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Program = BS(S.E)

Paper = Differential Equation

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Q 1 =

(a) Homogenous differential equation :—

It Involves only derivatives of y and terms Involving y and they are set to 0 as in this example.

$$\frac{d^4y}{dx^4} + x\frac{d^2y}{dx^2} + y^2 = 0$$

⇒ Non-homogenous differential equation :-

Same as homogenous differential equation except they can have terms involving only x on right side as given.

$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} + y^2 = 6x + 3$$

Q1 =

(b) Solve the following 2nd order linear homogenous/non homogenous differential equation?

(i) $4y'' - 6y' + 7y = 0$

Sol:-

First finding $\sqrt{\ }$

So,

$$4\lambda^2 - 6\lambda + 7 = 0$$

$$\lambda = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$$

$$\lambda = \frac{6 \pm \sqrt{36 - 112}}{8}$$

$$\lambda = \frac{6 \pm \sqrt{-76}}{8}$$

$$\lambda = \frac{6}{8} + \frac{2\sqrt{19}}{8} i$$

$$\lambda = \frac{3}{4} \pm \frac{\sqrt{19}}{4} i$$

$$\lambda_1 = \frac{3}{4} + \frac{\sqrt{19}}{4} i \quad , \quad \lambda_2 = \frac{3}{4} - \frac{\sqrt{19}}{4}$$

So it has the complex conjugate roots.

$$Q_1(x) = e^{\lambda_1 x} \cos \lambda_1(x)$$

$$Q_2(x) = e^{\lambda_2 x} \sin \lambda_2(x)$$

$$\Rightarrow y = C_1 e^{3/4 x} \cos i\sqrt{19}/4 (x) + e^{3x/4}$$

$$\sin i\sqrt{19}/4 (x) \; C_2$$ Ans

(b)

(ii) $y'' - 4y' - 12y = 3e^{\wedge}(5x)$

Sol:-

The characteristics equation and its roots :

$$y^2 - 4y - 12 = (y-6)(y+2) = 0$$

$$y_1 = -2, \quad y_2 = 6$$

The Complementory Solution is then :-

$$y_c(t) = C_1 e^{-2t} + C_2 e^{6t}$$

Q2: Solve the following IVP for the 2nd order linear equations.

(i) $16y'' - 40y' + 25y = 0 \quad y(0) = 3, \; y'(0) = -9/4$

Solution:-

The characteristic equation & root are as given:

$16y^2 - 40y + 25 = (4y^2 - 5)^2 = 0 \quad y_1 = \frac{5}{4}, \; y_2 = \frac{5}{4}$

$\Rightarrow$ The general solution and derivative are:

$\Rightarrow y(t) = C_1 e^{5t/4} + C_2 e^{5t/4}$.

$\Rightarrow y'(t) = 5/4 \; C_1 e^{5t/4} + C_2 e^{5t/4} + \frac{5}{4} C_2 t e^{5t/4}$

$\Rightarrow$ Now put it in initial position:

$3 = y(0) = C_1$

$-\frac{9}{4} = y'(0) = \frac{5}{4} C_1 + C_2$

The solution for IVP is then:

$y^t = 3e^{5t/4} - 6te^{5t/4}$.

Ans

Q2:

(ii) $y'' + 14y' + 49y = 0 \quad y(-4) = 1 \; y'(-4) = 5$

Sol:-

The characteristic equation and its root are:

$$y^2 + 14y + 49 = (y+7)^2 = 0 \quad y_1 = 7, y_2 = -7$$

The general solution and its derivatives are:

$$y(t) = C_1 e^{-7t} + C_2 t e^{-7t}$$

$$y't = -7C_1 e^{-7t} + C_2 e^{-7t} - 7C_2 t e^{-7t}$$

Putting in the Initial Condition.

