Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Mid – Term Examinationspring2020
Date:

Course Code:	MTH 102			Course Title:	Calculus and analytic geometry		
Prerequisite:				Instructor:	Sir. HIMAYATULLAH		
Module:	3	Program:	BEE	Total Marks:	30	Time Allowed:	90 min

Student Details

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Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	. **Identify** $\lim\limits_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have nay horizontal tangent if so where ?	Marks 10 CLO1 C1

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Question No 1:

Part (a).

Identify

$$\lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Solution:

$$\lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Appling Lim on Function

$$\lim_{h\to 0} F(x) = \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

$$= \frac{\sqrt{2+0} - \sqrt{2}}{0}$$

$$\lim_{h\to 0} F(x) = \frac{\sqrt{2} - \sqrt{2}}{0}$$

$$\lim_{h\to 0} F(x) = \frac{0}{0} \longrightarrow (i)$$

By direct applying Limit it give us 0/0. so Frist we have rationalize The expression Then apply limit.

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(2)

$$F(x) = \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

multiply & divide $\sqrt{2+h} + \sqrt{2}$

$$F(x) = \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

we know that

$$(a+b)(a-b) = a^2 - b^2$$

$$\Rightarrow F(x) = \frac{(\sqrt{2+h})^{\cancel{2}} - (\sqrt{2})^{\cancel{2}}}{h(\sqrt{2+h} + \sqrt{2})}$$

$$F(x) = \frac{\cancel{2} + h - \cancel{2}}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h} + \sqrt{2})}$$

$$F(x) = \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

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By applying lim

$$\lim_{h \to 0} F(x) = \lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

$$\lim_{h \to 0} F(x) = \frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$\boxed{\lim_{h \to 0} F(x) = \frac{1}{2\sqrt{2}}} \rightarrow \text{ANS}$$

x —— x —— x —— x —— x —— x

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Part (b):-

$$y = (x + 1/x)(x - 1/x + 1)$$

Find d/dx.

Solution:-

$$y = (x + 1/x)(x - 1/x + 1)$$

Taking d/dx both side.

$$\frac{d}{dx} y = \frac{d}{dx}\left[(x + 1/x)(x - 1/x + 1)\right]$$

By Product rule.

$$y' = (x + 1/x)\, \frac{d}{dx}\left(x - \frac{1}{x} + 1\right)$$

$$= + (x - 1/x + 1)\, \frac{d}{dx}(x + 1/x)$$

$$y' = (x + 1/x) \cdot \left[\frac{d}{dx}(x) - \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}(1)\right] + (x - 1/x + 1)$$

$$\left[\frac{d}{dx}(x) + \frac{d}{dx}(1/x)\right]$$

$$y' = \left(x + \frac{1}{x}\right)\left[(1) - \frac{d}{dx}(x^{-1}) + 0\right]$$

$$+ \left[x - \frac{1}{x} + 1\right]\left[(1) + \frac{d}{dx}(x^{-1})\right.$$

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$$y' = [x + \frac{1}{x}] [-1 + x^{-2}] + [x - \frac{1}{x} + 1] [1 - x^{-2}]$$

$$y' = (x + 1/x) [1 + 1/x^2] + [x - 1/x + 1] [1 - 1/x^2]$$

$$\cancel{y' = x + \frac{1}{x}}$$

$$\cancel{y' = x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x^3} + x}$$

$$y'' = \cancel{x} + \frac{\cancel{x}}{x^2} + \frac{1}{\cancel{x}} + \frac{1}{x^3} + x - \frac{\cancel{x}}{x^2} - \frac{1}{x} + \frac{1}{x^3}$$

$$+1 - 1/x^2$$

$$y' = x + 1/x^3 + x + 1/x^3 + 1 - 1/x^2$$

$$2x + \frac{2}{x^3} - 1/x^2 + 1$$

$$\boxed{y' = 2x^{-3} - x^{-2} + 2x + 1} \rightarrow \text{ANS.}$$

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Question 2:

Given That:

height (s) $= 160t - 16t^2$

velocity $= 160$ m/sec

Find:

Height (s) = ?

vecocity ($\vec{v}$) & speed v

at $s = 256$ ft

acceleration (a) at $t = 5$ sec.

Solution:

Velocity ($\vec{v}$)

we know that

$$\vec{v} = \frac{ds}{dt}$$

Here $s = 160t - 16t^2$

$$d/dt(s) = d/dt(160t - 16t^2)$$

$$d/dt(s) = 160 - 32t \rightarrow (i)$$

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Also given that

$s = 256$ & $s = 160t - 16t^2$

$\Rightarrow 160t - 16t^2 = 256$

$160t - 16t^2 - 256 = 0$

$-16t^2 + 160t - 256 = 0$

$16t^2 - 160t + 256 = 0$

$16(t^2 - 10t + 16) = 0$

$t^2 - 10t + 16 = 0$

$t^2 - 8t - 2t + 16 = 0$

$t(t-8) - 2(t-8) = 0$

$(t-8)(t-2) = 0$

$t - 8 = 0$, $t - 2 = 0$

$t = 8$ sec $t = 2$ sec

Put these value of (t) in eq (1)

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Now velocity at $s = 256$

when $t = 2$ sec

~~$v(t) = 160$~~

$$v(t) = 160 - 32t$$

$$v(2) = 160 - 32(2)$$

$$v(2) = 160 - 64$$

$$v(2) = 96 \text{ m/sec}$$

$\boxed{v = 96 \text{ ft/s}}$ is the velocity of rock on its way up.

