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Subject	Linear algebra
Assignment	Mid term
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Q no.1: consider the given matrix as the augmented matrix of a linear system .

Explain in your word the next elementary row operation that should be performed in order to solve the system . where ID3 is the 3rd digit in your ID and ID last digit of your id inverse.

inverse -

$$\left[\begin{array}{ccccc} 1 & \text{ID}_3 & 3 & 0 & 5 \\ 0 & 1 & \text{ID}_{\text{last}} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & \text{ID}_3 \end{array} \right] \quad \left. \vphantom{\begin{array}{ccccc} 1 & \text{ID}_3 & 3 & 0 & 5 \\ 0 & 1 & \text{ID}_{\text{last}} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & \text{ID}_3 \end{array}} \right\} \text{ID} = (6001)$$

$$\left[\begin{array}{ccccc} 1 & \text{ID}_3 & 3 & 0 & 5 \\ 0 & 1 & \text{ID}_{\text{last}} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & \text{ID}_3 \end{array} \right]$$

$$\left[\begin{array}{ccccc} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$R_2 + R_3$

$$\left[\begin{array}{ccccc} 1 & 0 & 3 & 0 & 5 \\ 0+0 & 1+0 & -1+1 & 0+0 & 7-6 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

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$$\begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_1 - 3R_3$$

$$\begin{bmatrix} 1 & 0 & 3-3 & 0 & 5+18 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

This is the final linear system

$$x_1 = 23, \quad x_2 = 1, \quad x_3 = -6$$

$$x_4 = 0$$

Verification

$$23 - 18 = 5$$

$$5 = 5 \quad \text{— true}$$

$$1 - (-6) = 7$$

$$1 + 6 = 7$$

$$7 = 7 \quad \text{— true}$$

Q no.2: Find the elementary row operation that transform the first matrix into (a) second and reverse row operation that transform the second matrix into first?

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Matrix 1 + Matrix 2

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & -5+8 & -4-1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

This is now matrix 2.

Matrix 2 to matrix 1

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

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$$R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0+2 & 3+(2) & 5+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

So this is matrix (1).

Q no.2: (part B)

Below given the same matrix. Find which one is the row echelon form and which is reduced row echelon form. Explain in your own word for each selection in detail?

a. $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \Pi & 0 & 0 \\ 0 & 0 & -\Pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

b. $\begin{bmatrix} 1 & 0 & \Pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

Echelon form:

- All nonzero rows are above any rows of all zeros.
- Each leading entry of a row is in a column to the right of the leading entry of the rows above it.
- All entries in a column below a leading entry are zero .

c. $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

d. $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Reduced Echelon form:

- The leading entry in each non-zero row is 1.
- Each leading 1 is the only non-zero entry in this column.

Q no. 3: (a)

The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.

Ans:

Reduced row (column) echelon form:

A matrix is said to be in reduced row (column) echelon form when it satisfies the following conditions.

- The matrix satisfies conditions for a row (column) echelon form.
- The leading entry in each row (column) is the only non-zero in its column (rows).

For example

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row(column)echelon form:

A matrix is said to be row and column echelon form when it satisfies the following condition.

- The first non-zero element in each row (column) called the leading entry is 1.
- Rows (column) with all zero elements, if any, are below (after) the rows (column) having a non-zero element.

For example

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q no.3: (b)

Find an echelon form for the below matrix using row operations. Where ID2 is 2nd digit in your ID e.g. if your ID is 12345 ID2 = 2, ID3=3, ID_first_last is the first and last digit of your ID i.e.15

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10 \text{ first last} \end{bmatrix}$$

My Id is (600)

$$\text{sol: } = \begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10 \text{ first last} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix}$$

$$2R_1 + 3R_4$$

$$\begin{bmatrix} 5 & 0 & 49 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix}$$

$$R_3 \longleftrightarrow R_4$$

$$\begin{bmatrix} 5 & 0 & 49 \\ 1 & 8 & -1 \\ 1 & -4 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$

$$4R_3 + 2R_2$$

$$\begin{bmatrix} 5 & 0 & 49 \\ 2 & 8 & -1 \\ 8 & 0 & 42 \\ 0 & 0 & 0 \end{bmatrix}$$

$$5R_3 - 2R_1$$

$$\begin{bmatrix} 5 & 0 & 49 \\ 0 & 8 & 93 \\ 8 & 0 & 42 \\ 0 & 0 & 0 \end{bmatrix}$$

$$5R_3 - 8R_1$$

$$\begin{bmatrix} 5 & 0 & 49 \\ 0 & 8 & 93 \\ 0 & 0 & -182 \\ 0 & 0 & 0 \end{bmatrix}$$



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This is an echelon form

