Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	Module:	6th
Instructor:	Peer Mehr Ali Shah	Total Marks:	50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	Marks 7 CLO 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	Marks 7 CLO 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6 CLO 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z \rvert > \lvert a \rvert$$	Marks 6 CLO 2
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\lvert H\left(\frac{\pi}{4}\right)\right\rvert^2 = \frac{1}{2}$.	Marks 6 CLO 3

Q No: 1 (a)

Determine the response $y(n)$, $n \geq 0$ of The System.

Sol: The Characteristics equation is:

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \text{ hence.}$$

$$y_h(n) = c_1 2^n + C_2 n 2^n$$

The particular solution is:

$$y_p(n) = K(-1)^n u(n).$$

Substituting The solution into difference equation we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$; $K(1+4+4) = 2 \Rightarrow K = 2/9$

The Total solution is.

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

From the Initial Condition we obtain.

$$y(-1) = y(-2) = 0$$

$$C_1 + 2/9 = 0$$

$$C_1 = -2/9$$

$$2C_1 + 2C_2 - \frac{2}{9} = 0$$

$$2C_2 = 2/9 - 2C_1$$

$$\boxed{C_1 = -2/9}$$

$$2C_2 = 2/9 - 2(-2/9)$$

$$= 2/9 + 4/9$$

$$\frac{2+4}{9}$$

$$2C_2 = 6/9$$

$$C_2 = \frac{6}{9 \times 2} \Rightarrow 3/9$$

$$\boxed{C_2 = 1/3}$$

X————————X

Q No: 1 (b)

Determine the Impulse response and Unit step response ...

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution: $\lambda^{n} - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$

$$\lambda^{n-2}\left[\lambda^{2} - 0.7\lambda + 0.1\right) = 0$$

$$\cancel{\lambda^2 - 0.6\lambda}$$

$$\lambda^{2} - 0.5\lambda - 0.2\lambda + 0.1 = 0$$

$$\lambda(\lambda - 0.5) - 0.1(\lambda - 0.5) = 0$$

$$(\lambda - 0.5)(\lambda - 0.1) = 0$$

$$\lambda = 0.5 \qquad \lambda = 0.1$$

General form of the solution To be. Homogeneous equation is.

$$y_h(n) = c_1\lambda_1^{n} + c_2\lambda_2^{n}.$$

4

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{2}^n$$

With $(z(n) = \delta(n)$ we have.

$$y(0) = 2.$$

$$y_+(1) - 0.7\, y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence $c_1 + c_2 = 2$ and.

Implicit response:

$$\frac{1}{2} c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$c_1 + \frac{2}{5} c_2 = \frac{14}{5}$$

These equation yield.

$$c_1 = \frac{10}{3} \qquad c_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$$

step response is:

$$s(n) = \sum_{k=0}^{n} h(n-k) \Rightarrow \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-1} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{1}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^n (2^{n+1} - 1)\, u(n) - \frac{1}{3}\left(\frac{1}{5}^n (5^{n+1} - 1\right) u(n)\right.$$

Q NO: 2 (a)

Determine the Calual signal $x(n)$

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol: $\frac{X(z)}{z} = \frac{z^2}{(2z-1)(z-1)^2}$

$$\frac{X(z)}{z} = \frac{A_1}{2z-1} + \frac{B_1}{z-1} + \frac{C}{(z-1)^2}$$

Find A, B, and C.

