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Subject: Numerical Analysis

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Q1:=> (a)

Solution:

=x=x=

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

This matrix is the matrix of co-efficient of system

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -3 & 1 & 1 \end{bmatrix}$$

Subtract 2 x row 1 from row 2

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ -3 & 1 & 1 \end{bmatrix}$$

Subtract -3 x row 1 from row 3

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & 7 & -2 \end{bmatrix}$$

Subtract $\frac{-7}{3}$ x row 2 from row 3

$$U = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

The lower trangular L matrix is formed

Q1 iii
(b)

**Solution:**

According to Given Condition

$$x + 2y - z = 3$$
$$2x + y - 2z = 3$$
$$-3x + y + z = -6$$

This is written in Tableau from as

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 1 & -2 & 3 \\ -3 & 1 & 1 & -6 \end{array}\right]$$

Two step are needed to eliminate coloumn 1

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 1 & 2 & 3 \\ -3 & 1 & 1 & 6 \end{array}\right] \rightarrow$$ subtract $2 \times$ row 1 from row 2

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -3 & 0 & -3 \\ 0 & 7 & -2 & -4 \end{array}\right]$$

Now elimate coloumn 2.

Subtract $-\frac{7}{3} \times$ row 2 from row 3

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -3 & 0 & -3 \\ 0 & 7 & -2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

Returning to the equation

$$x + 2y - z = 3$$
$$-3y = -3$$
$$-2z = -4$$

We can solve for the variables

$$x = 3 - 2y + z$$
$$-3y = -3$$
$$-2z = -4$$

$$x = 3, \quad y = 1, \quad z = 2$$

Q1 (c)

Solution:

Tableaus form is

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & -3 \\ -1 & 0 & -2 & 1 \\ 2 & 2 & 4 & 0 \end{array}\right]$$

under partial pivoting we compare

$|a_{11}| = 1$ with $|a_{21}| = 1$ & $|a_{31}| = 2$

and choose $a_{31}$ for the new pivot.

exchange rows 1 and 3

$$\left[\begin{array}{cc|cc} 1 & -1 & -3 & -3 \\ -1 & 0 & 1 & 1 \\ 2 & 2 & 0 & 6 \end{array}\right]$$

Exchange row 1 and row 3

$$\begin{bmatrix} 2 & 2 & 4 & 1 & 0 \\ -1 & 6 & -2 & 1 & 1 \\ 1 & -1 & 3 & 1 & -3 \end{bmatrix}$$

Subtract $\frac{-1}{2}$ x row 1 from row 2

$$\begin{bmatrix} 2 & 2 & 4 & 1 & 0 \\ 6 & 1 & 0 & 1 & 1 \\ 1 & -1 & 3 & 1 & -3 \end{bmatrix}$$

Subtract $\frac{1}{2}$ x row 1 from row 3

$$\begin{bmatrix} 2 & 2 & 4 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & -2 & 1 & 1 & -3 \end{bmatrix}$$

before eliminating coloumn 2 we must compare the current $|a_{22}|$ with the current $|a_{32}|$

$$\begin{bmatrix} 2 & 2 & 4 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & -2 & 1 & 1 & -3 \end{bmatrix}$$

Exchange row 2 from & row 3

$$\begin{bmatrix} 2 & 2 & 4 & 1 & 0 \\ 0 & -2 & 1 & 1 & -3 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

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Subtract $-\frac{1}{2} \times$ row 2 from row 3 → (5)

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Note all the three multipliers are less than 1 in absolute value

$$\frac{1}{2} x_3 = -\frac{1}{2}$$

$$-2x_2 + x_3 = -3$$

$$2x_1 + 2x_2 + 4x_3 = 0$$

We find that $x = [1, 1, -1]$

Q 2 (a)

Solution:

Given condition

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

The Gauss-Seidel iteration is

$$u_{k+1} = \frac{4 - v_k + w_k}{3}$$

$$v_{k+1} = \frac{1 - 2u_{k+1} - w_k}{4}$$

$$w_{k+1} = \frac{1 + u_{k+1} - 2v_{k+1}}{5}$$

Starting with $X_0 = [u_0, v_0, w_0] = [0, 0, 0]$ we calculate

$$\begin{bmatrix} u_1 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{4-0-0}{3} = 4/3 \\ \frac{1-8/3-0}{4} = -5/12 \\ \frac{1+4/3+5/6}{5} = \frac{19}{30} \end{bmatrix} = \begin{bmatrix} 1.3333 \\ -0.4167 \\ 0.6333 \end{bmatrix}$$

&

$$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{101}{60} \\ -3/4 \\ \frac{251}{300} \end{bmatrix} = \begin{bmatrix} 1.6833 \\ -0.7500 \\ 0.8367 \end{bmatrix}$$

The system is strictly diagonolly dominat and therefore the iteration will coverge of the solution

$[2, -1, 1]$

proved

Q2(b)



Solution:

Set $y_1 = A_1 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ Then $r_{11} =$

$\|y_1\|_2 = \sqrt{1^2+2^2+3^2} = 3$

and the first unit vector is

$$q_1 = \frac{y_1}{\|y_1\|_2} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

to find the second unit vector set

$$y_2 = A_2 - q_1 q_2^T A_2 = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix} 2 = \begin{bmatrix} -14/3 \\ 1/3 \\ 2/15 \end{bmatrix}$$

&

$$q_2 = \frac{y_2}{\|y_2\|_2} = 1/5 \begin{bmatrix} \frac{-14}{5} \\ 5/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} -14/15 \\ 1/3 \\ 2/15 \end{bmatrix}$$

