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Subject	Intro to Structural Dynamics and Earthquake Engg
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Q No 01

Given data

$E = 29,000$ ksi

$I = 150 \text{ in}^4$

δ_{st} = Deflection due to 7699 lb Static Load

Beam is pulled ½" downwords

Required

Natural time period of system

Develop and solve equation of motion

Draw graphs to slow the variation of displacement with time, and the variation of equivalent static forces with time.

Sol

General EOM for SDOF system is.

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

Since System is undamped ($c=0$) undergoing free vibration $p(t)=0$

Hence general EOM becomes

$$ku + m\ddot{u} = 0$$

$$k = \frac{3EI}{L^3} \Rightarrow \frac{3 \times 29000 \frac{k}{in^2} \times 150 in^4}{(10 \times 12 in)^3}$$

$$\boxed{k = 7.55208 \; k/in}$$

⇒ In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec.

$$k = 7.55208 \; k/In$$

$$= 90625 \; lb/ft$$

$$m = w/g$$

$$= 7699/32.2 = \boxed{239.09 \text{ slug}}$$

$$\omega_n = \sqrt{k/m} \Rightarrow \sqrt{\frac{90625}{239.09}}$$

$$\boxed{\omega_n = 19.469 \text{ rad/sec}}$$

$$T_n = \frac{2\pi}{\omega_n} \Rightarrow \frac{2\pi}{19.469}$$

$$\boxed{T_n = 0.322 \text{ Sec}}$$

Put m and k in eq (1)

$$90625u + 239.09\ddot{u} = 0$$

where k is lb/ft and m is in lb sec / ft^2

⇒ General solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\ddot{v}(0)}{\omega_n}\sin(\omega_n t)$$

$$= u(0) = \frac{1}{2''} = \frac{1}{24} \text{ ft } \& \ \ddot{u}(0) = 0$$

3

$u(t) = \left(\frac{1}{24}\right) \times \cos(19.469t) + 0 = \left(\frac{1}{24}\right) \times \cos(19.469t)$

Equivalent static force at any time "t" is

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.469t)}{24}$$

$$= 3776 \cos(19.469t)$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\frac{v(0)}{\omega_n}\right)^2\right]}$$

