Department of Electrical Engineering
Assignment
Date:
13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

Name:	**Syed M Zahoor**	**Student ID:**	**12595**

Q1.	(a)	Consider the following analog signal $□_□(□) = 3□□□100□□ + 4□□□200□□$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $□_□ = 100□□$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal □(□) we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $□(□) = \begin{cases} 0.5^{□}, & □ \geq 0 \\ 0, & □ < 0 \end{cases}$ This is signal is sampled at the rate $□_□ = 2□□$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $□[□] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$, $h[□] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $x(n) = \begin{cases} \square^{n+1}, & -3 \le n \le 5 \\ 0, & \text{otherwise} \end{cases}$ $h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} \left(\frac{1}{4}\right)^{n}, & n \ge 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} \left(\frac{1}{2}\right)^{n} - 3^{n}, & n \ge 0 \\ 0, & \text{otherwise} \end{cases}$	Marks 10 CLO 2

Digital Signal Processing

Name : Syed. M. Zahoor

ID :- 12595

(a) Q(1) Consider The following analog Signal

$$X_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine The minimum Sampling rate required to avoid aliasing.

Ans

According to Sampling Therem

$f_1 = 100Hz$, $F_2 = 200Hz$

$f_s \geq f_{max}$ $\qquad f = \frac{\omega}{2\pi}$

So

f_2 is max (greater than f_1)

$f_s > 2 \times 100$

$f_s = 200Hz$

(ii) Suppose That The Signal is Sampled at The rate $f_s = 100Hz$ what is the discrete-time signal obtained after Sampling? Also ~~Example~~ Explain The effect of This Sampling rate on The newly generated discrete time signal.

Solution :

$f_s = 100Hz$

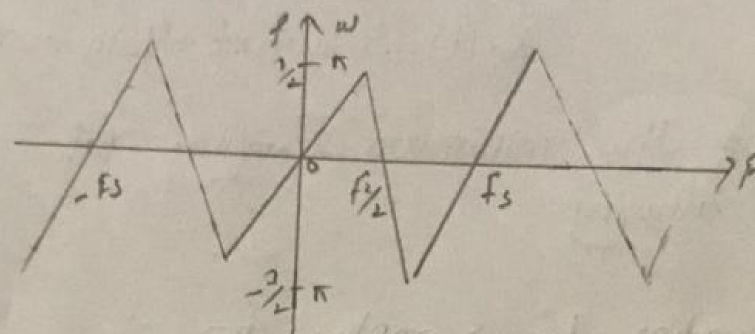
$f = \frac{100}{2} = 50Hz$

This is The max Frequency that can be represented uniquely by The Sampled signal

As

$$x_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



The effect of Sampling rate on the newly generated discrete time signal is that

There will be no disloy phenomenon mean There will not present unwanted component in the reconstructor of The signals. The recostruit orignal signals

(iii) what is The analog Signal $y_a(t)$ we can reconstruct from The Samples if we use ideal Interpolation?

Ans:

folding frequency $= \frac{f_s}{2} = \frac{100}{2}$

$= 50HZ$

$f_1 = 50Hz$, $f_2 = 100HZ$

Both frequency are either equal or greater The folding frequency

Hence for ideal Interpolation we can construct The original signal.

$$X_a(t) = 3 \cos 100\pi + 4 \sin 200\pi t$$

Since only The frequency compounts at 100Hz are present on The Sampled signals The analog signal we can remove or reconstruct is

$$y_a(t) = 3 \cos 100\pi t \quad \text{Ans.}$$

(b) consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n & n \geq 0 \\ 0, & n < 0 \end{cases}$$

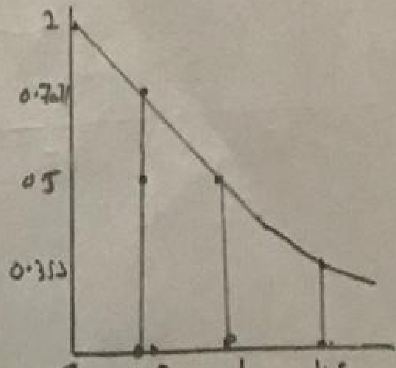
This is signal is sampled at the rate $F_s = 2 HZ$.

