h[n-K]$$







$$y[0] = \sum_{K=-1}^{0} x[-1]\,h(-1) + x(0)\,(h(0)$$

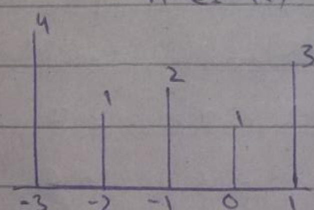
$= 2 \times 1 + (1)(3)$

$= 2 + 3$

$= 5$

For $n = 1$

$h(1-K)$



$$y[1] = \sum_{K=-1}^{1} x[n]\, h[1-K]$$

$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$

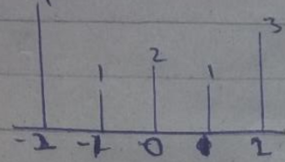
$= (2)(2) + (1)(1) + (3)(-2)$

$= 4+1-6$

$= -1$

$n = 2$

$h = [2-K]$



$$Y[2] = \sum_{K=-1}^{2} x[n]\,h[2-K]$$

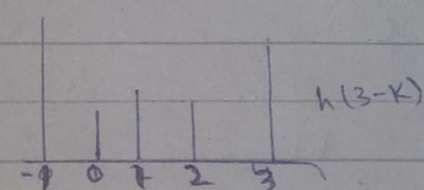
$= x(-1)h(-1)+x(0)+h(0)+x(1)h(1)+x(2)h(2)$

$= 2(1)+(1)(2)+(-2)(1)+(3)(3)$

$= 2+2-2+9$

$= 11$

$n = 3$



$$Y[3] = \sum_{K=-1}^{3} x(n)\,h(3-K)$$

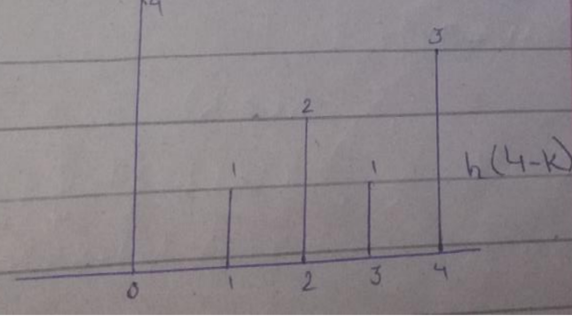
$= x(-1)h(-1)+x(0)h(0)+x(1)h(1)+x(2)h(2)+x(3)h(3)$

$= 2\times4+1\times1+(2)(2)+3\times1+(-4)(3)$

$= 4+1-4+3-12$

$= -8$

$n = 4$



$y(4) = \sum_{k=0}^{3} x(n)\,h(4-k)$

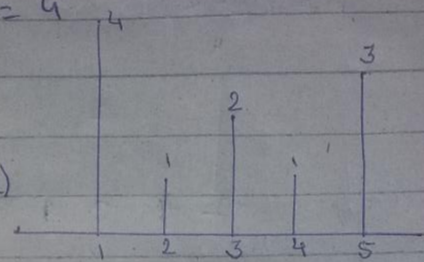
$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