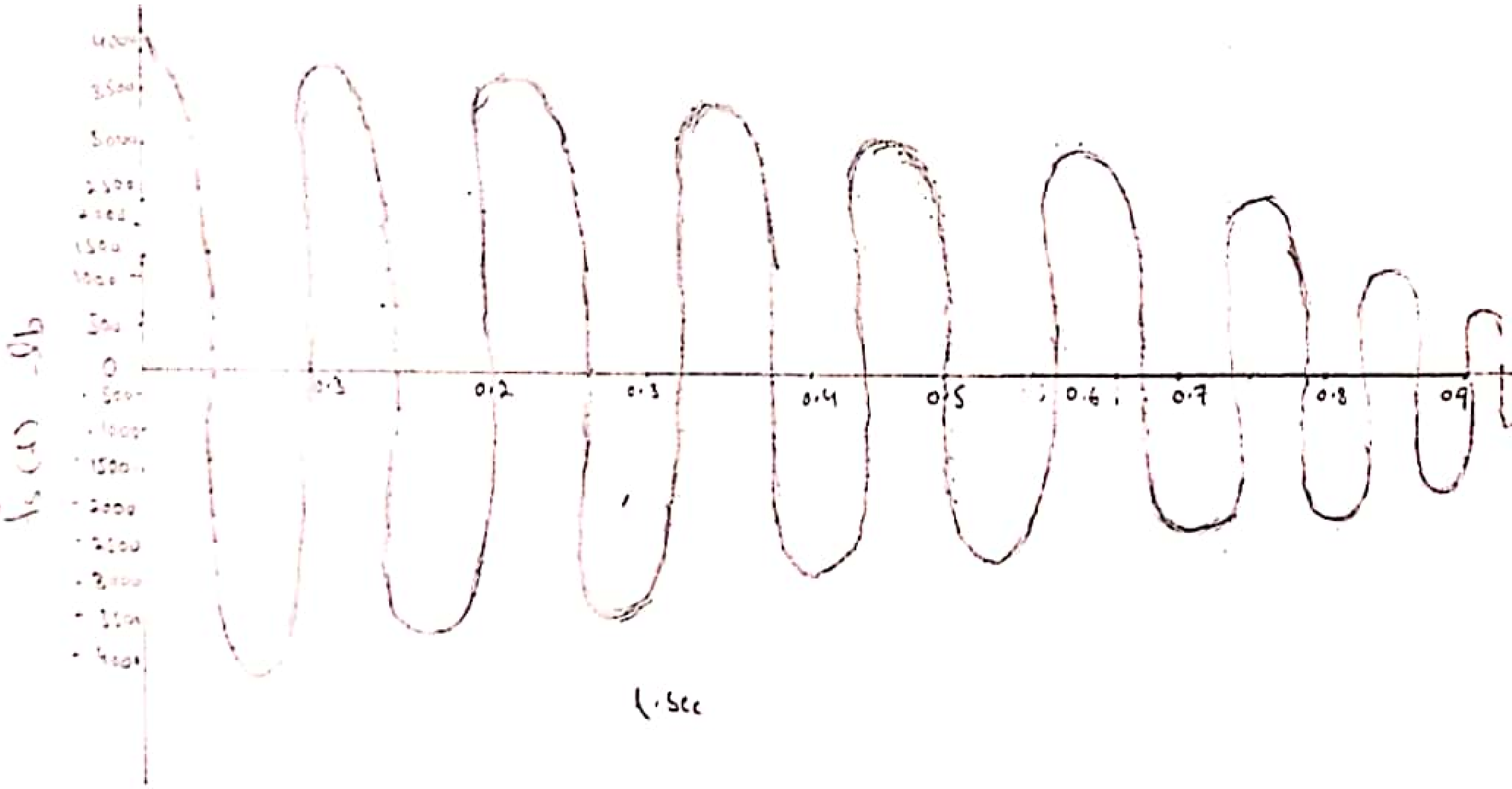
$$= \sqrt{((1/24)^2 + 0}$$

$$= 1/24 \text{ ft}$$

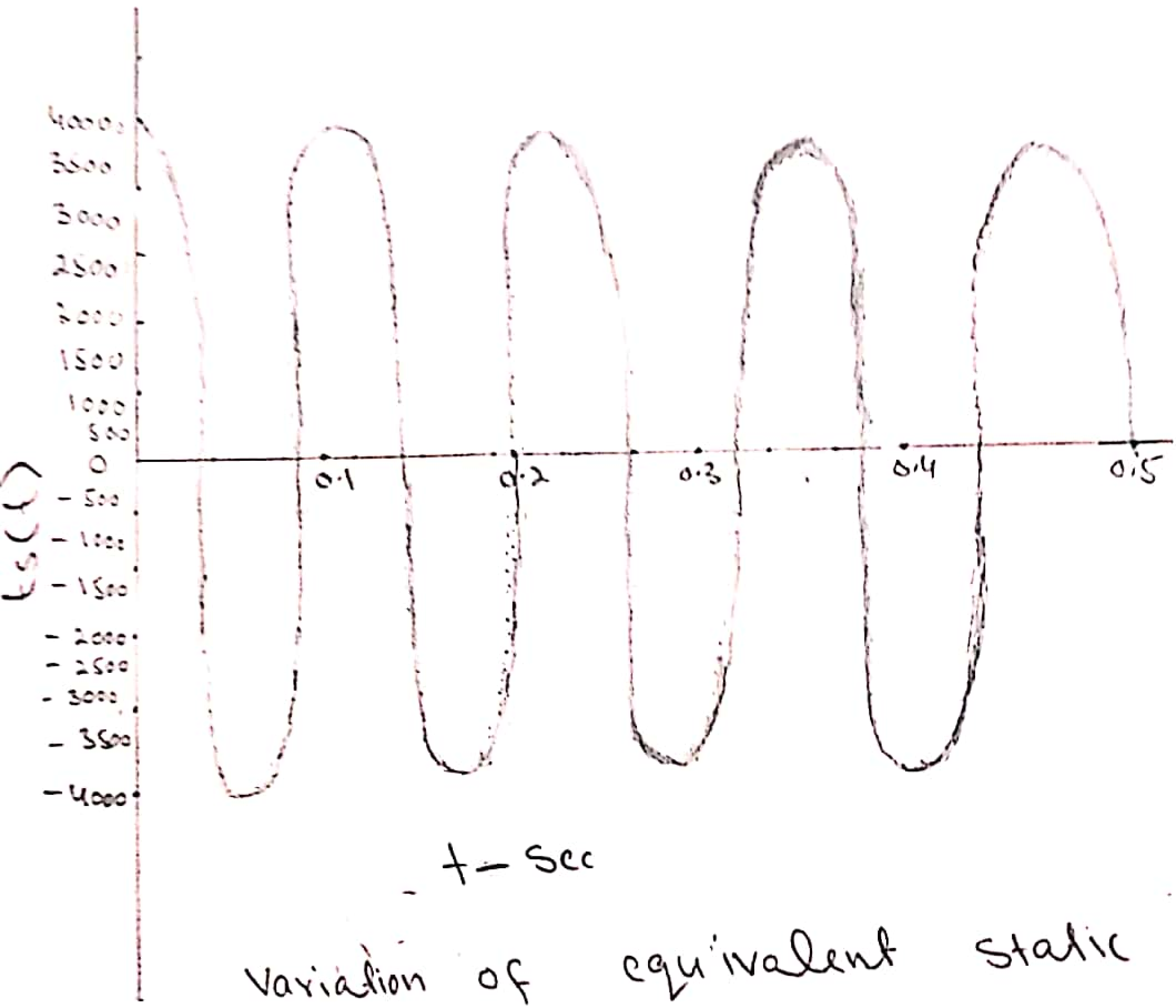
Amplitude of equivalent static force f_{so}

