Date: 20-Aug-20

Mid-Term Paper (Summer 2020)

CALCULUS & ANALYTICAL GEOMETRY

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Q1: (a) Differentiate $\frac{3x^3-5x^2+5}{x^2+1}$ w.r.t

Solution:-

$$f(x) = \frac{3x^3-5x^2+5}{x^2+1}$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{3x^3-5x^2+5}{x^2+1}\right)$$

Now, by quotion rule

$$\frac{dy}{dx} = \frac{(x^2+1)(9x^2-10x)-(3x^3-5x^2+5)(2x)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{9x^4-\cancel{10x^3}+9x^2-\cancel{10x}-6x^4+\cancel{10x^3}-\cancel{10x}}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{3x^4+9x^2}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \boxed{\frac{3x^4+9x^2}{(x^2+1)^2}}$$ Answer.

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Q1 (b) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ w.r.t x

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(x^2+1)^2}{x^2-1}\right)$$

$$\frac{dy}{dx} = \frac{(x^2-1)\,{}^{d}/_{dx}\,(x^2+1)^2 - (x^2+1)^2\,{}^{d}/_{dx}\,(x^2-1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-1)\,2(x^2+1)\,{}^{d}/_{dx}\,(x^2) - (x^2+1)^2\,(2x)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2x\,(x^2-1)(x^2+1) - 2x\,(x^2+1)^2}{(x^2-1)^2}$$

$$y' = \frac{2x\,(x^2+1)\,\left(2(x^2-1) - (x^2+1)\right)}{(x^2-1)^2}$$

$$y' = \frac{2x\,(x^2+1)\,[2x^2-2-x^2-1]}{(x^2-1)^2}$$

$$\boxed{y' = \frac{2x\,(x^2+1)(x^2-3)}{(x^2-1)^2}}$$ Answer.

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Q3 (a) Find Integration $\int \frac{1}{\sqrt{x^3}}\,dx$

Solution:- $\int \frac{1}{(x^3)^{1/2}}\,dx$

$$= \int \frac{1}{x^{3/2}}\,dx$$

$$= \int x^{-3/2}\,dx$$

$$= \frac{x^{-3/2+1}}{-3/2+1} + C$$

$$= \frac{x^{\frac{-3+2}{2}}}{-3+2/2} + C$$

$$= \frac{x^{-1/2}}{-1/2} + C$$

$$\boxed{= \frac{-2}{\sqrt{x}} + C}$$ Answer.

Q3 (b) Find Integration $\int \frac{1}{(6x+7)^6}\,dx$

Solution:-

let $6x+7 = U$

PTO

Now we diff. w.r.t x;

$\frac{du}{dx} = 6 \Rightarrow du = 6dx$

$\frac{du}{6} = dx$ Substitute:-

$= \frac{1}{6} \int \frac{1}{u^6} du$

$= \frac{1}{6} \int u^{-6} du = \frac{1}{6} \frac{u^{-6+1}}{-6+1} + c$

$= \frac{u^{-5}}{-30} + c$ So,

$U = 6x + 7$ putting in eq

$\boxed{= \frac{1}{-30(6x+7)^5} + c}$ Answer

Q2 (b) Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1-x}{1+x}}$ using chain rule

Solution:-

Differentiate w.r.t "x"

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$$\frac{dy}{dx} = \frac{d}{dx}\left(\left(\frac{1-x}{1+x}\right)^{1/2}\right)$$

$$y' = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$y' = \frac{1}{2} \cdot \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$y' = \frac{1}{2} \cdot \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{-1 \cancel{-x} - 1 \cancel{+x}}{(1+x)^2}$$

$$y' = \frac{1}{\cancel{2}} \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\cancel{-2}}{(1+x)^2}$$

$$\boxed{y' = \frac{-\sqrt{1+x}}{\sqrt{1-x} \cdot (1+x)^2}}$$ Answer.

Q3 (a) Find $\frac{dx}{dy}$ if $y = \left(1+2\sqrt{x}\right)^3 \cdot x^{2/3}$

Solution:-

$$\frac{d}{dx}\left[\left(2\sqrt{x}+1\right)^3 x^{\frac{2}{3}}\right]$$

$$= \frac{d}{dx}\left[\left(2\sqrt{x}+1\right)^3\right] \cdot x^{\frac{2}{3}} + \left(2\sqrt{x}+1\right)^3 \cdot \frac{d}{dx}\left[x^{2/3}\right]$$

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$$= 3\left(2\sqrt{x}+1\right)^2 \cdot \frac{d}{dx}\left[2\sqrt{x}+1\right] \cdot x^{\frac{2}{3}} + \left(2\sqrt{x}+1\right)^3 \cdot \frac{2}{3} x^{\frac{2}{3}-1}$$

$$= 3\left(2\sqrt{x}+1\right)^2 \left(2 \cdot \frac{d}{dx}\left[\sqrt{x} + \frac{d}{dx}[1]\right] x^{2/3} + \frac{2\left(2\sqrt{x}+1\right)^3}{3\sqrt[3]{x}}\right.$$

$$= 3\left(2\sqrt{x}+1\right)^2 \left(2 \cdot \frac{1}{2} x^{\frac{1}{2}-1} + 0\right) x^{2/3} + \frac{2\left(2\sqrt{x}+1\right)^3}{3\sqrt[3]{x}}$$

$$\boxed{= 3\left(2\sqrt{x}+1\right)^2 \sqrt[6]{x} + \frac{2\left(2\sqrt{x}+1\right)^3}{3\sqrt[3]{x}}}$$ Answer.