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Section	A
Subject	Applied Calculus
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Q No 1

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

(a) State any point of discontinuity

(b) find if they exist

i. $\lim_{t \to 3} g$

Sol

To check the discontinuity of function at

$g(t) = t^2$ $\quad$ $t = 0$

$g(0) = 0^2$

$\boxed{0}$ defined

for R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)$

$= \lim_{h \to 0} 1 + h^2 + 2h$ $\quad$ $h = 0$

Apply limit $1 + 0^2 + 2(0)$

$\boxed{= 1}$

for L.H.L

$$\lim_{h \to 0} g(1-h) = 2t+3$$

$$= \lim_{h \to 0} 2(1-h) = 2t+3$$

$$\lim_{h \to 0} 2(1-h)+3$$

$$= \lim_{h \to 0} 2-2h+3$$

Apply limt

$2-2(0)+3$

$\boxed{=5}$

R.H.L $\neq$ L.H.L $= g(t) = 5$

Now at $t=4$

$$g(4) = 2(4)+3$$

$$= 8+3$$

$$= 11$$

for R.H.L

$$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h)+3$$

$$= \lim_{h \to 0} 2+2h+3$$

Apply limt

$2+2(0)+3 = 5$

for L.H.L $\lim g(1-h) = 12$

$g(4) =$ R.H.L $\neq$ L.H.L point of discontinuity is at $t=4$

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(b) find if they exist

(i) $\lim_{t \to 3} g$

for $g(t) = t^2$

RHL $\lim_{h \to 3} g(1+h)$ $\lim (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limit $1 + 3^2 + 2(3) = 16$

LHR $\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h-3} 2(1+h) + 3$

$= \lim_{h-3} 2 - 2h + 3$

Apply limit

$2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

RHL # LHL (do not exist since LHL is -ve)

Q.No2

(i) find the Maclaurin's Series for

Sol

$$y(x) = x^2 + \sin x$$

Since we know that Maclaurin's Series for function $y(x)$ is given by

$$y(x) = y(0) + xy'(0) + \frac{(x-0)^2 y''(0)}{2!} + \frac{x^3}{3!} y'''(0) + \cdots$$

Now

put $x = 0$

$$y(x) = y(0) + (x-0)y'(0) + \frac{(x-0)^2 y''(0)}{2!} + \frac{x^3 y'''(0)}{3!} + \cdots$$

$$y(x) = y(0) + xy'(0) + \frac{x^2 y''(0)}{2!} + \frac{x^3 y'''(0)}{3!} + \cdots$$

Now find

$$y(0) = ?$$

$$y(x) = x^2 + \sin x$$

$$y(0) = 0 + \sin 0$$

$$= 0 + 0$$

$$\boxed{y(0) = 0}$$

$$y(x) = x^2 + \sin x$$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$$y'(x) = 2x + \cos x$$

$$y'(0) = 2(0) + \cos 0$$

$$y'(0) = 2(0) + \cos 0$$

$$\boxed{y'(0) = 1}$$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$y''(x) = 2 + (-\sin x)$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$y''(0) = 2 - 0$$

$$\boxed{y''(0) = 2}$$

Now

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x \Rightarrow y'''(0) = 0 - \cos 0$$

$$\boxed{y'''(0) = -1}$$

$$Y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cancel{x} \text{-----}$$

$$= x + \frac{2x^2}{2!} - \frac{x^3}{3!} \text{----}$$

$$= x + x^2 - \frac{x^3}{3!} \text{----}$$

So $$y(x) = x + x^2 - \frac{x^3}{3!} + \text{---}$$

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QNo 3 (i) find y'' given

$$1 + xy = x^2 + y^2$$

(ii) find y' by using logarithmic differentiation

$$y = x^3(1+x)^9 e^{6x}$$

(i) find y'' given

$$1 + xy = x^2 + y^2$$

Sol

$$1 + xy = x^2 + y^2$$

by Differentiation

$$\Rightarrow \frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\Rightarrow \frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}y^2$$

$$\Rightarrow 0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x^{2-1}\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\Rightarrow \frac{dy}{dx}(x-2y) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{2x - xy - 4y + 2yy' - (2x - 4xy' - 4 + 2yy')}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{2x - xy - 4y + 2yy' - 2x + 4xy' + y - 2yy'}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{4xy' - 3y - xy'}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{4x\left(\frac{2x-y}{x-2y}\right) - 3y - x\left(\frac{2x-y}{x-2y}\right)}{(x-2y)^2}$$

$$\Rightarrow y'' = \frac{4x(2x-y) - 3y(x-2y) - x(2x-y)}{(x-2y)(x-2y)^2}$$

$$\Rightarrow y'' = \frac{8x^2 - 4xy - 3xy + 6y^2 - 2x^2 + xy}{(x-2y)^3}$$

$$\Rightarrow \boxed{y'' = \frac{6x^2 + 6y^2 - 6x^2y}{x-2y^3}}$$

D (ii) find y' by using logarithmic defferentation

$$Y = x^3(1+x)9e^{6x}$$

Sol

$$Y = x^3(1+x)9e^{6x}$$

taking ln to both side

$$\ln y = \ln[x^3(1+x)^9 e^{6x}]$$

$$\Rightarrow \ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\Rightarrow \ln y = 3\ln x + 9\ln(1+x)^9 + 6x\ln e$$

$$\Rightarrow \ln y = 3\ln x + 9\ln(1+x) + 6$$

Differentiating both Sides

$$\Rightarrow \frac{1}{y}\cdot y' = 3\cdot\frac{1}{x} + 9\cdot\frac{1}{x} + 6$$

$$\Rightarrow \frac{y'}{y} = \frac{3}{x} + \frac{9}{1+x} + 6$$

$$\Rightarrow y' = y\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$

$$\Rightarrow y' = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$