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Subject : DSP.

Paper : Final

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ORTANT

IMPORTANT

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p-1

Q.1 (a)

Determine the response $y(n), n \geq 0$ of the system. Describe second order

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - (n-1)$$

i/p $x(n) = (-1)^n u(n)$

Initial condition: $y(-1) = y((-2) = 0$.

Soln:

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

$$y_h(n) = \cancel{(1)}\; C_1 2^n + C_2 n 2^n$$

The particular solution is.

$$y_p(n) = K(-1)^n u(n).$$

$$K[-1]^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $K(1+4+4) = 2$.

$K = 2/9$ The total solution is

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$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

From the initial conditions we obtain.

$$y(0) = 1, \quad y(1) = 2.$$

Thus

$$C_1 + \frac{2}{9} = 1.$$

$$\boxed{C_1 = \frac{7}{9}}$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2.$$

$$\boxed{C_2 = \frac{1}{3}}$$

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Q 9 (B)

Determine the impulse response and unit step response.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2).$$

Soln:

The characteristic equation

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$\lambda = 1/2, 1/5$, Hence,

$$y_h(n) = C_1 \left(\tfrac{1}{2}\right)^n + C_2 \left(\tfrac{1}{5}\right)^n$$

with $x(n) = \delta(n)$ we have,

$$y(0) = 2.$$

$$y(1) = 1.4.$$

Hence

$$C_1 + C_2 = 2.$$

And,

$$\tfrac{1}{2} C_1 + \tfrac{1}{5} = 1.4.$$

$$1.4 = 7/5.$$

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$$c_1 + \tfrac{2}{5} \cdot c_2 = 14/5$$

These equation yeild.

$$c_1 = \frac{10}{3}, \quad c_2 = -4/3.$$

$$h(n) = \left[\tfrac{10}{3} (1/2)^n - 4/3 \, (1/5)^n \right] u(n)$$

The step response is.

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2} \right)^{n-k} - 4/3 \sum_{k=0}^{n} (1/5)^{n-k}.$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2} \right)^n \sum_{k=0}^{n} 2^k - 4/3 \, (1/5)^n \sum_{k=0}^{n} 5^k$$

$$s(n) = \frac{10}{3} (1/2)^n (2^{n+1} - 1) \, u(n) - 1/3$$

$$(1/5)^n (5^{n+1} - 1) \, u(n).$$

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Q2 (a)

Determine the Causal signal $x(n)$ having the z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Soln.

Taking inverse and z-transform.

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{C\,(z^{-1})}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$.

Hence:

$$x(n) = \{4(2)^n - 3 - n\}\,u(n).$$

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Q #2 B P-6

Determine the Circular Convolution of the following two sequences.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}.$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}.$$

Solu.

Each sequence consists of four nonzero points for the purpose of illustrating the operations involved in circular convolution it is desired to graph each sequence as point on a circle

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with x_1 and x_2 as specified.

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-m)]_N$$

$x_2(-n)_4$ is simply sequence $x_2(n)$.

xpling $x_1(n)$ with $x_2(-n)$ 4 point.

$$x_3(0) = 14.$$

For $m = 1$ we have.

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$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4.$$

it is easily verified that.

$x_2(1-n)_4$. is simply the seq.

$x_2(-n)_4$ rotated counter clockwise by one unit.

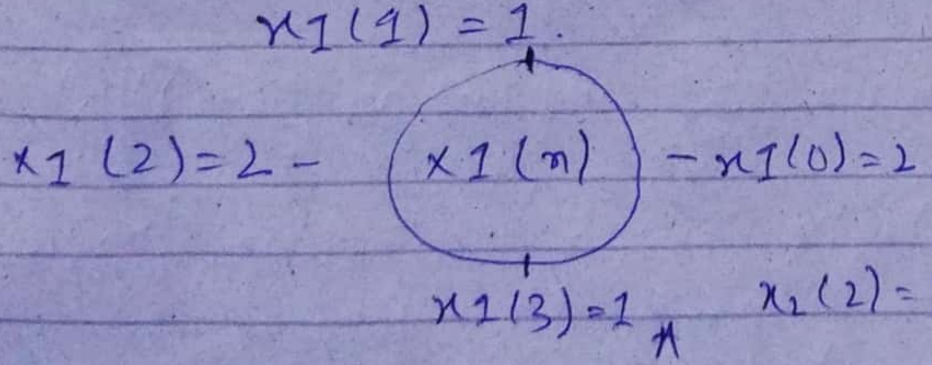
$x_3(1) = 16$.