$t = 8$ sec

$$v(t) = 160 - 32t$$

$$v(8) = 160 - 32(8)$$

$$= 160 - 256$$

$\boxed{v(8) = -96 \text{ ft/sec}}$ is the velocity of rock on it way down

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Speed OF rock

we know that

$$\text{speed}(v) = |\text{velocity}(\vec{v})|$$

speed at $t = 2$ sec

$$v = |\vec{v}|$$

$$v_2 = |96|$$

$$\boxed{v = 96\ \cancel{ft}/\text{sec}}$$

speed at $t = 8$ sec

$$v = |\vec{v}|$$

$$v = |-96|$$

$$\boxed{v = 96\ ft/\text{sec}}$$

Hence speed of rock at
$s = 256$ FS
is 96 ~~ft~~/sec

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(10) (10)

Height the rock go (s)

$$S = 160t - \cancel{8}t^2 \longrightarrow (ii)$$

we know that
at the hight velocity of
rock will be zero.

$$V = 160 - 32t = 0$$

$$160 - 32t = 0$$

$$+32t = +160$$

$$32t = 160$$

$$t = 160/32$$

$$\boxed{t = 5 \text{ sec}}$$

put $t = 5$ in eq ii

$$S = 160(5) - 16(5)^2$$

$$S = 800 - 16 \times 25$$

$$S = 800 - 400$$

$$\boxed{S = 400 \text{ Ft}}$$

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acceleration of rock

we know that

$$\vec{a} = dv/dt$$

$$v = 160 - 32t$$

$$dv/dt = \frac{d}{dt}(160 - 32t)$$

$$dv/dt = 0 - 32$$

$$\boxed{\vec{a} = -32\ Ft/sec^2}$$

is the $\vec{a}$ of rock

Now acceleration at 5 sec.
we know that
velocity at 5 sec is equal to zero because at 5 sec the rock will reach at ~~his~~ maximum height $s = 400Ft$. we know that at max height the velocity will be zero.
so
at $t = 5$ the acceleration is only due to gravity which is equal to $9.8\ m/sec^2$

$$\Rightarrow \quad \boxed{a(5) = 9.8\ m/sec^2}$$

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Result:-

* Height the rock will go

$\boxed{s = 400 Ft}$ → ANS

* velocity of rock at $s = 256$ Ft on the way up & down.

velocity on the way up

$\boxed{V = 96 Ft/sec}$ → ANS

$\vec{V}$ on the way down

$\boxed{V = -96 Ft/sec}$ → ANS

* speed at $s = 256$ Ft

$\boxed{|V| = 96 Ft/sec}$ → ANS

* accelaration :-

$\boxed{a = -32 Ft/sec}$ → ANS

accelaration at $t = 5$sce

$\boxed{a(5) = 9.8 m/sec}$ → ANS

x — x — x — x — x — x

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QNo 3:

$$y = x^4 - 2x^2 + 2$$

Does curve have any horizontal tangent if so where.?

Solution:-

$$y = x^4 - 2x^2 + 2$$

We know that Tangent of a curve is dy/dx

Taking dy/dx both side.

$$\frac{d}{dx} y = \frac{d}{dx} (x^4 - 2x^2 + 2)$$

$$y' = d/dx\,(x^4) - \frac{d}{dx}(2x^2) + d/dx\,(2)$$

$$y' = 4x^3 - 2\,d/dx\, x^2 + 0$$

$$y' = 4x^3 - 4x + 0$$

$$y' = 4x^3 - 4x \quad \rightarrow (i)$$

If The tangent is horizontal Then The slope of curve should be zero. $\frac{d}{dx}y = 0$

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so we can write $y' = 0$

$\Rightarrow y' = 4x^3 - 4x = 0$

$\Rightarrow 4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$4x = 0$, and $x^2 - 1 = 0$

$x = 0$, $x^2 = 1$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

$\Rightarrow x = 0,\ x = 1,\ \&\ x = -1$

so This result tell us that
If $x = 0$, $x = 1$, and $x = -1$
Then slope should be horizontal.
now put these values of
$x = \{0, 1, -1\}$ in original equation
to find the y-coordinate..

If $x = 0$

$F(0) = x^4 - 2x^2 + 2$

$= (0)^4 - 2(0)^2 + 2$

$0 - 0 + 2$

$\boxed{F(0) = 2}$

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IF $x = 1$

$$F(1) = (1)^4 - 2(1)^2 + 2$$

$$= 1 - 2 + 2$$

$$\boxed{F(1) = 1}$$

IF $x = -1$

$$F(-1) = (-1)^4 - 2(-1)^2 + 2$$

$$= 1 - 2(1) + 2$$

$$= 1 - 2 + 2$$

$$\boxed{F(-1) = 1}$$

The coordinate of horizontal tangent are

$\boxed{(0,2), (1,1) \text{ and } (-1,1)}$ → ANS

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THE END

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