$A = 4$

$B = -3$

$C = -1$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$

Continue -

$$X(z) = \frac{1}{4}\frac{1}{(1-z^{-1})} + \frac{3}{4}\frac{1}{1-z^{-1}} + \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

By Applying The Inverse z-Transform.

$$x(n) = \frac{1}{4}(-1)^n u(n) - \frac{3}{4}u(n) - \frac{1}{2}n u(n) =$$

$$\left[\frac{1}{4}(-1)^n - \frac{3}{4} + \frac{n}{2}\right]u(n)$$

x ——————— x

QNo: 2(b)

Evaluate the Inverse - z - Transform

$$X(z) = \frac{1}{(1-a\bcancel{z^{-1}})(\ \bcancel{1-z^{-1}}\ )} \quad \bcancel{>}|z| > (a)$$

Solution: We have

$$x(n) = \frac{1}{2\pi j} \oint \frac{z^{n-1}}{C\, 1 - az^{-1}} dz.$$

$$= \frac{1}{2\pi j} \oint \frac{z^n dz.}{C\, z - a}$$

where C is a circle at radius greater than $|a|$. We shall evaluate This Integral.

(1) If $n \geq 0$ $f(z)$ has only zeros and hence no Poles Inside C. The only Pole Inside C is $z = a$ Hence.

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

If $n < 0$ $f(z) = z^{n}$ has an nth-order Pole at $z=0$ which is also Inside C. Thus there are Contribution from both boles. For $n=-1$ we have.

$$X(-1) = \frac{1}{2\pi j} \oint \frac{1}{c z^{2}(z-a)} dz.$$

$$\frac{d}{dz}\left(\frac{1}{z-a}\right)\Bigg|_{z=0} + \frac{1}{z^{2}}\Bigg|_{z=a} = 0$$

By continuting in the same way we can show that $X(n)=0$ for $n<0$ Thus.

$$X(n) = a^{n} u(n).$$

Q No: 3 (a)

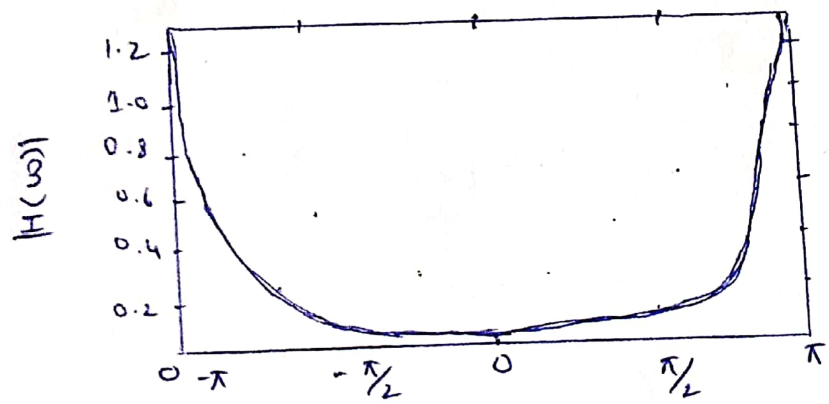
A Two-pole low pass has the system response ----- $H(z) = \frac{b_0}{(1 - pz^{-1})^2}$ ----

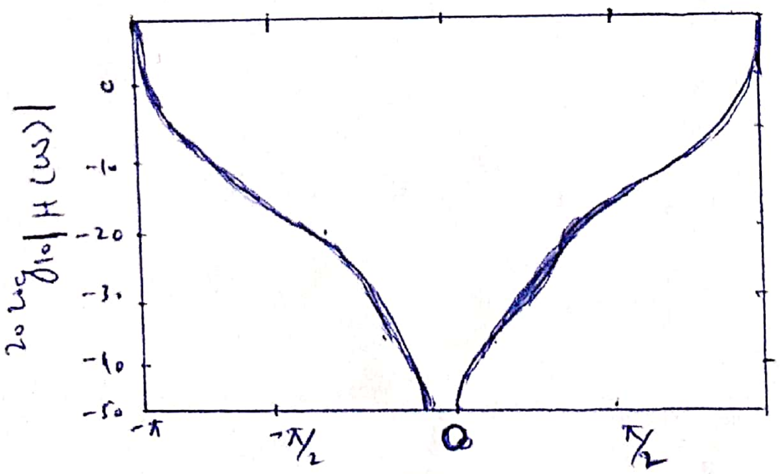
Sol:

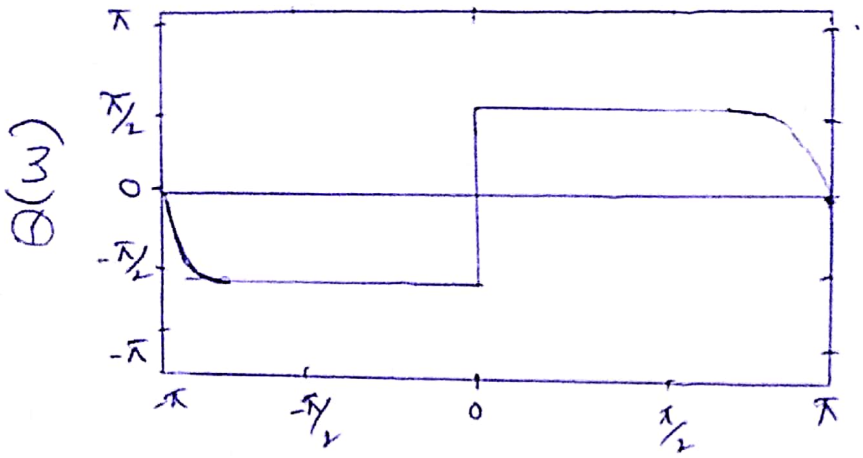
At $w = 0$ we have.

$$H_0(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-p e^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + j p\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + j p/\sqrt{2})^2}$$

Hence $$= \frac{(1-p)^4}{\left[(1-p/\sqrt{2})^2 + p^2/2\right]^2} = \frac{1}{2}$$

or equivalently:

$$\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}\,p$$

The value of $p = 0.32$ satisfy this equation. The system f/n for the desired filter is.

$$H(z) = \frac{0.46}{(1 - 0.32 z^{-1})^2}$$

The same principle can be applied for design of ~~low~~ band pass filter.

X ———————— X

QNo: 3 (b)

Design a Two-Pole bandpass filter...

Solution: Clearly, The filter Must have Poles at $P_{1,2} = re^{\pm j\pi/2}$ and zeros at $z=1$ and $z=-1$ Consequently, The system fun is:

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The Gain factor is determined by evaluating The frequency response $H(\omega)$ of The filter at $\omega = \pi/2$. Thus we have.

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-\gamma^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+\gamma^4+2\gamma^2\cos(8\pi/9)}$$

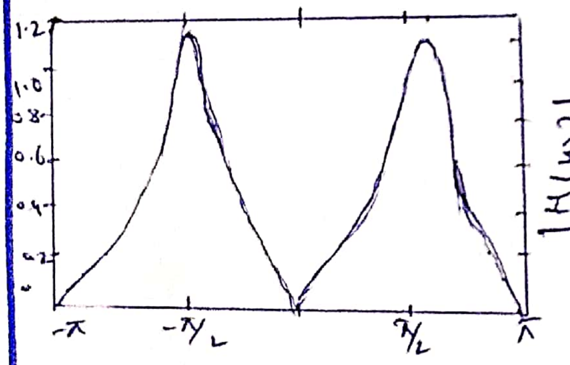
$$= \frac{1}{2}$$

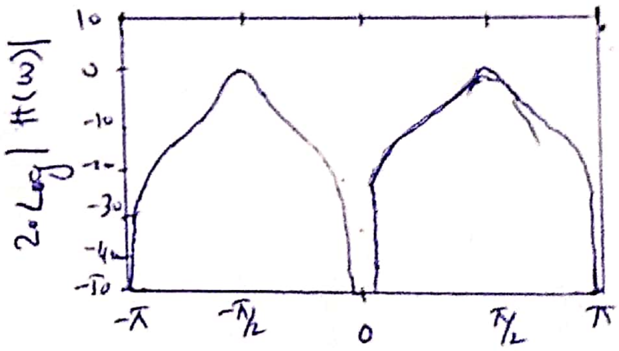
or $1.94(1-\gamma^2)^2 = 1 - 1.88\gamma^2 + \gamma^4$

