Date: ____________

NAME :- MUHAMMAD TALHA

ID :- 7965

SECTION :- "B"

DEPARTMENT :- CIVIL ENGINEERING

ASSIGNMENT :- DIFFERENTIAL EQUATIONS (01)

SUBMITTED TO :- MAM SHUMAILA

DATE :- 17 JUNE - 2020

SUBJECT NAME :- DIFFERENTIAL EQUATIONS

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①

Question :- 01

$$x^3y''' + 2x^2y'' + 2y = 10x + \frac{10}{x}$$

Solution :-

$$x^3y''' + 2x^2y'' + 2y = 10x + \frac{10}{x}$$

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10x + \frac{10}{x}$$

$$x^3D^3y + 2x^2D^2y + 2y = 10x + \frac{10}{x}$$

$$(x^3D^3 + 2x^2D^2 + 2)y = 10x + \frac{10}{x} \quad -①$$

Let

$x = e^t$

$t = \ln x$

$xD = \Delta$

$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$

putting in equation, - ①

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10e^t + \frac{10}{e^t}$$

$$(\Delta^3 - 3\Delta^2 + \cancel{2\Delta} + 2\Delta^2 - \cancel{2\Delta} + 2)y = 10e^t + 10/e^t$$

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$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$

$\cancel{(m^3 - m^2 + 2)\,y = 10e^t + \frac{10}{e^t}}$.

Complementary Solution y_c, $m^3 - m^2 + 2 = 0$

Using Synthetic Division

$$\begin{array}{r|rrrr} & 1 & -1 & 0 & 2 \\ -1 & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$\Delta^2 - 2\Delta + 2 = 0$

Now using Quadratic formula

$a = 1, \; b = -2, \; c = 2$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2a}$$

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$$m = \frac{2 \pm 2\sqrt{-1}}{2} \qquad \because i = \sqrt{-1}$$

$$m = \frac{2 \pm 2i}{2}$$

$$m = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$m = 1 \pm i$$

Roots are Complex,

$$y_c = e^{x}(C_1 \cos t + C_2 \sin t)$$

particular Integral

$$y_p = \frac{1}{\Delta^3 - \Delta^2 + 2}\left(10e^{t} + \frac{10}{e^{t}}\right)$$

$$= \frac{10e^{t}}{\Delta^3 - \Delta^2 + 2} + \frac{10e^{-t}}{\Delta^3 - \Delta^2 + 2}$$

$$= \frac{10e^{t}}{1^3 - 1^2 + 2} + \frac{10e^{t}}{1^3 - 1^2 + 2}$$

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$$= \frac{10e^{t}}{2} + \frac{10e^{-t}}{2}$$

$$= 5e^{t} + 5e^{-t}$$

$$(D^3 - D^2 + 2)y = 10e^{t} + \frac{10}{e^{t}}$$

Complementary Solution y_c :-

$$m^3 - m^2 + 2 = 0$$

By Synthetic Division,

$$\begin{array}{r|rrrr}
 & 1 & -1 & 0 & 2 \\
-1 & & -1 & 2 & -2 \\
\hline
 & 1 & -2 & 2 & \boxed{0}
\end{array}$$

$$m^2 - 2m + 2 = 0$$

Now By Quadratic formula,

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$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1,\ b = -2,\ c = 2$

$$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$m = \frac{2 \pm \sqrt{4-8}}{2}$$

$$m = \frac{2 \pm \sqrt{-4}}{2}$$

General solution,

$$y = y_c + y_p$$

$$y = e^{t}(c_1 \cos t + c_2 \sin t) + 5e^{t} + 5e^{-t}$$

put $e^t = x \Rightarrow t = \ln x$

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$$y = e^{x}\left(c_1 \cos(\ln x) + c_2 \sin(\ln x)\right) + 5x + \frac{5}{x}$$

Result :-

So the Required General solution is,

$$y = e^{x}\left(c_1 \cos(\ln x) + c_2 \sin(\ln x)\right) + 5x + 5/x$$

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QUESTION :- 02

$$x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - \frac{5xdy}{dx} - 15y = x^4$$

Solution :-

$$x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - \frac{5xdy}{dx} - 15y = x^4$$

$$x^3D^3y + 4x^2D^2y - 5xdy - 15y = x^4$$

$$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4 - (1)$$

Let $x = e^t$

$t = \ln x$

$xD = \Delta$

$x^2D^2 = \Delta(\Delta-1)$

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$

putting in eq (1)

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta(\Delta-1)) - 5\Delta - 15)y = e^{4t}$$

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 4\Delta^2 - 4\Delta - 5\Delta - 15)y = e^{4t}$$

$$(\Delta^3 + \Delta^2 - 7\Delta - 15)y = e^{4t}$$

Complementary Solution y_c

$$m^3 + m^2 - 7m - 15 = 0$$

By Synthetic Division

	1	1	-7	-15
3		3	12	15
	1	4	5	0

$$m^2 + 4m + 5 = 0$$

By Quadratic formula,

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, \ b = 4, \ c = 5$

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$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{(2)(1)}$$

$$m = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$= \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$m = -2 \pm i$$