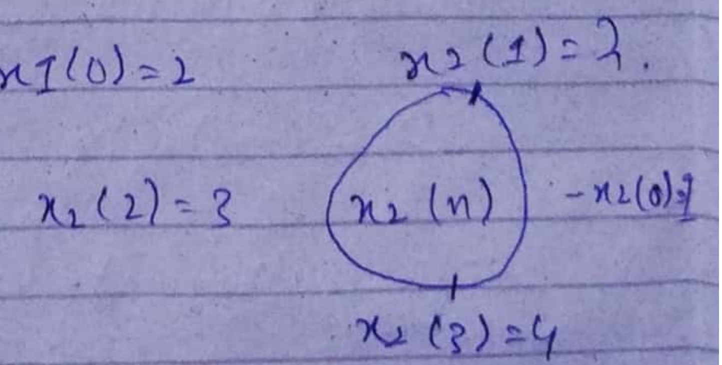
For $m=2$ we have.

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4.$$

Now

$x_2(2-n)_4$ is the folded sequence.





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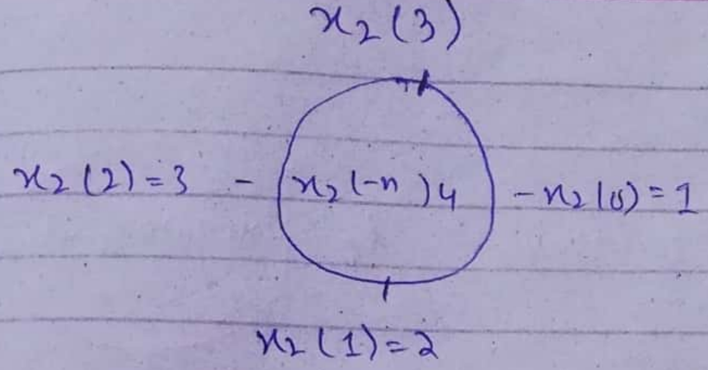
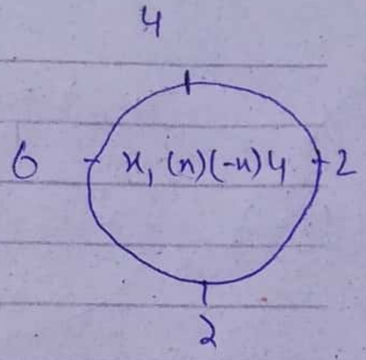
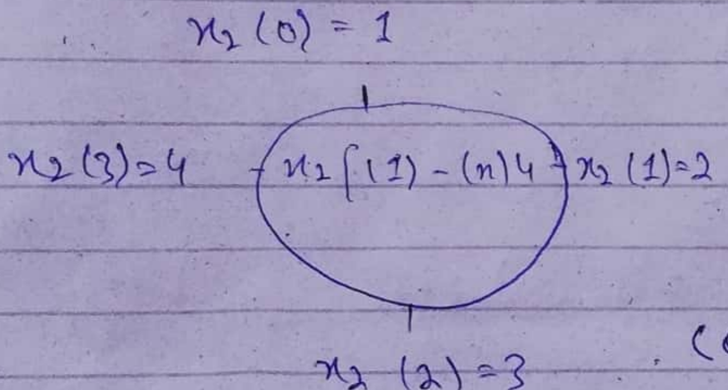
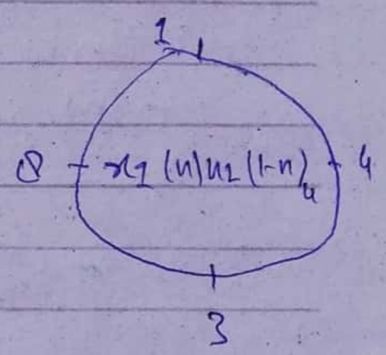




Folded seq. (B) Product seq.