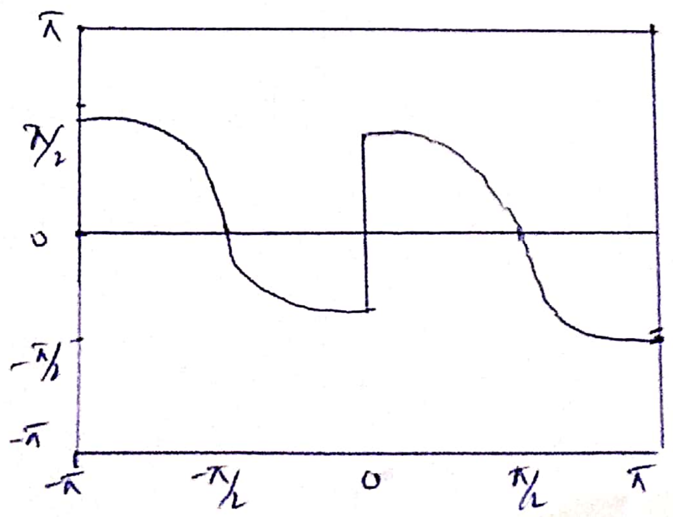
The value of $\gamma^2 = 0.7$ satisfy this equation,

The system f/n for the designed filter is.

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$







X ——— X

Q No: 4 (a)

A finite duration Sequence of Length L is given as:

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine N point DFT for this sequence for $N \ge L$.

Sol: The fourier Transform of the sequence is:

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \Rightarrow$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The Magnitude and Phase of $X(\omega)$ are illustrated in figure 1. for $L = 10$. The N-point DFT of $x(n)$ is simply $X(\omega)$

evaluated at the set of N equally spaced frequencies $w_k = 2\pi k/N$ $k = 0, 1, 2 \ldots N-1$

Hence :

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \qquad k = 0, 1 \ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