$$ku_o = 90625 \times 1/24$$

$$ku_o = 3776$$



Variation of Equivalent static forces with time



Variation of equivalent static forces with time

QNo : 02

Given data

ζ (Damping ratio) of Reinforced Concrete with considerable cracking =

$= 3-5\%$

$= 3\%$

Using Data of beam given in Question # 1

Required

⇒ Develope and Solve the equation showing variation in equivalent Static force with time

⇒ Draw graph to Show the variation of displacement with time and variation of equivalent static force with time.

Sol::

EOM for damped free vibrations is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{— (1)}$$

from Question 1

$K = 90625 \text{ lb/ft}$ and $m = 239.09 \text{ lb sec}^2/\text{ft}$

$\omega_n = 19.469 \text{ rad/sec}$

$$c = \zeta \times 2m\omega_n$$

$$c = (0.03) \times 2(239.09)(19.469)$$

$$c = 279.29 \text{ lb.sec/ft}$$

Put values in eq (1)

$$90625u + 279.29\dot{u} + 239.09\ddot{u} = 0$$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{v}(0) + u(0)\zeta\omega_n\right] \times \sin\omega_D t\right]$$

$$\omega_D = 19.469 \text{ rad/sec}$$

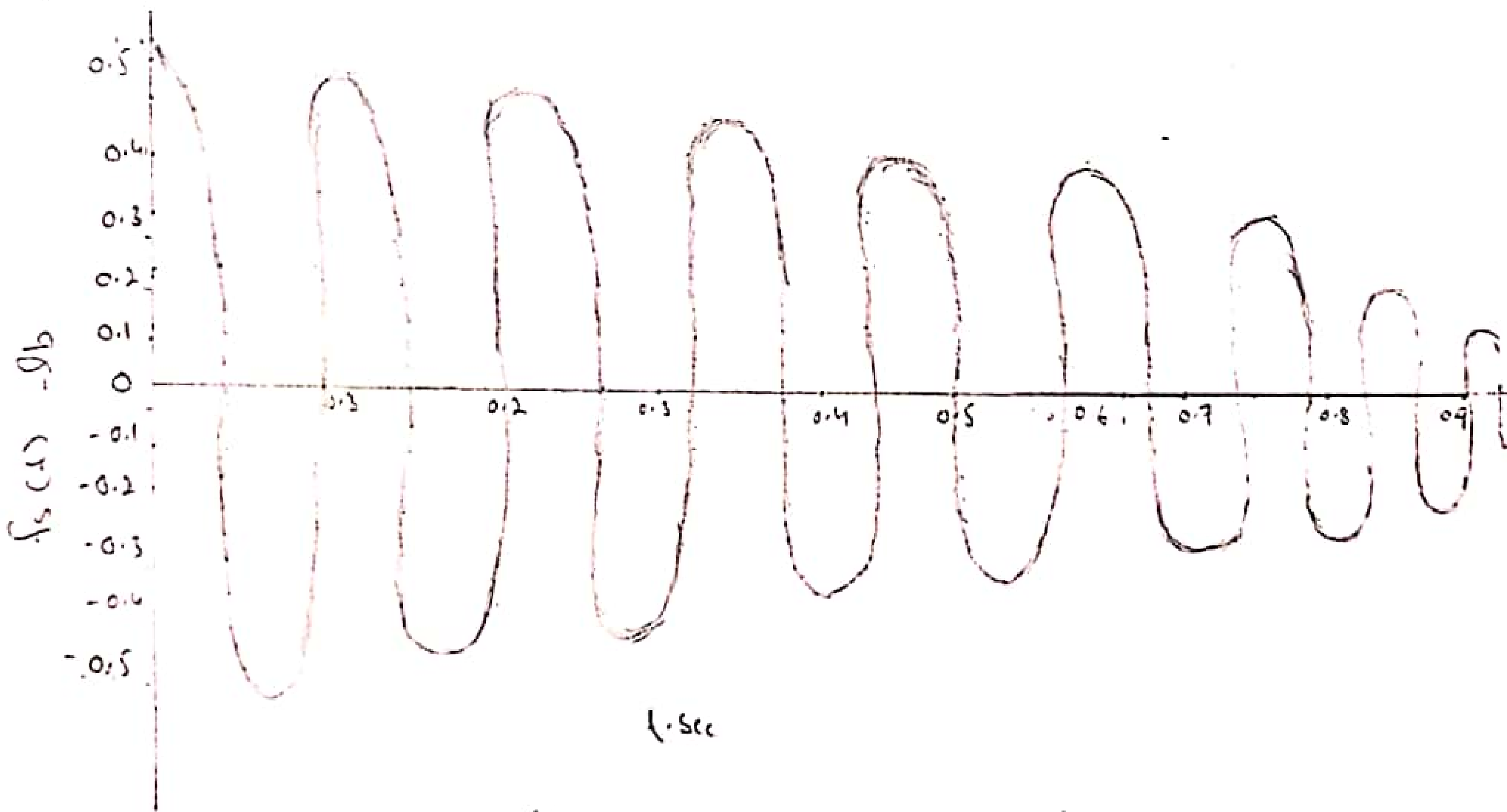
$$u(t) = e^{-0.03\times19.469t}\left[\tfrac{1}{24}\times\cos(19.469t) + \frac{1}{19.469}\times\left\{0+\frac{1}{24}\times0.03\times19.469\times\sin(19.469t)\right)\right.$$