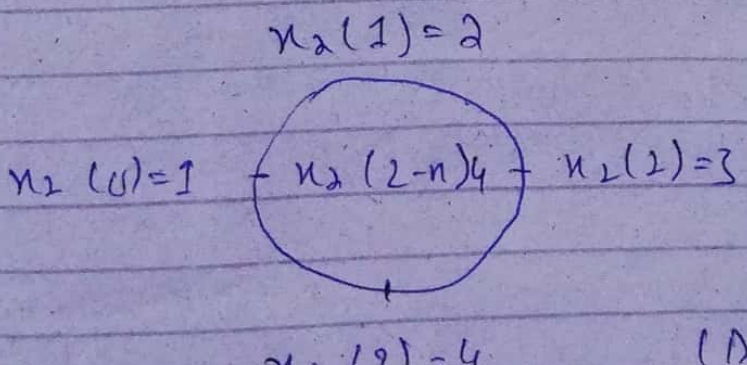


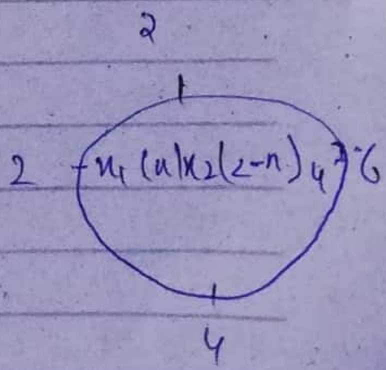


(C)

Product seq.

Folded seq rotated by one unit in time





(D)

Folded seq. rotated by two unit in time

Product seq.

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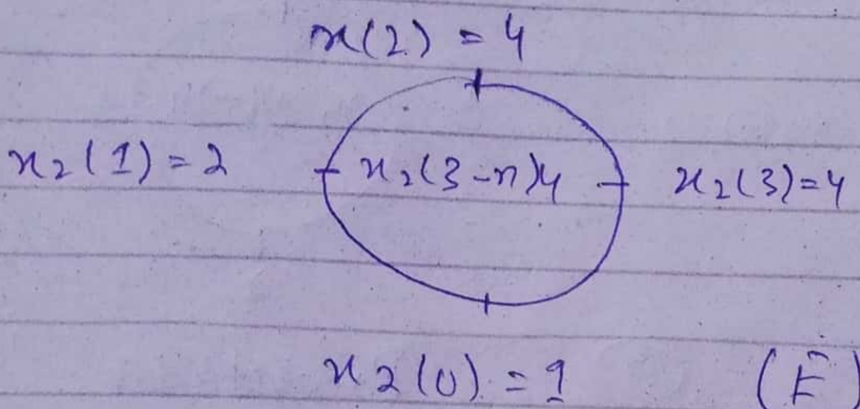
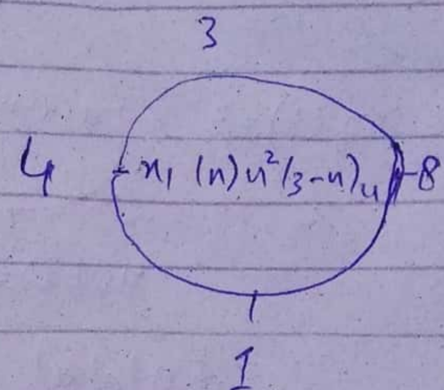


(F)



=) Folded seq. rotated by three unit in time.

Product seq.

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Q#3 (a)

$$H z = \frac{b_0}{(1-p z^{-1})^2}$$

Determine b_0 and p such that $H(w)$ satisfies the conditions:

$$H(0) = 1 \quad \text{and} \quad \left| H(\pi/4) \right|^2 = 1/2$$

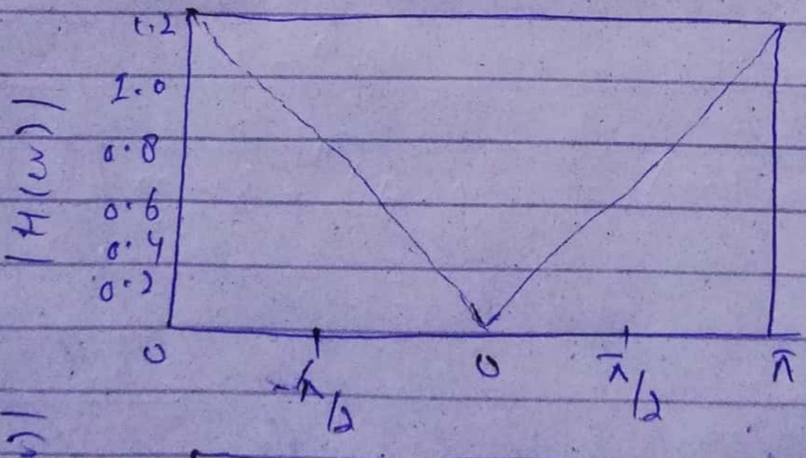
Soln:

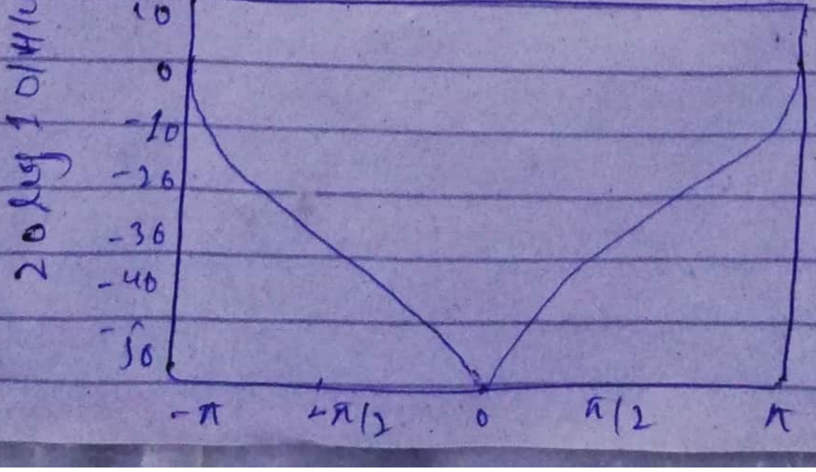
At $w=0$ we have,

$$H(0) = \frac{b_0}{(1-p)^2} = 1.$$

Hence.

$$b_0 = (1-p)^2$$





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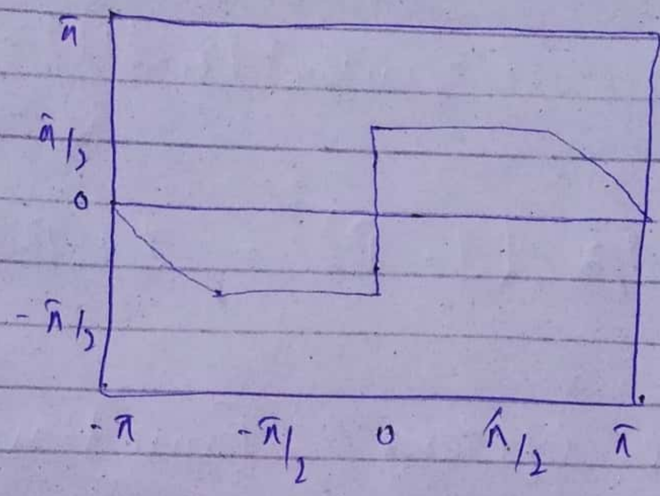

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-p z^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{[1 - p\cos(\pi/4) + jp\sin(\pi/4)^2]}$$

$$\frac{(1-p)^2}{[1 - p/\sqrt{2} + jp/\sqrt{2}]^2}$$

Hence

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2}$$

$$= 1/2$$

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=> Equivallently.

$$\sqrt{2}\cdot(1-P)^2 = 1+P^2-\sqrt{2}P$$

The system Function for desired filter

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q3(B)

Design a two-pole bandpass filter center of its pass band at $\omega = \pi/2$

frequency response at $\omega = 0$ and $\omega = \pi$ magnitude response is $1/\sqrt{2}$ at $\omega = 4\pi/9$.

Solⁿ: The filter must have poles at $P_{12} = re$

And zero at $z = 1$ and $z = -1$

Consequently the same system function is

$$\Rightarrow H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$H(z) = G\frac{z^2 - 1}{z^2 + r^2}$$

$\Rightarrow$ The gain factor is determined evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$.