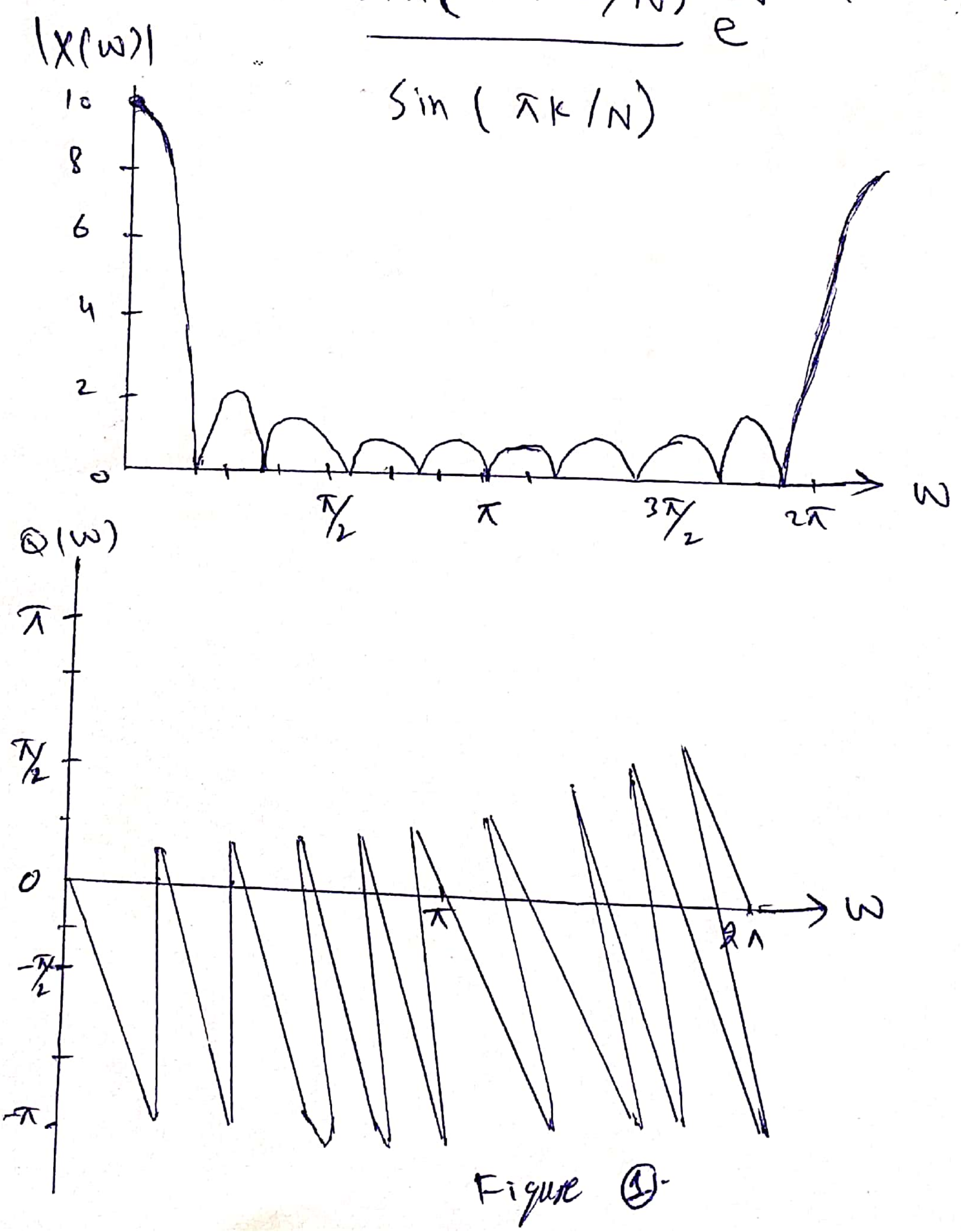


Figure (1).

If N is selected such that N = L

Then DFT becomes.

$$X(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, 3 \ldots L-1 \end{cases}$$

Thus there is only one non zero value in the DFT. This is apparent from observation of $X(w)$ since $X(w) = 0$ at frequencies $w_k = 2\pi k/L$ $k \neq 0$. The reader should verify that $x(n)$ can be recoverd from $X(k)$ by performing an L-Point IDFT.

X ———————— X

QNo: 4 (b)

Perform The circular Convolution of the following Two sequencies.....

$$X_1(n) = (\underset{\uparrow}{2}, 1, 2, 1)$$

$$X_2(n) = (\underset{\uparrow}{1}, 2, 3, 4)$$

Solution: Each sequence consists of four non zero points. for The purposes of illustrating the operation involved in Circular Convolution, it is desirable To graph each sequence as points on a circle. Thus The seqeces $X_1(n)$ and $X_2(n)$ are graphed as illustrated in figure. We NoTe that The sequences are graphed in a Counter clock wise direction on a Circle. Now $X_3(m)$ is obtained by ciralarly convolving $X_1(n)$ and $X_2(n)$ as specified by. begining with $m=0$ we have -

$m = 0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle illustrted in Fig. The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$. Point by point. This sequence is illustrated in Fig. -- . Finally we sume the values in the product sequence to obtain.

$$x_3(0) = 14.$$

for $M = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((\cancel{1}-n))_4.$$

It is easily verified that $x_2(1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counterclockwisie by one unit in time as illustrated. This rotated sequence multiplies.