(i) Draw The Sampled Signal.

$$f_s = \frac{1}{T} = T = 1/f_s$$

$$= 1/2 = 0.5 \text{ sec}$$

X_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



(ii) The Samples of The Signals are Intended to carry 3 bits per Sample Determine The quantization level and quantization resolution to quantized The sampled Signal achieved in parti:

Ans

$L = 2^n$

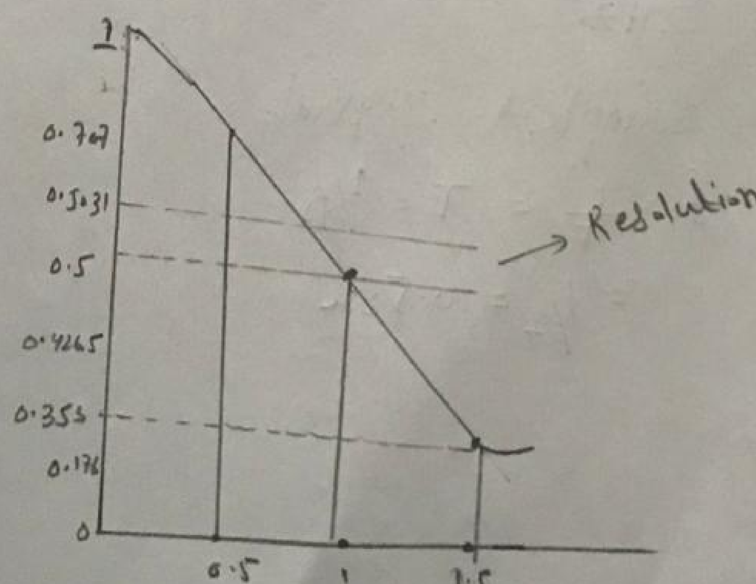
$n = bits = 3$

$L = 2^3 =$ 8 Levels

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1 - 0}{8}$$

$$= 0.125$$



(iii)

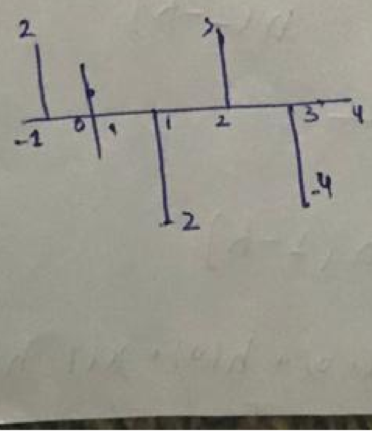
	Dist signal	injection	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.1	-0.1

Q2 (a) Determin The response of The system to The following Input signal with given Impulse response.

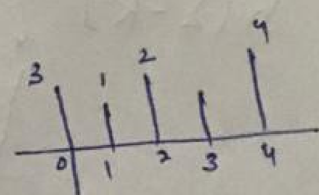
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution:-

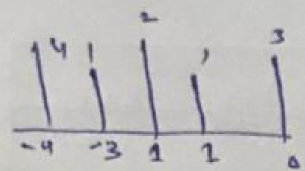
$$y[n] = \sum_{1 \infty \infty}^{\infty} x[k]\, h(n-k)$$



h(k)



h(-k] = folded Signal



h[-k]

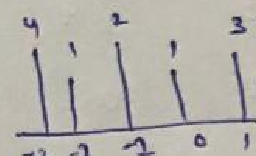
$$y[0] = \sum_{k=1}^{0} x(-1] h(-1) + 1(0)(h(0)$$

= 2X1 + (1)(3)

