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SUBJECT :- Differential Equation

DEPARTMENT :- ELECTRICAL ENGINEERING

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Question # 1

(1)

Q1 Estimate general solution of

$$4y'' - 20y' + 25y = 0$$

Solution:-

$$4m^2 - 20m + 25 = 0$$

$$4m^2 - 10m - 10m + 25 = 0$$

$$2m(2m-5) - 5(2m-5) = 0$$

$$(2m-5)(2m-5) = 0$$

$$(2m-5)^2 = 0$$

$2m-5=0$ and $2m-5=0$

$2m=5$ $\quad$ $2m=5$

$m=\frac{5}{2}$ $\quad$ $m=\frac{5}{2}$

$$y_c = (c_1 + c_2 x)e^{\frac{5}{2}x}$$

$$y_c = c_1 e^{\frac{5}{2}x} + c_2 x e^{\frac{5}{2}x}$$

Answer.

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Question # 2

(2)

(2) Estimate the general solution of

$$y' = (x+2)\,y^2$$

Ans

$$y' = (x+2)y^2$$

$$\Rightarrow \frac{dy}{dx} = (x+2)y^2$$

$$\Rightarrow \int \frac{1}{y^2}\,dy = \int (x+2)\,dx$$

$$\Rightarrow \int y^{-2}\,dy = \int (x+2)\,dx$$

$$\Rightarrow \int \frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C$$

$$\Rightarrow \frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + C$$

$\Rightarrow$ Multiplying both sides by -1

$$\Rightarrow y^{-1} = -\left(\frac{x^2}{2} + 2x + C\right)$$

$$\Rightarrow y = -\left(\frac{1}{x^2/2 + 2x + C}\right)$$

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Question # 3

(3)

Q③ Calculate the initial value problem

$$y'' + 2y' + y = 0$$
$$y(0) = 4 \quad y'(0) = -6$$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$(m+1)^2 = 0$

$m+1 = 0$ and $m+1 = 0$

$m = -1 \quad m = -1$

$y = C_1 e^{-x} + C_2 x e^{-x}$

$y = (0) = 4$

$x = 0 \quad y = 4$

$4 = C_1 e^{-0} + C_2(0)$

$\boxed{4 = C_1}$

$y' = -C_1 e^{-x} + C_2(-xe^{-x} + e^{-x})$

$y' = -C_1 e^{-x} + -xe^{-4} C_2 + C_2 e^{-x}$

Applying the condition $y'(0) = -6$

$-6 = -C_1(1) + C_2$

$-6 = -C_1 + C_2$

Since $\boxed{C_1 = 4}$

$-6 = -4 + C_2$

$-6 + 4 = C_2$

$\boxed{-2 = C_2}$

Hence

$y = 4e^{-x} - 2xe^{-x}$

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Question # 4 (4)

(1)(4) Analyze the general solution of
$$x^2y'' + 3xy' + y = 0$$

Solution :-

$$a = 3 \;,\; b = 1$$

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \;,\; m = -1$$

roots are real & equal

So,
$$y = (C_1 + C_2 \ln x)\, x^m$$

Answer.

Question #5

(5)

Q5 Examine the method of undetermined coefficient method for $y''+y'-6y=6x^3-3x^2+12x$

Step 1 : we first solve the homogenous Equation

$$y''+y'-6y=0$$

$$m^2+m-6=0$$
$$m^2+3m-2m-6=0$$
$$m(m+3)-2(m+3)=0$$
$$(m+3)(m-2)=0$$
$$y_c=C_1e^{-3x}+C_2e^{-2x}$$

Step 2 Now since the input function $g(x)$ is a quadratic polynomial assume a particular solution that is also in the form of a quadratic polynomial