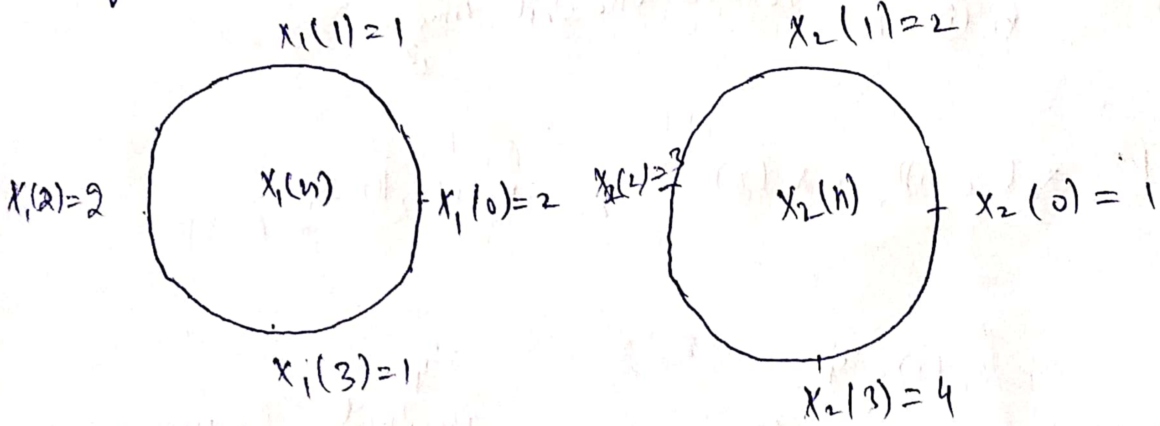
$x_1(n)$ To yield The product Sequence.

$$X_3(1) = 16$$

$m = 2$. we have

$$X_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4.$$

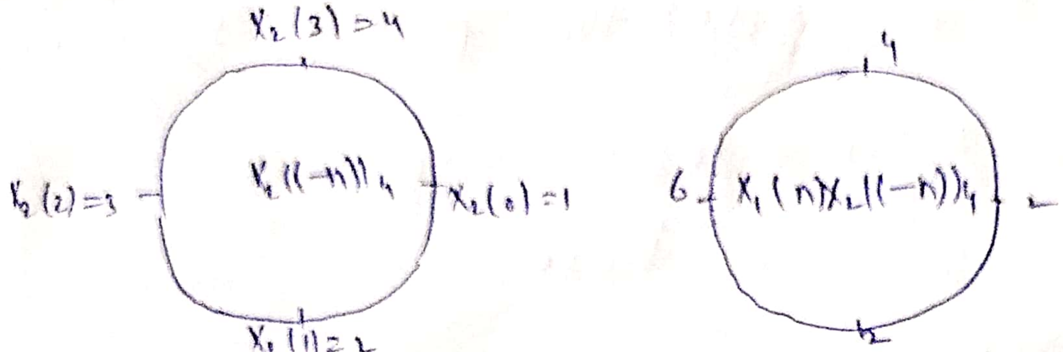
Now $x_2((2-n))_4$ is the folded Sequence in Fig. rotated Two Units of time in The counterclockwise direction. The resultant sequence is illustrated in Figure _



Folded sequence

Product sequence.