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$$\Rightarrow H = (\pi/2) = G \frac{2}{1-r^2} = 1.$$

$$G = \frac{1-r^2}{2}.$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$.

we have.

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2.$$

$\Rightarrow$ Equivalently.

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its freq. response is illustrated.

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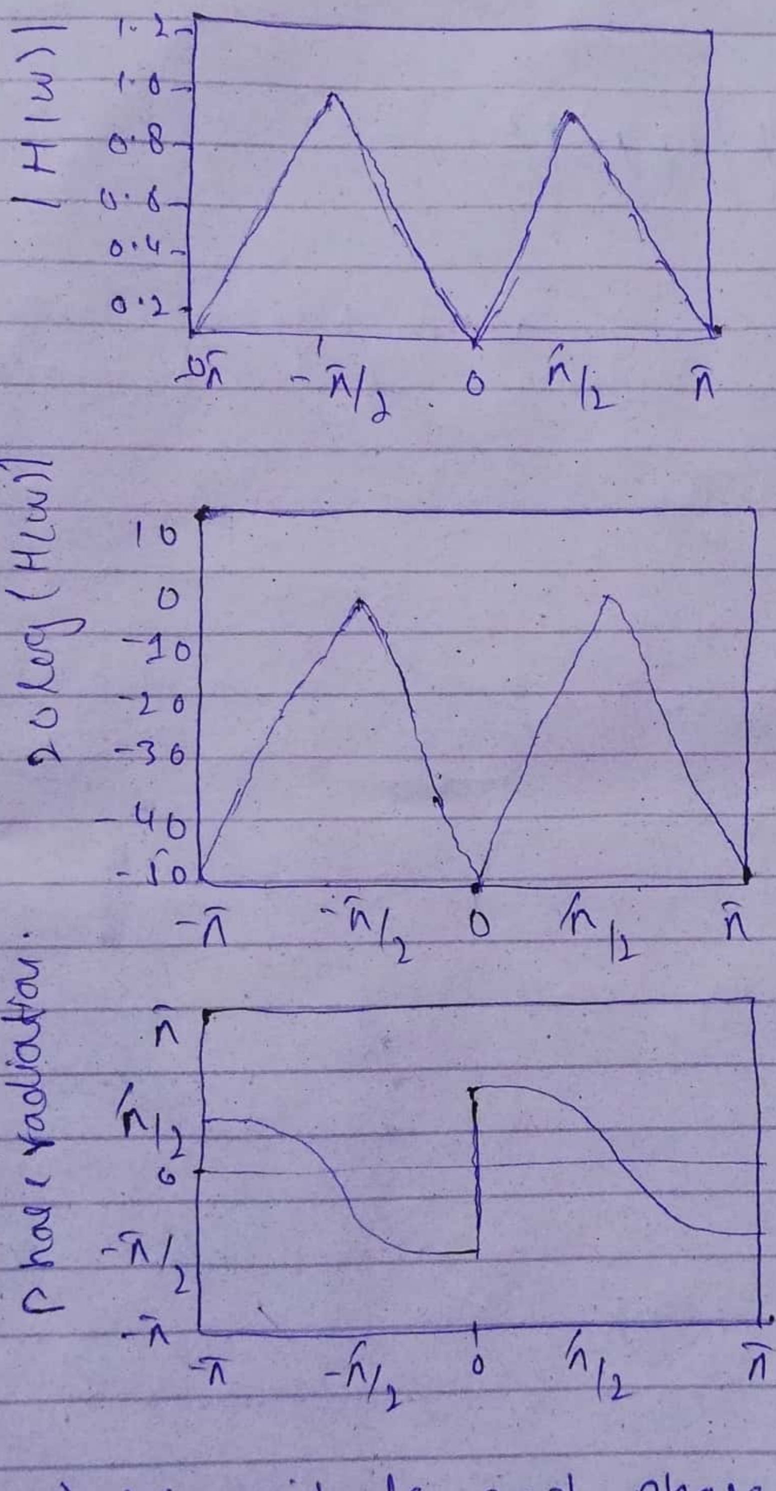

⇒ Magnitude and phase response of a simple bandpass filter is

$$H(z) = 0.15 \left[(1 - z^{-2}) / (1 + 0.7z^{-2}) \right].$$

THANK

YOU