Day: M T

Name : Syed Waleed Shah
I.D : 7497
Submitted to : Ma'am Shumaila Mazhar
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Subject : Numerical Analysis

Question no 1

Solution

$\frac{dy}{dx} = 2x$ $\quad y(0) = 1$

$h = 0.1$

$0 \leq x \leq 5$

Euler Method

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.1 f(0, 1)$

$= 1 + 0.1(0)$

$y_1 = 1 + 0 = 1$

$y_2 = y_1 + hf(x, y)$

$= 1 + 0.1 f(0.1, 1)$

$= 1 + (0.1)(0.2)$

$= 1 + 0.02$

$y_2 = 1.02$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.02 + 0.1 f(0.2, 1.02)$

$= 1.02 + 0.1(0.4)$

$= 1.02 + 0.04$

$y_4 = y_3 + hf(x_3, y_3)$

$= 1.06 + 0.1 f(0.3, 1.06)$

$= 1.06 + 0.1(0.6)$

$y_4 = 1.12$

$y_5 = y_4 + hf(x_4, y_4)$

$= 1.12 + 0.1 f(0.4, 1.12)$

$= 1.12 + 0.1(0.8)$

$y_5 = 1.2$

~~$y(0.5)$~~

By Improve Eular Method

$y_{m+1} = y_m + hf\left(x_m + \frac{1}{2}h, y_m + \frac{1}{2}hf(x_m, y_m)\right)$

$= f(x_0, y_0) = f(0,1) = 0$

$= x_0 + \frac{1}{2}h = 0 + \frac{0.1}{2} = 0.05$

$y_0 + \frac{1}{2}hf(x_0, y_0) = 1 + \frac{0.1}{2} \times 0 = 1$

$f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hf(x_0, y_0)\right) = f(0.05, 1) = 0.1$

$= y_1 = y_0 + hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hf(x_0, y_0)\right) =$

$1 + 0.1 \times 0.1 = 1.01$

$f(x_1, y_1) = f(0.1, 1.01) = 0.2$

$x_1 + \frac{1}{2}h = 0.1 + \frac{0.1}{2} = 0.15$

$y_1 + \frac{1}{2}hf(x_1, y_1) = 1.01 + \frac{0.1}{2} \times 0.2 = 1.02$

$f + \frac{1}{2}hf(x_1, y_1) = 1.01 + \frac{0.12}{2} \times 0.2 = 1.02$

$f\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}hf(x_1, y_1)\right) = f(0.15, 1.02) = 0.3$

$= y_2 = y_1 + hf\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}hf(x_1, y_1)\right) = 1.01 + 0.1 \times 0.3$

$= 1.04$

$f(x_2, y_2) = f(0.2, 1.04) = 0.4$

$x_2 + \frac{1}{2}h = 0.2 + \frac{0.1}{2} = 0.25$

$y_2 + \frac{1}{2}hf(x_2, y_2) = 1.04 + \frac{0.1}{2} \times 0.4 = 1.06$

$f(x_2 + \frac{1}{2}h, y_2 + \frac{1}{2}hf(x_2, y_2)) = f(0.25, 1.06)$

$= 0.5$

$0.1 = 1.01$

$= y_3 = y_2 + hf(x_2 + \frac{1}{2}h, y_2 + \frac{1}{2}hf(x_2, y_2)) = 1.04 + 0.1 \times 0.5$

$= 1.09$

$f(x_3, y_3) = f(0.3, 1.09) = 0.6$

$x_3 + \frac{1}{2}h = 0.3 + \frac{0.1}{2} = 0.35$

$y_3 + \frac{1}{2}hf(x_3, y_3) = 1.09 + \frac{0.1}{2} \times 0.6 = 1.12$

$f(x_3 + \frac{1}{2}h, y_3 + \frac{1}{2}hf(x_3, y_3)) = f(0.35, 1.12) = 0.7$

$y_4 = y_3 + hf(x_3 + \frac{1}{2}h, y_3 + \frac{1}{2}hf(x_3, y_3)) = 1.09 + 0.1 \times 0.7$

$= 1.16$

$f(x_4, y_4) = f(0.4, 1.16) = 0.8$

$x_4 + \frac{1}{2}h = 0.4 + \frac{0.1}{2} = 0.45$

$y_4 + \frac{1}{2}hf(x_4, y_4) = 1.16 + \frac{0.1}{2} \times 0.8 = 1.2$

$f(x_4 + \frac{1}{2}h, y_4 + \frac{1}{2}hf(x_4, y_4)) = f(0.45, 1.2) = 0.9$

$y_5 = y_4 + hf(x_4 + \frac{1}{2}h, y_4 + \frac{1}{2}hf(x_4, y_4)) = 1.16 + 0.1$

$\times 0.9 = 1.25$

$y(0.5) = 1.25$

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Q2 .....

Given data:

$y=0 \;,\; x=0 \;,\; h=0.2 \cdot \quad 0 \le x \le 0.6$

$y_{n+1} = y_n + k$

1st Integration

$n=0$

$y_1 = y_0 + k \;,\; k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_0, y_0)$

$k_1 = h(x_0^2 - x_0 - y_0)$

$k_1 = 0.2 \cdot (0^2 - 0 - 0)$

$\boxed{k_1 = 0}$

$k_2 = hf(x_0 + h/2 \;,\; y_0 + h/2)$

$= 0.2 f(x_0 + h/2 \;,\; y_0 + h/2)$

$= 0.2 f\left(0 + \frac{0.2}{2} \;,\; 0 + \frac{0.2}{2}\right)$

$= 0.2 f(0.1, 0.1)$

$= 0.2 (0.1^2 + 0.1 - 0.1)$

$\boxed{k_2 = 0.0020}$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$= 0.2f\left(0 + \frac{0.2}{2}, \frac{0 + 0.002}{2}\right)$$

$$= 0.2f(0.1, 0.001)$$

$$= 0.2(0.1^2 + 0.1 - 0.001)$$

$$\boxed{k_3 = 0.0218}$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$= 0.2f(0 + 0.2, 0 + 0.218)$$

$$= 0.2f(0.2, 0.0218)$$

$$= 0.2(0.2^2 + 0.2 - 0.0218)$$

$$\boxed{k_4 = 0.0436}$$

$$k = \frac{1}{6}(0 + 2(0.002) + 2(0.0218 + 0.0436))$$

$$k = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$\boxed{y_1 = 0.0152}$$

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Q3 .....

Q Solution

Given Data:

$a = 0$

$b = 10$

$n = 10$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

x	0	1	2	3	4	5	6	7	8	9	10
	10.1	17.2	24.4	34.6		50.9		60.3		62.1	
				↓		↓		↓		↓	
				29.2		41.2		57.8		61.2	

Using formula

$$\int f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \cdots f(x_9) + f(x_{10})\right]$$

$$= \frac{1}{2}\left[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 62.1\right]$$

$$= 412.9 \text{ Ans}$$

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Q4 $\int_2^3 \ln(x^3+1)\,dx$

Use 10 strips

$SI = n = 10$

$h = \frac{3-2}{10} = 0.1$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777

	x_8	x_9
x	1.8	1.9
$f(x)$	1.922	2.062

Now using formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4(f(x_1) + f(x_3) \cdots) + 2(f(x_2 \cdots) + f(x_n))\right]$$

$$\frac{0.1}{3}\left[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + 2(1.003 + 1.320 + 1.628 + 1.922) + 2.062\right]$$

$= 1.184$ Ans.