$$y_p=Ax^3+Bx^2+Cx+D$$

we seek to determine specific coefficients A, B, C and D

$$y_p'=3AX^2+2BX+C$$

$$y_p''=6AX+2B$$

Substituting y_p and its derivative in differential Equactions

$$y''+y'-6y=6AX+2B+3AX^2+2BX+C-6(AX^3+BX^2+CX+D)$$

$$=6AX+2B+3AX^2+2BX+C-6AX^3-6BX^2-6CX-6D$$

$$-6A=6$$

$$A=\frac{6}{-6}$$

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$\boxed{A = -1}$

$3A - 6B = -3$

$3A - 6B = -3$ As $\boxed{A = -1}$

$3(-1) - 6B = -3$

$-3 - 6B = -3$

$-6B = -3 + 3$

$-6B = 0$

$B = \frac{0}{-6}$

$\boxed{B = 0}$

$6A + 2B - 6C = 12$

$6(-1) + 2(0) - 6C = 12$

$-6 + 0 - 6C = 12$

$-6C = 12 + 6$

$-6C = 18$

$C = \frac{18}{-6}$

$\boxed{C = -3}$

$2B + C - 6D = 0$

$2(0) - 3 - 6D = 0$

$0 - 3 - 6D = 0$

$-6D = 3$

$D = \frac{-3}{6}$

$\boxed{D = \frac{-1}{2}}$

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$$Y_P = Ax^3 + Bx^2 + Cx + D$$

$$Y_P = -x^3 + 0 \cdot x^2 - 3x - \frac{1}{2}$$

$$Y_P = -x^3 - 3x - \frac{1}{2}$$

Step 3 $\quad Y = Y_C + Y_P$

$$Y = C_1 e^{-3x} + C_2 e^{-2x} - x^3 - 3x - \frac{1}{2} \quad \text{Ans.}$$

Question #6

Q(6) Examine the method of variation of parameters for $y'' - 4y' + 4y = x^2 e^{2x}$.

Solution :- Since the auxiliary equation is

$$m^2 - 4m + 4 = 0$$
$$m^2 - 2m - 2m + 4 = 0$$
$$m(m-2) - 2(m-2) = 0$$
$$(m-2)(m-2) = 0$$
$$m - 2 = 0 \qquad m - 2 = 0$$
$$m = 2 \qquad m = 2$$

$$y_c = C_1 e^{2x} + C_2 x e^{2x}$$

$$y_1 = e^{2x} \text{ and } y_2 = x e^{2x}$$

We next compute Wronskian

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$$W(e^{2x}, xe^{2x}) = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x}+e^{2x} \end{vmatrix}$$

$$= 2xe^{4x} + e^{4x} - 2xe^{4x}$$

$$= \cancel{2x}e^{4x} + e^{4x} - \cancel{2x}e^{4x}$$

$$= e^{4x}$$

Now as $f(x) = x^2 e^{2x}$

$$W_1 = \begin{vmatrix} 0 & xe^{2x} \\ x^2e^{2x} & 2xe^{2x}+e^{2x} \end{vmatrix}$$

$$= -x^3e^{4x}$$

$$W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & x^2e^{2x} \end{vmatrix}$$

$$W_2 = x^2e^{4x} - 0$$

$$W_2 = x^2e^{4x}$$

$$U_1' = \frac{W_1}{W}$$

$$U_1' = -\frac{x^3e^{4x}}{e^{4x}}$$

$$U_1' = -x^3$$

$$U_2' = \frac{W_2}{W}$$

$$U_2' = \frac{x^2e^{4x}}{e^{4x}}$$

$$U_2' = x^2$$

It follows that

$$U_1' = -x^3$$

$$\int U_1' = -\int x^3$$

$$U_1 = -\frac{x^4}{4}$$

and $U_2' = x^2 \Rightarrow \int U_2' = \int x^2$

$$U_2 = \frac{x^3}{3}$$

Therefore, $y_p = \left(-\frac{x^4}{4}\right)e^{2x} + \left(\frac{x^3}{3}\right)xe^{2x}$

$$y_p = \left(-\frac{x^4}{4} + \frac{x^4}{3}\right)e^{2x}$$

Hence $y = y_c + y_p$

$$y = C_1e^{2x} + C_2xe^{2x} + \left(-\frac{x^4}{4} + \frac{x^4}{3}\right)e^{2x}$$

Answer

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Question # 7

(10)

Q(7) Identify an ODE

$$y'' + ay' + by = 0 \text{ for basis } 1, e^{-3x}$$

Solution:-

Basis are $1, e^{-3x}$

$y = c_1 e^{x} + c_2 e^{-3x}$

Putting values

$\lambda = 0 \quad , \quad \lambda = -3$

$\lambda_1 - 0 = 0 \, , \; \lambda + 3 = 0$

$\lambda (\lambda + 3) = 0$

$\lambda^2 + 3\lambda = 0$

$a = 3 \quad b = 0$

$y'' - 3y' + 0y = 0$

$y'' - 3y' = 0$ Answer.

END