= 5

for n = 1

h(1-k]



$$y[1] = \sum_{k=-1}^{1} x[n][h(1-k]$$

$= x(-1)h(-1] + x(0)h(0) + x(1)h(1)$

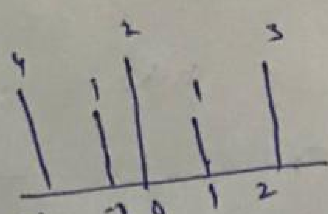
$\Rightarrow (2)(2) + (1)(1) + (3)(-2)$

= 4 + 1 - 6

= -1

n = 2

h(2-k]



h(2-k)

$$y(2) = \sum_{k=1}^{3} x(n] h(2-k]$$

$x(-1)h(-1) + x(0) + h(0) + x(1)h(1)$

(7)

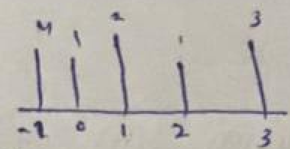
$x(-1)h(-1) + x(0) + h(0) + x(1)(1)$

$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$

$= 2 + 2 - 2 + 9$

$= 11$

$n = 3$



$$y(3) = \sum_{k=-1}^{3} x(n)\, h(3-k]$$

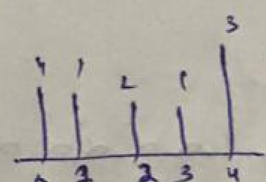
$\Rightarrow x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(x)(h(2)$

$\Rightarrow 2\times4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 4 + 1 - 4 + 3 - 12$

$= -8$

$n = 4$



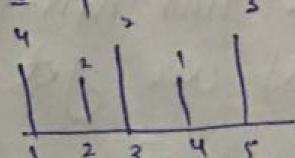
$$f(4) = \sum_{k=0}^{4} x(n)\, h(4-k]$$

$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

$= 4 - 2 + 6 - 4$

$= 4$

$n = 5$



$$y(5) = \sum_{k=1}^{5} x(n)\, h(5-k]$$

$x(1)h(1) + x(2)h(2) + x(3)h(3)$

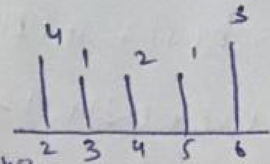
$= (-2)(4) + 3(1) + (-4)(2)$

$= -8 + 3 - 8$

$= 13$

$\underline{n = 6}$

⑧



$$y(6) = \sum_{k=3}^{k=3} x(2)\,h(2) + x(3)\,h(3)$$

$$= (3)(4) + (1)(-4)$$

$$= 8$$

(b) Compute the convolution $y(n)$ of the following Signal.

$$x(n) = \begin{cases} a^{n+1} & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Solution:

we have

$$x(n) = x(k) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}, 0, a \ldots\}$$

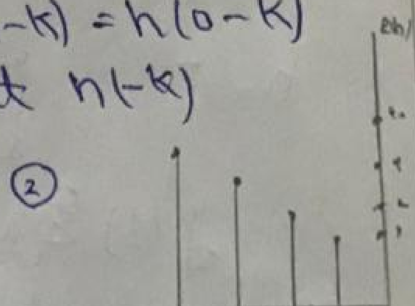
$$h(n) = h(K) = \{\ldots 0, 1, 2, 4, 8, 16, 0 \ldots\}$$

To find $y(n)$:

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K)$$

for $n=0$ first to find $h(n-K) = h(0-K)$ so by inverting $h(k)$ we get $h(-k)$

$$\Rightarrow h(-K) = \{16, 8, 4, 2, \underset{\uparrow}{1}\} \quad \text{— ②}$$



So $$y(0) = \sum_{K=-\infty}^{\infty} x(K) \times h(-K)$$

$$y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times 4) + (1 \times 2) + (\alpha \times 1)$$

$$y(0) = 8\alpha^{-2} + 4\alpha^{-1} + \alpha + 2$$

$$= a - 2 + 4\alpha^{2} + 4\alpha^{-2}$$

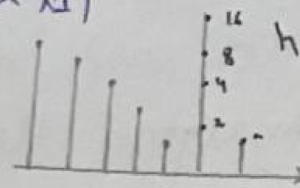
(9)

for $n = 1$ - $h(1-k) = \{16, 8, 4, 2, 1\}$

So $y(1)$ $(\alpha^{-2} \times 16) + (\alpha^{-2} \times 8) + (1 * 4) + (\alpha \times 2) + (\alpha^2 \times 1)$