$$u(t) = e^{-0.584t}\left[0.041\times\cos(19.469t) + 0.00125\times\sin(19.469t)\right)$$

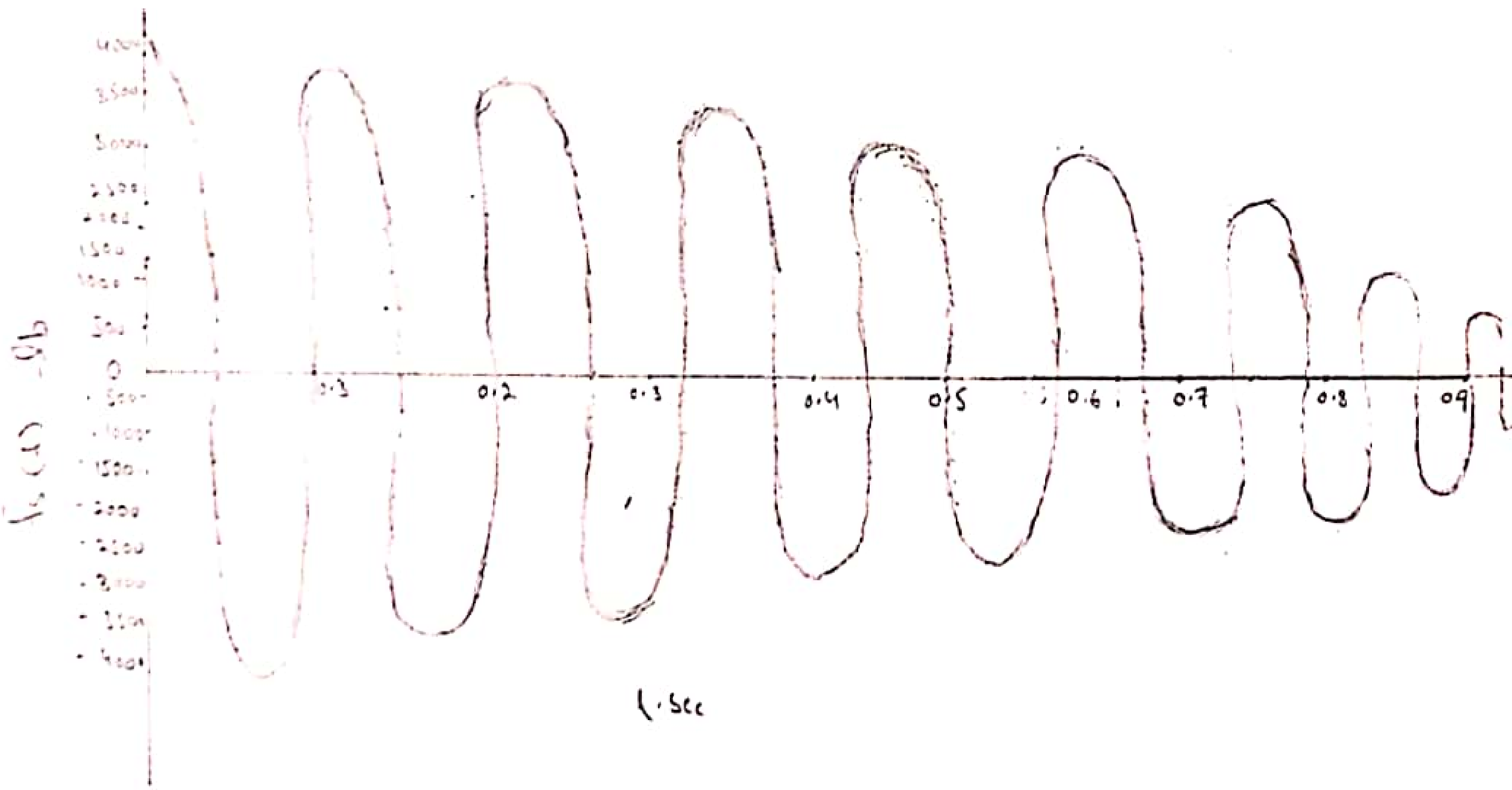
$$f_s(t) = K\cdot u(t) \Rightarrow 90625\times u(t)$$

