Q1. $f(t) = 1+t, \quad -\pi \leq t \leq \pi$

Sol:

Here we use the formula

$$f(u) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad \text{— (i) eq ①}$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2} \right) \right)$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(2\pi + \frac{\cancel{2}\pi^2}{\cancel{2}} \right)$$

$$\Rightarrow \boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

$$\Rightarrow a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

$$\Rightarrow a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt}{n} \frac{dt}{dt} (1+t) \right) \right)$$

$$\Rightarrow a_n = \frac{1}{\pi} \left(\cancel{(1+t) \frac{\sin nt}{n}} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$\Rightarrow a_n = \frac{-1}{n^2 \pi} \left(\cos n\pi - \cos n(-\pi) \right)$$

$$\Rightarrow a_n = \frac{-1}{n^2 \pi} \left(-1 - (-1) \right) \quad \Rightarrow \boxed{a_n = 0}$$

* $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin nt \, dt$

$\Rightarrow b_n = \frac{1}{\pi} \left((1+t) \int_{-\pi}^{\pi} \sin nt - \int (\int \sin nt - \frac{d}{dt} (1+t) dt \right)$

$\Rightarrow b_n = \frac{1}{\pi} \left(\frac{(1+t)(-\cos nt)}{n} \right) \Big|_{-\pi}^{\pi} - \int \left(-\frac{\cos nt}{n} \right) (1) \right)$

$\Rightarrow b_n = \frac{-1}{n\pi} \left((1+\pi)(\cos n\pi) \right) - \left((1+(\pi)) (\cos nt) \right)$

$\Rightarrow b_n = \frac{-1}{n\pi} \left(\cancel{\cos n\pi} + \pi \cos n\pi - \cos n\pi + \pi \cos n\pi \right)$

$\Rightarrow b_n = \frac{-1}{n\cancel{\pi}} (2\cancel{\pi} \cos n\pi)$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$\Rightarrow \boxed{b_n = \frac{2}{n} (-1)^{n+1}}$

So equation becomes

$$f(u) = \frac{1}{2\pi} (2\pi + \pi^2) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t.$$

Q:2:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} \text{ Eigen value} = ?$$

Sol:.

Step #01: we have

$(A - \lambda I)X = 0$ A = Given Matrix

Step #02:-

we have; the characteristics equation is given by

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step #03:

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal element}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal minor}\end{matrix}\right| \lambda - |A| = 0 \quad -(B)$$

Sum of Diagonal element $= 1 + 1 + 2 = 4$

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B):

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \longrightarrow (C)$

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By putting values in (C):

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

using Quadratic formula

$\lambda = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = -4$

$c = -3$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

we have eigen values;

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$ Required solution.

Q#3:-

Solution:

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_4 \; R_2}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+\cancel{4/5} & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & 1 & 0 & -1 \end{array}\right] \xrightarrow{-1/5 \times R_3}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \xrightarrow{5 \times R_3} \text{ and } 5 \times R_4$$

$$\left(\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right) \xrightarrow{5R_3} \text{ and } \underrightarrow{5R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad C_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = (3/4, 31/21, -11/21, 1/3)$$

$$x = 3/4, \quad y = 31/21, \quad z = -11/21 \quad m = 1/3$$

Ans

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Q#4: verify

$u(x,t) = \sin(x + \partial t)$

Solution:- Given that

$u(x,t) = \sin(x + \partial t)$

Diff w.r.t x partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x + \partial t)$$

$$\frac{\partial u}{\partial x} = \cos(x + \partial t) \frac{\partial}{\partial x}(x + \partial t)$$

$$\frac{\partial u}{\partial x} = \cos(x + \partial t) \quad (1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x + \partial t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x + \partial t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x + \partial t) \cdot \frac{\partial}{\partial x}(x + \partial t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x + \partial t)(1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x + \partial t)}$$

and $u(x,t) = \sin(x+2t)$

Differentiating w.r.t to 't'

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)\,(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t)(0+2)$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

As we that one dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

for the orbitrary constant $c = \pm 2$.

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for The orbitary constant $c = 2$