$= 1\times 4 + (-2)\times 1 + 3(2) + (-4)(1)$

$= 4 - 2 + 6 - 4 = 4$

$n = 5$

$h(5-k)$



$y(5) = \sum_{k=1}^{3} x(n)\,h(5-k)$

$= x(1)h(1) + x(2)h(2) + x(3)h(3)$

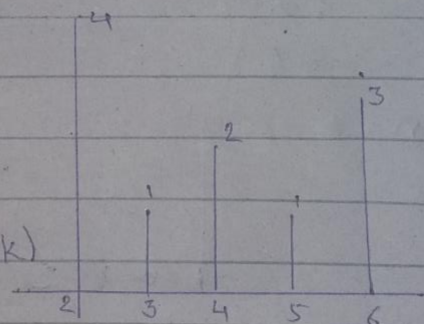
$= (-2)(4) + 3(1) + (-4)(2)$

$= -8 + 3 - 8$

$y(5) = -13$

$n = 6$

$h(6-k)$



$y(6) = \sum_{k=2}^{k=3} x(2)h(2) + x(3)h(3)$

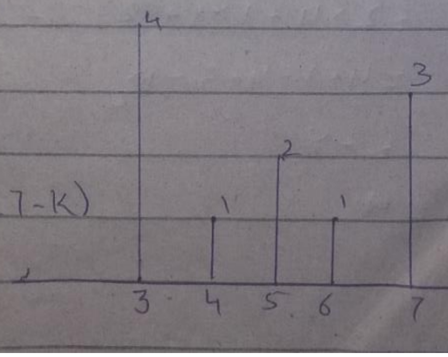
$= 3(4) + 1(-4)$

$= 12 - 4$

$y(6) = 8$

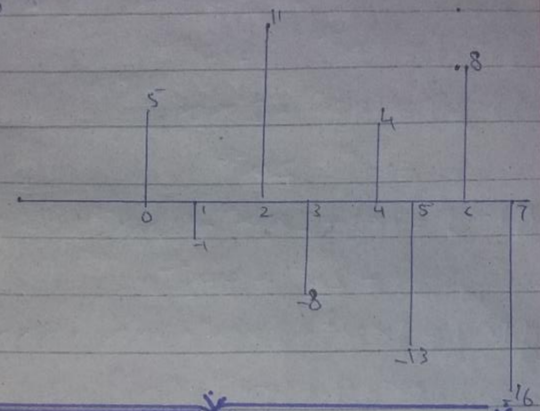
$n = 7$

$h(7-k)$



$y(7) = x(3)\,h(3)$
$= 4 \times (-4)$
$= -16$

$y(n) =$



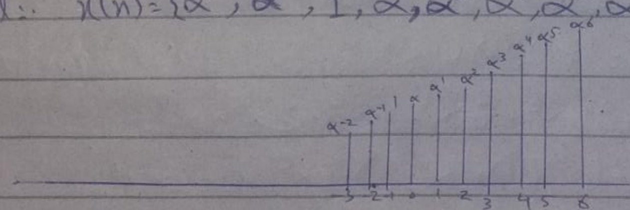
Q No 2 (b)

compare the following $y(n)$ of The following Signal

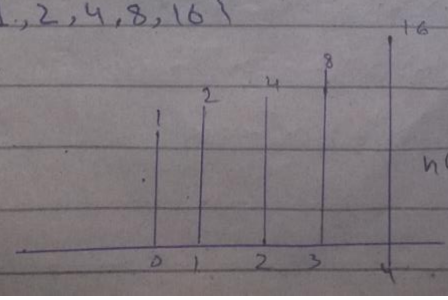
$$x(n) \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{else} \end{cases}$$

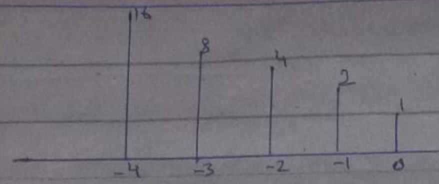
$$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0 & \text{else} \end{cases}$$

Sol:- $x(n) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6\}$



$h(x) = \{1, 2, 4, 8, 16\}$





$h(-n)$

$y[-3] = 1\alpha^{-2} \quad = \alpha^{-2}$

$y[-2] = 1\times\alpha^{-1} + 2\times\alpha^{-2}$

$= \alpha^{-1} + 2\alpha^{-2}$

$y[-1] = 1\times1 + 2\times\alpha^{-1} + 4\times\alpha^{-2}$

$= 1 + 2\alpha^{-1} + 4\alpha^{-2}$

$y[0] = 1\times\alpha^{1} + 2\times\alpha^{4} + 4\times\alpha^{-1} + 8\times\alpha^{-2}$