$m = -2+i$, $m = -2-i$

So Roots are complex,

$$y_c = e^{-2x}(c_1 \cos t + c_2 \sin t)$$

particular Integral,

$$y_p = \frac{1}{D^3 + D^2 - 7D - 15} \cdot e^{4t}$$

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$$y_p = \frac{1}{(4)^3+4^2-7(4)-15} \cdot e^{4t}$$

$$y_p = \frac{1}{37} \cdot e^{4t}$$

General solution :-

$$y = y_c + y_p$$

$$y = e^{-2x}\left(c_1 \cos t + c_2 \sin t\right) + \frac{1}{37} e^{4t}$$

Replace,

$$t = \ln x \quad , \quad e^t = x$$

$$y = e^{2x}\left(c_1 \cos \ln x + c_2 \sin \ln x\right) + \frac{1}{37} x^4$$

Result :-

So, the required general solution is,

$$y = e^{-2x}\left(c_1 \cos \ln x + c_2 \sin \ln x\right) + \frac{1}{37} x^4$$

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Question:- 03

$$x^2y'' + 2xy' - 6y = 10x^2$$

$$y(1) = 1$$
$$y'(1) = -6$$

Solution:-

$$x^2y'' + 2xy' - 6y = 10x^2$$

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$$

$$x^2D^2y + 2xDy - 6y = 10x^2$$

$$(x^2D^2 + 2xD - 6)y = 10x^2 \quad - (1)$$

$$xD = \Delta$$
$$x^2D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$$

$$x = e^t \Rightarrow t = \ln x$$

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putting in equation - 1

$$(\Delta^2 - \Delta + 2(\Delta) - 6)y = 10e^{2t}$$

$$(\Delta^2 + \Delta - 6)y = 10e^{2t}$$

Complementary solution,

$$m^2 + m - 6 = 0$$

$$m^2 + 3m - 2m - 6 = 0$$
$$m(m+3) - 2(m+3) = 0$$
$$(m+3)(m-2) = 0$$

$$m_1 = -3 \quad , \quad m_2 = 2$$

So, Roots are real and distinct

$$y_c = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$y_c = C_1 e^{-3t} + C_2 e^{2t}$$

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for particular solution,

$$y_p = \frac{1}{D^2+D-6} \cdot 10e^{2t}$$

$$y_p = \frac{1}{(2)^2+2-6} \cdot 10e^{2t}$$

$$y_p = \frac{10e^{2t}}{0}$$

y_p = Not possible

for General solution,

$$y = y_c + y_p$$

$$y = C_1e^{-3t} + C_2e^{2t}$$

$$x = e^t \Rightarrow t = \ln x$$

$$y = C_1x^{-3} + C_2x^2.$$

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Question No :- 4

$$x^2y'' + 7xy' + 5y = x^5$$

$$y(0) = 2 \ , \ y'(1) = 2$$

Solution :-

$$x^2y'' + 7xy' + 5y = x^5$$

$$x^2 \frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$x^2 D^2y + 7xDy + 5y = x^5$$

$$(x^2D^2 + 7xD + 5)y = x^5 \quad - \ 1$$

$$x = e^t \Rightarrow t = \ln x$$

$$xD = \Delta$$
$$x^2D^2 = \Delta(\Delta-1)(\Delta^2-\Delta)$$
$$(\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$$

$$(\Delta^2 + 6\Delta + 5)y = e^{5t}$$

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Complementary Solution,

$m^2 + 6m + 5 = 0$

$m^2 + 1m + 5m + 5 = 0$

$m(m+1) + 5(m+1) = 0$

$(m+1)(m+5) = 0$

$m_1 = -1, \quad m_2 = -5$

So, Roots are real and distinct

$y_c = C_1 e^{m_1 t} + C_2 e^{m_2 t}$

$y_c = C_1 e^{-1t} + C_2 e^{-5t}$

particular Integral

$y_p = \dfrac{1}{D^2 + 6D + 5} e^{5t}$

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$$y_p = \frac{1}{5^2+6(5)+5} e^{5t}$$

$$y_p = \frac{1}{60} e^{5t}$$

General solution,

$$y = y_c + y_p$$

$$y = C_1 e^{-t} + C_2 e^{-5t} + \frac{e^{5t}}{60}$$

$$x = e^t$$

$$y = C_1 x^{-1} + C_2 x^{-5} + \frac{x^5}{60}$$

Result :-

so, Required general solution,

$$y = C_1 x^{-1} + C_2 x^{-5} + x^5/60$$

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Question:- 05

$$(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$$

Solution:-

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$(x+1)^2 D^2y - 3(x+1)Dy + 4y = x^2$$

$$((x+1)^2 D^2 - 3(x+1) + 4)y = x^2 \quad - (1)$$

$$x = e^t \quad , \quad t = \ln x$$

$$(x+1)D = \Delta$$
$$(x+1)^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - 1.$$

put in eq (1),

$$(\Delta^2 - \Delta - 3\Delta + 4)y = e^t$$

Complementary solution

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$$m^2 - 4m + 4 = 0$$
$$m^2 - 2m - 2m + 4 = 0$$
$$m(m-2) - 2(m-2) = 0$$
$$(m-2)(m-2) = 0$$
$$m_1 = 2 \;,\; m_2 = 2$$

So, Roots are real and repeated.

$$y_c = (C_1 + C_2 x)^{mx}$$
$$y_c = (C_1 + C_2 x)^{2x}$$

For particular Integral.

$$y_p = \frac{1}{D^2 - 4D + 4} \cdot e^{2t}$$

$$y_p = \frac{1}{(2)^2 - 2(2) + 4} \cdot e^{2t}$$

particular solution is Not possible so,

$$y = y_c + y_p$$
$$y = (C_1 + C_2 x)^{2x}$$

Result :-

Required General solution,

$$y = (C_1 + C_2 x)^{2x}$$