$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^2$

$= \alpha^2 + 2a + 4 + 8\alpha^{-1} + 16\alpha^{-2}$

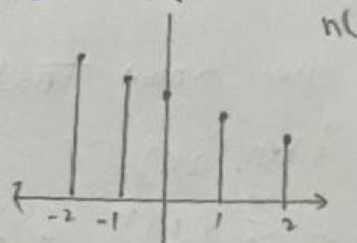


now for $n = 2$

$h(2-k) = \{16\ 8, 4, 2, 1\}$

$y(2) = \{(\alpha^{-1} \times 16) + (\overset{\uparrow}{1} \times 8) + (\alpha \times 4) + (\alpha^2 \times 2) + (\alpha^3 \times 1)$

$= 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$

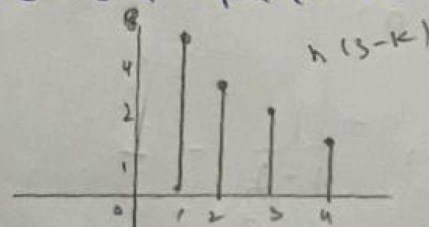


Similarly for $n = 3$

$h(3-k) = \{16, 8, 4, 2, 1\}$

$y(3) = (1 \times 16)\ (\alpha \times 8) + (\alpha^2 + 4) + (\alpha^3 \times 2) + (\alpha^4 \times 1)$

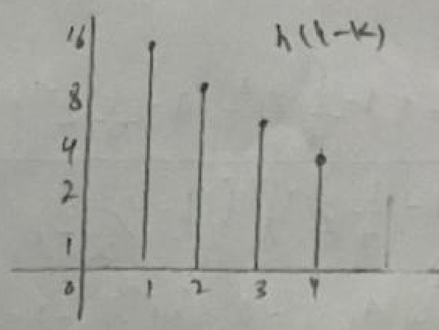
$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$



Now $h'(4-k) = \{16, 8, 4, 2, 1\}$

$y(4) = (\overset{\uparrow}{\alpha} \times 16) + (\alpha^2 \times 8) + (\alpha^3 + 4) + (\alpha^4 + 2) + (\alpha^5 + 1)$

$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$

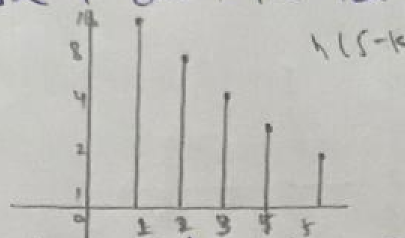


8(9)(b)

$\Rightarrow h(5-k) = \{0, 16, 8, 4, 2, 1\}$

$y(5) = (\alpha \times 0) + (\alpha^2 \times 16) + (\alpha^3 \times 8) + (\alpha^4 \times 4) + (\alpha^5 \times 2) + (\alpha^6 \times 1)$

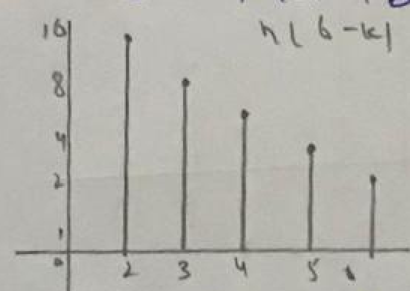
$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$



Similarly if we calculate for rest of the values of n up till there are any common values we get.