$= \alpha + 8 + 4\alpha^{-1} + 8\alpha^{-2}$

$y[1] = 1\times\alpha^{2} + 2\times\alpha^{1} + 4\times1 + 8\times\alpha^{-1} + 16\times\alpha^{-2}$

$= \alpha^{2} + 2\alpha^{1} + 4 + 8\alpha^{-1} + 16\alpha^{-2}$

$y[2] = 1\times\alpha^{3} + 2\times\alpha^{2} + 4\times\alpha^{1} + 8\times\alpha^{1} + 16\times\alpha^{-1}$

$= \alpha^{3} + 2\alpha^{2} + 4\alpha^{1} + 8 + 16\alpha^{-1}$

$y[3] = (1)(16) + (\alpha)(8) + (\alpha^{2})(4) + (\alpha^{3})(2) + (\alpha^{4})(1)$

$= 16 + 8\alpha + 4\alpha^{2} + 2\alpha^{3} + \alpha^{4}$

$y[4] = (\alpha)(16) + (\alpha^{2})(8) + (\alpha^{3})(4) + (\alpha^{4})(2) + \alpha^{5}(1)$

$= 16\alpha + 8\alpha^{2} + 4\alpha^{3} + 2\alpha^{4} + \alpha^{5}$

$y[5] = (\alpha^{2})(16) + (\alpha^{3})(8) + (\alpha^{4})(4) + (\alpha^{5})(2) + (\alpha^{6})(1)$

$= 16\alpha^{2} + 8\alpha^{3} + 4\alpha^{4} + 2\alpha^{5} + \alpha^{6}$

$y[6] = (\alpha^{3})(16) + (\alpha^{4})(8) + (\alpha^{5})(4) + (\alpha^{6})(2)$

$= 16\alpha^{2} + 8\alpha^{4} + 4\alpha^{5} + 2\alpha^{6}$

$y[7] = (16)(\alpha^{4}) + 8(\alpha^{5}) + 4(\alpha^{3})$

$= 16\alpha^{4} + 8\alpha^{5} + 4\alpha^{6}$

$y[8] = 16(\alpha^5) + 8(\alpha^6)$

$= 16\alpha^5 + 8\alpha^6$

$y[9] = 16\alpha^6$

$y[10] = 0$

There is no overlap in $y[10]$.

Q No 3

i) $x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3), & n < 0 \end{cases}$

Sol:-

$x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3)^{-n}, & n < 0 \end{cases}$

writing in The form of z-transform

$X(z) = \sum_{n=0}^{\infty} (1/4)^{n} z^{-n} + \sum_{n=-\infty}^{0} (1/3)^{n} z^{n} - 1$

using geomatrig Series

$= \frac{1}{1-1/4 z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^{n} - 1$

$= \frac{1}{1-1/4 z^{-1}} + \frac{1}{1-1/3} - 1$

$= \frac{1-1/4 z^{-1} + 1-1/1^{-1}}{(1-1/4 z^{-1})(1-1/3 z^{-3})} - 1$

$= \frac{1-1/3 z + 1-1/4 z^{-1} - (1+1/3 z - 1/4 z^{-1} + 1/12 z^{-1} - z^{-1})}{(1-1/4 z^{-1})(1-1/3 z)}$

$= \frac{1-1/3 z + 1 - 1/4 z^{3} - 1 + 1/4 z^{3} + 1/12}{(1-1/4 z^{-1})(1-1/3 z)}$

$= \frac{1+1/12}{(1-1/4 z^{-1})(1-1/3 z)}$

$$= \frac{13/12}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Hence The ROC is $\frac{1}{4} < |z| < 3$

ii) $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0 & \text{else where} \end{cases}$

Sol:-

In The form of Z transform

$$X(z) = \sum_{n=0}^{\infty} (\frac{1}{2})^n z^{-n} - \sum_{n=0}^{\infty} 6 \cdot 3^n z^{-n}$$

using geometic Series

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{1-3z^{-1}-1+\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{-5/2\, z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

The ROC is $|z| > 3$