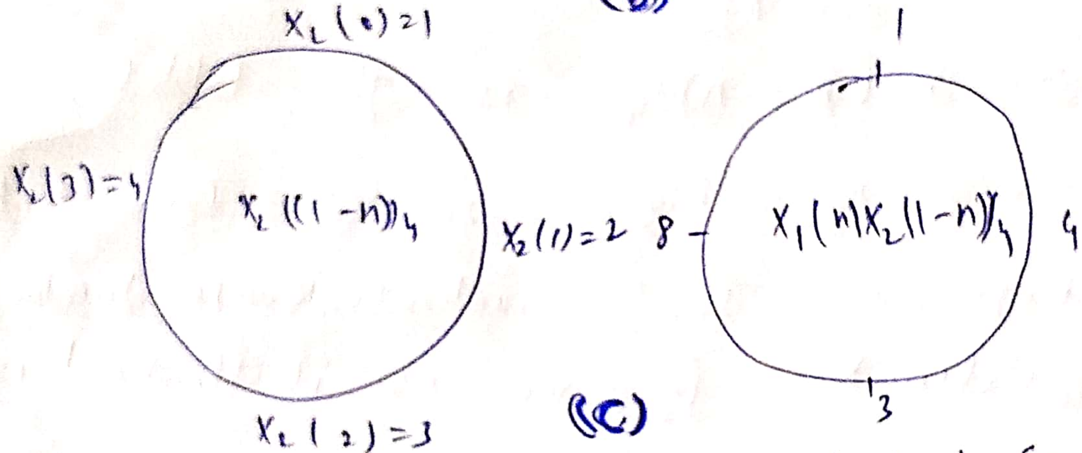
(a)



Folded Sequence

Product sequee.

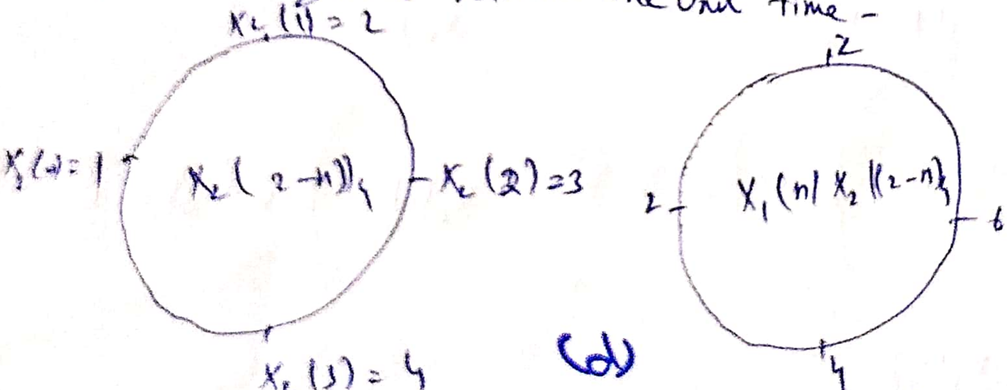
(b)



(c)

Product Sequence

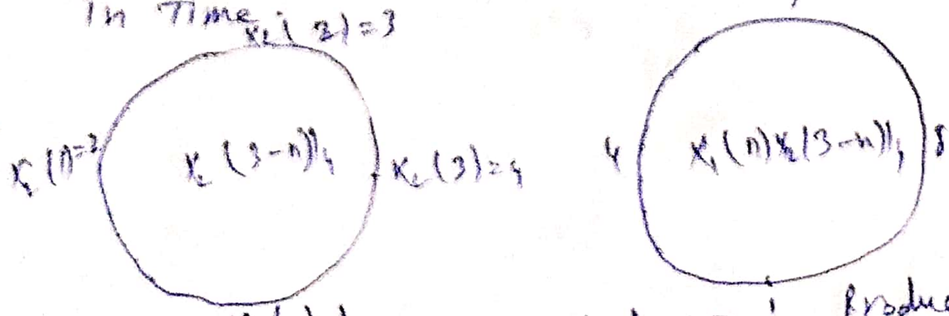
Folded Sequence rotated by One Unit time.



(d)

Folded Sequence by Two Unit in Time.

Product Sequence



Product Sequence.

Folded Sequence rotated by 3 Units in Time.

Along with product sequence $x_1(n)\,x_2(2-n)_4$

By summing the four terms in the product sequence.

$$x_3(2) = 14$$

for $M=3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\,x_2(3-n)_4 .$$

$$x_3(3) = 16 .$$

$$x_3(n) = (\underset{\uparrow}{14}, 16, 14, 16)$$

The end.