Mam Shamil

NAME Haris Iqbal

ID 7926 (A)

Subject Numerical Analysis

EXAM Summer

=> Question 1 – 1

EULERS Method:

$dy/dx = 2x \; ; \; y(0) = 1$

Solution

$x_0 = 0$

$y_0 = 1$

$h = 0.1$

$f(x, y) = 2x$

$f(x_0, y_0) = 2x_0$

$f(x_1, y_1) = 2x_1$

$f(x_2, y_2) = 2x_2$

As we know that

$x_{n+1} = x_n + h$

$y_{n+1} = y_n + hf(x_n, y_n)$

Put $n=0$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$ $\boxed{x_3 = 0.3}$ $\boxed{x_4 = 0.4}$

1st iteration.

Put $n=0$

$y_{n+1} = y_n + h\,(x_n, y_n)$

$y_{0+1} = y_0 + hf\,(x_0, y_0)$

$y_1 = 1 + 0.1\,(2(0))$

$y_1 = 1 + 0.1(0)$

$\boxed{y_1 = 1}$

2nd iteration:

$$y_2 = y_1 + hf(x_1, y_1)$$

$$y_2 = 1 + 0.1(2(0.1))$$

$$y_2 = 1 + 01(0.2)$$

$$\boxed{y_2 = 1.02}$$

⇒ ## 3rd iteration:

$$y_3 = 1.2 + 0.1(2(0.2))$$

$$y_3 = 1.2 + 0.1(0.4)$$

$$y_3 = 1.2 + 0.04$$

$$\boxed{y_3 = 1.24}$$

Q2 Runge Kutta Method.

$$\frac{dy}{dx} = x^2 + x - y$$

Where:-

Solution:-

$$\frac{dy}{dx} = x^2 + x - y$$

$$f(x,y) = x^2 + x - y$$

$$f(x_0, y_0) = x_0^2 + x_0 - y_0$$

$$f(x_1, y_1) = x_1^2 + x_1 - y_1$$

$$f(x_2, y_2) = x_2^2 + x_2 - y_2$$

As we know that

$$x_{n+1} = x_n + h$$

$$y_{n+k} = y_n + k \quad \text{——— (1)}$$

So;

$$x_{n+1} - x_n + h$$

Put $n = 0$

$x_1 = x_0 + h$

$x_1 = 0 + 0.2$

$x_1 = 0.2$

Put $n = 1$

$x_2 = 0.2 + 0.2$

$\boxed{x_2 = 0.4}$

Put $n = 1$

$x_2 = 0.2 + 0.2$

$\boxed{x_2 = 0.4}$

Put $n = 2$

$x_3 = 0.4 + 0.2$

$\boxed{x_3 = 0.6}$

Put $n = 3$

Now

=> We know that :-

$k_1 = hf(x_0, y_0)$

$k_1 = 0.2(x_0^2 + x_0 - y_0)$

$k_1 = 0.2(0+0-0)$

$\boxed{k_1 = 0}$

Now $= k_2$

$k_2 = hf\left(x_0 + h/2, y_0 + \frac{k_1}{2}\right)$

$k_2 = 0.2\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$

$k_2 = 0.2\left(\underset{x_0}{0.1}, \frac{0.1}{y_0}\right)$

$k_2 = 0.2\left((0.1)^2 + (0.1) - 0.1\right)$

$k_2 = 0.2(0.01 + 0.01 - 0.1)$

$k_2 = 0.2(0.01)$

$\boxed{k_2 = 0.002}$

Now k_3

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_3 = 0.2\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$$

$$k_3 = 0.2\ (\underset{x_0}{0.1}, \underset{y_0}{0.001})$$

Now

$$k_3 = 0.2\left((0.1)^2 + (0.1) - (0.001)\right)$$

$$k_3 = 0.2\ (0.01 + 0.1 - 0.001)$$

$$k_3 = 0.2\ (0.109)$$

$$\boxed{k_3 = 0.0218}$$

Now k_4

$$k_4 = hf\left(x_0 + h, y_0 + k_3\right)$$

$$k_4 = 0.2\ (0 + 0.2, 0 + 0.0218)$$

$$k_4 = 0.2\ (\underset{x_0}{0.2}, \underset{y_0}{0.0218})$$

$$k_4 = 0.2((0.2)^2 + (0.2) - (0.0218)$$

$k_4 = 0.2\,(0.4 + 0.2) = 0.6814$

$k_4 = 0.2\,(0.578)$

$\boxed{k_4 = 1.1564}$

$k = \frac{1}{6}\,(k_1 + 2k_2 + 3k_3 + k_4)$

$k = \frac{1}{6}\,(0.2 + 2[0.002 + 3(0.0218) + 1.1564)$

$k = \frac{1}{6}\,(02 + 0.004 + 0.6654 + 1.1564)$

$k = \frac{1}{6}\,(1.4258)$

$\boxed{k = 0.237}$

Q3 Given DATA:

$a = 0, \quad b = 10, \quad n = 10$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution:

x	0	1	2	3	4	5	6	7	8	9	10
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$f(x)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	68.3	81.2	62.1

Using Formula.

$$\int f(x)\,dx = h/2\,[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \ldots f(x_9) + f(x_{10})$$

$$= 1/2\,[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8) + 62.1]$$

$$\Rightarrow \boxed{412.9 \text{ Ans}}$$

Q4 $\int_2^3 \ln(x^3+1)\,dx$

use 10 Strips

Solution:

$$\Delta x = \frac{b-a}{2n}$$

$$\Delta x = \frac{3-2}{2(5)} = \frac{1}{10} = 0.1$$

n	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
$f(x)$	2.14	2.32	2.45	2.57	2.69	2.81	2.92	3.02	3.13	3.23	3.33

$$A = \frac{\Delta x}{3}\left[f(x_0) - 2\left[f(x_1)+f(x_2)+f(x_4)+f(x_6)\right] + 4\left[f(x_1)+f(x_3)+f(x_7)+f(x_9)\right] + f(x_{10})\right]$$

$$A = 0.1/3\left[2.14 + 2\left[2.45 + 2.69 + 2.92 + 3.13\right] - 4\left[2.32 + 2.57 + 2.81 + 3.02 + 3.23\right] + 3\right]$$

$$A = 0.03\left[2.14 + 22.38 + 55.8 + 3.33\right]$$

$$A = 2.511 \Rightarrow \int_2^3 \ln(x^3+1)\,dx = 2.511$$