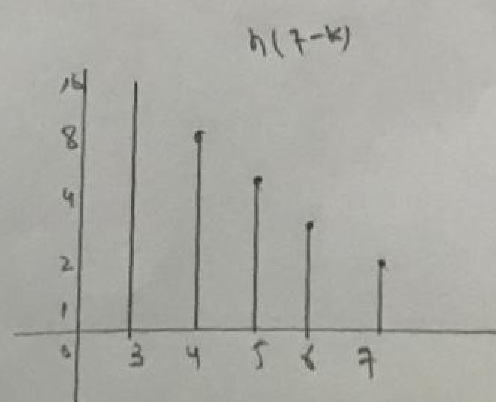
$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha 6$

$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$



$y(7) = 0 + 0 + 0 + 16\alpha^4 + 8\alpha^5 + 4\alpha^6$

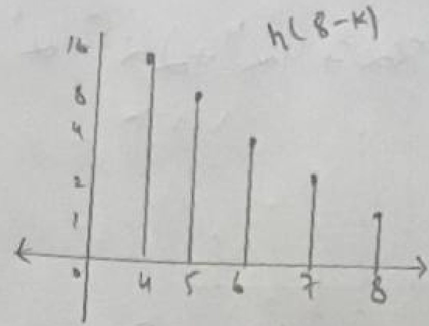
$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$



(9) ©

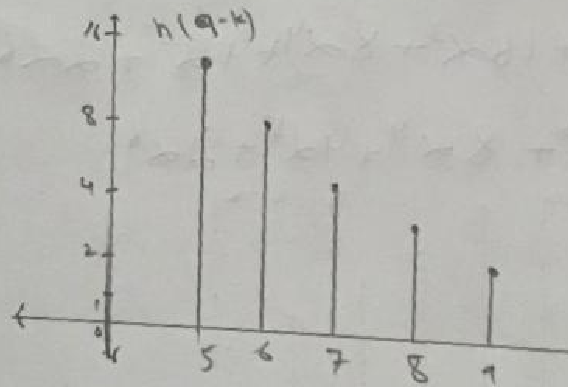
$y(8) = 0+0+0+0+16\alpha^5+8\alpha^6$

$\Rightarrow 16\alpha^5+8\alpha^6$



$y(9) = 0+0+0+0+0+16\alpha^6$

$= 16\alpha^6$



Q5 Determine The Z-transform of The following Signal and also sketch its Region of Convergence (ROC).

(1) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} & n < 0 \end{cases}$

Solution

As we know that

Z-transform

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z + \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n z^n - 1$$

using geometric Series

$$\Rightarrow \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n - 1$$

$$\Rightarrow \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}} - 1$$

$$= \frac{1-\frac{1}{\frac{1}{4}z^{-1}} + 1 - \frac{1}{1} - 1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-3}\right)^{-1}}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

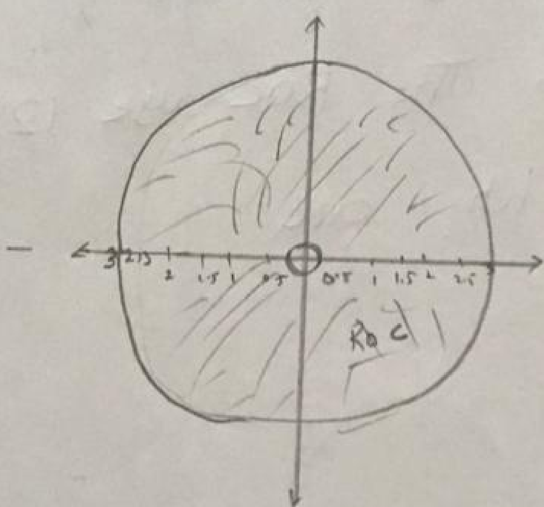
$$= \frac{1 - \frac{1}{3}z + 1 - \frac{1}{4}z^{1} - 1 + \frac{1}{3} + \frac{1}{4}z + \frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$= \frac{\frac{13}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Hence The ROC is $\frac{1}{4} < |z| < 3$

The sketch is under



(ii) $x(n) = \begin{cases} (1/2)^n - 3^n, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

Solution

using The Z-transform pair eq

i.e $x(n) = \alpha^n u(n) \longleftrightarrow X(z) = \frac{1}{1-\alpha z^{-1}} \rightarrow$ Eq (B)

By putting values

$$X_2(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$= \frac{\frac{-5}{2} z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

A seen the ROC use $|z|>2$

The sketch are