$$f_s(t) = e^{-0.584t}\left[(90625\times0.041)\cdot\cos(19.469t) + (90625\times0.00125)\times\sin(19.469t)\right)$$

$$\boxed{f_s(t) = e^{-0.584t}\left[3715.62\cos(19.469t) + 113.28\sin(19.469t)\right)}$$



variation of displacement with time



Variation of Equivalent static forces with time

Q.No : 03

Given data

Force = 60 Kips

Displacement of tank $= \left(\frac{ID}{1000}\right)'' = \frac{7699''}{1000}$

$= 7.699''$

Time taken to complete 7 cycles

$= 3.57$ sec

Amplitude of displacement $= 2.286$ cm

$= 0.9''$

Required

(a) Damping ratios

(b) Natural Period of un-damped vibration

(c) Stiffness of Structures

(d) weight of tank

(e) Damping Co-efficient

(f) number of cycles to reduce the displacement complitude to $0.5''$

Solution:

→ Displacement of tank, u_1 =

→ After 7 cycles i-e, After $j = 7$, u_{j+1} =
$u_8 = 0.9''$

(a) Damping ratio = ζ = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.699}{0.9}\right]$$

$$\boxed{\zeta = 0.0488 = 4.88\%}$$

(b) Natural Period of undamped Vibration = T_n = ?

As, the 7 cycles of vibrations are completed in 3.57 sec

⇒ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec

$T_D = 0.51$ sec

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\zeta)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$\Rightarrow T_n = 0.5094$$

$$\boxed{\Rightarrow T_n = 0.51 \text{ sec}}$$

(c) Stiffness of structure, K = ?

$$K = \frac{60 \times \cos 60^\circ}{7.699} = 3.89 \text{ k/in}$$

$$K = 3.89 \times 12 \times 1000$$

$$\boxed{K = 46680 \text{ lb/ft}}$$

(d) weight of tank, $W = ?$

$$\omega_n = \sqrt{k/m} = \sqrt{k/{W/g}} = \sqrt{k \cdot g / W}$$

$$\Rightarrow \omega_n^2 = \frac{k \times g}{W_n^2}$$

Also, $\omega_n = 2\pi / T_n$

$$W = kg \Big/ \left(\frac{4\pi^2}{T_n^2}\right)$$

$$W = \left[\frac{46680 lb}{ft} \times \frac{32.2 ft}{sec^2}\right] \times \frac{(0.51)^2}{4\pi^2}$$

$$W = 9913.06 \text{ lb}$$

$$\boxed{W = 9.91k}$$

(e) Damping Co-efficient, $C = ?$

It is known that

$$\zeta = C / 2m\omega_n$$

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$\Rightarrow C = \zeta \times 2m\omega_n$

$= \zeta \times 2m \times \left(2\pi/T_n\right)$

$$\Rightarrow C = \frac{(0.0488) \times 4 \times \pi \times \left(\frac{9913.06}{32.2}\right)}{0.51}$$

$$\boxed{C = 369.99 \text{ lb.sec/ft}}$$

(f) number of cycles to reduce the displacement amplitude to 0.5", J = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow j = \frac{1}{2\pi \times 0.0488} \ln\left[\frac{7.699}{0.5}\right]$$

$$\boxed{j = 8.92 \text{ or } \approx 9 \text{ cycles}}$$

x ——— x ——— x ——— x