Since

$r_{12} = q_1^T A_2$ & $r_{22} = \|y_2\| = 5$

Result in Matrix form.

$$A = \begin{bmatrix} 1 & -11 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1/3 & -11/15 \\ 2/3 & 1/3 \\ 2/3 & 2/5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} = QR$$

proved

Q2(c)



Solution:

Set
$$V = W - x = \begin{bmatrix} 5 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

& define the projection in matrix

$$P = \frac{VV^T}{V^TV} = \frac{1}{20}\begin{bmatrix} 4 & -8 \\ -8 & 16 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.4 \\ -0.4 & 0.8 \end{bmatrix}$$

Then
$$H = I - 2P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & -0.8 \\ -0.8 & 1.6 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

Check That H moves x to W

& vice versa

$$Hx = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} = x$$

proved

Q3(a)

Solution:

According to given condition first we Take the derivate

$f'(x) = 3x^2+1$

using formula

$$x_{i+1} = x_i - \frac{x_i^3 + x_i - 1}{3x_i^2+1}$$

$$\Rightarrow \frac{2x_i^3+1}{3x_i^2+1}$$

Iterating the formula from Intal Guess $x_0 = 0.7$ Yield

$$x_1 \Rightarrow \frac{2x_0^3+1}{3x_0^2+1}$$

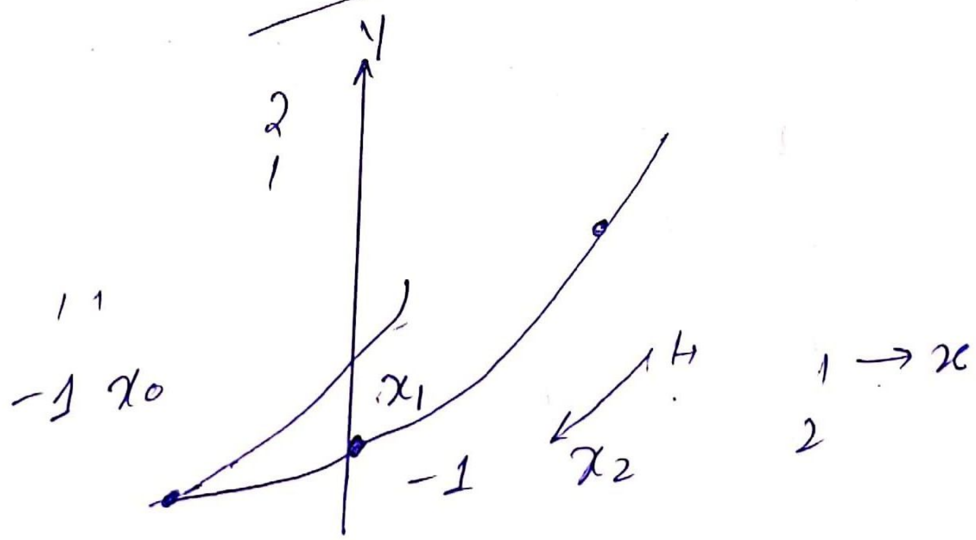
$$\Rightarrow \frac{2(0.7)^3+1}{3(-0.7)^2+1}$$

$$= 0.1271$$

$$x_2 = \frac{2x_1^3+1}{3x^2+1} \Rightarrow 09577$$

| $i$ | $x_i$ | $\varepsilon_i = (x_i - r)$ | $e_i / e^2 - 1$ |
|---|---|---|---|
| 0 | -0.700000 | 13823780 | |
| 1 | 0.1271251 | 0.55520230 | 0.2906 |
| 2 | 0.957678 | 0.275350 | 08933 |
| 3 | 0.7348277 | 0.052499 | 0.6924 |
| 4 | 0.68459 | 0.0026397 | 0.8214 |
| 5 | 0.682332 | 0.0000437 | 0.8527 |
| 6 | 0682321 | 0.0000000 | 0.8541 |
| 7 | 0.682327 | 0.0000000 | |

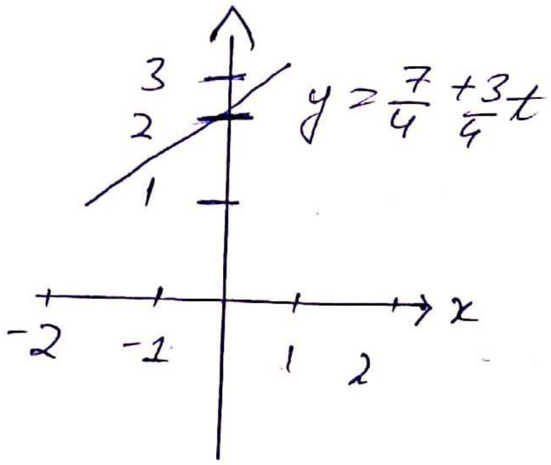
## Newton Method



## Q3 (b)



## Solution:-




The model is $y = c_1 + c_2 t$ and the goal to find best $c_1$ & $c_2$ Substitution of data paints into the model Yieds.

$$c_1 + c_2(1) = 2$$
$$c_1 + c_2(-1) = 1$$
$$c_1 + c_2(1) = 3$$

Matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

(PTO)

The System has no solution $(c_1 . c_2]$ for two Sp sates season, The points are not collinear The equation are in Constant. so the best Solution is

$$(c_1, c_2] = [\tfrac{7}{4}, 3/4) \text{ Therefore}$$

$$(c_1, c_2) = [\tfrac{7}{4}, 3/4). \text{ Therefore}$$

The obest line is

$$y = 7/4 + 3/4 t$$

proved

The End