$$-1 = y(-4) = C_1 e^{28} - 4C_2 e^{29}$$

$$5 = y(-4) = -7C_1 e^{28} + C_2 e^{28} + 28 C_2 e^{28}$$

$$= -7C_1 e^{28} + 29 C_2 e^{28}$$

It gives the following Constant:

$$C_1 = -9e^{-28}$$

$$C_2 = -2e^{-28}$$

The Solution for IVP is:

$$y(t) = -9e^{28} e^{-7t} - 2te^{-28} e^{-7t}$$

$$y(t) = -9e^{-7(t-4)} - 2te^{-7(t+4)}$$

Q2 :-

(iii) $y'' - 4y' + 9y = 0 \quad y(0) = 0, \; y'(0) = -8$

Sol:- The characteristic equation for this D.E is :

$$y^2 - 4y + 9 = 0$$

The r of equation are :

$$y_1 = 2 \pm \sqrt{5}\, i$$
$$y_2 = 2 \pm \sqrt{5}\, i$$

The general solution to D.E is

$$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t).$$

Appying intial condition along with derivative

$$y(t) = C_2 e^{2t} \sin(\sqrt{5}t)$$
$$y'(t) = 2i\,2e^{2t} \sin(\sqrt{5}t) + \sqrt{5}\, C_1 e^{2t} \cos(\sqrt{5})$$
$$-8 = y'(0) = \sqrt{5}(2) = C_2 = -8/5$$

Solution is :-

$$y(t) = -\frac{8}{5} e^{2t} \sin$$

Ans

Q2 :

(iv) $y'' - 8y' + 17y = 0 \quad y(0) = -4, \; y'(0) = -1$

Sol :-

The characteristic equation and its root are :-

$$y^2 - 8y + 17 = 0$$

$$y_1 = 4 + i$$

$$y_2 = 4 - i$$

The general Solution and derivative are :-

$$y(t) = c_1 e^{4t} \cos(t) + c_2 e^{4t} \sin(t).$$

$$y'(t) = 4c_1 e^{4t} \cos(t) - c_2 e^{4t} \cos(t)$$

$$y'(t) = 4c_1 e^{4t} \cos(t) - c_1 e^{5t} \cos(t) + 4\,2c^{4t} \sin(t) + c_2 e^{4t} \cos(t),$$

By Applying the Initial Condition gives as :

$$-y = y(0) = c_1$$

$$-1 = y(0) = 4c_1 + c_2$$

So the Solution is :-

$$y(t) = -4e^{4t} \cos(t) + 15e^{4t} \sin(t).$$

Ans

Q3:

A. Find the laplace transform of the given functions.

1:- $f(t) = 6(e^\wedge - 5t) + e^\wedge 3t + 5(t^\wedge 3) - 9$

Sol:-

$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

$$F(s) = 6\frac{1}{3-(-5)} + \frac{1}{3 \cdot 3} + 5\frac{3}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{3+5} + \frac{1}{3 \cdot 3} + \frac{30}{s^4} - \frac{9}{s}$$

(ii) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Sol:-

$$g(s) = \frac{4s}{s^2+4^2} - \frac{9 \cdot 4}{s^2+(4)^2} + \frac{2s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

Ans

(iii) $h(t) = e^\wedge 3t + \cos(6t) - e^\wedge(3t)\cos(6t)$

Sol:- $H(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

Ans

Q4:- Solve the following IVP using Laplace form.

1):- $y'' - 10y' + 9y = 5t, \quad y(0) = -1, \quad y'(0) = 2.$

Soln

First taking transform of every term.

$\mathcal{L}\{y''\} - 10\mathcal{L}\{y\} + 9\mathcal{L}(y) = \mathcal{L}\{5t\}$

By formula.

$$s^2 y_{(s)} - sy(0) - y'(0) - 10(s y_{(s)}) - y(0) + 9y_{(s)} = \frac{5}{s^2}$$

Now, put in initial condition.

$(s^2 - 10s + 9)\, y(s) + s + 12 = 5/s^2.$

Solve for $y(s)$

$$y(s) = \frac{5}{s^2(s+9)(s+1)} + \frac{19 - s}{(s-9)(s-1)}$$

$$y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$$

The partial fraction of transform will be

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$$5 + 12s^2 - s^3 = A\, s(s-9)(s-1) + B(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9).$$

Solve for constants.

$s = 0 \qquad 5 = 9B \qquad \Rightarrow B = 5/9$

$s = 1 \qquad 16 = -8D \qquad \Rightarrow D = -2$

$s = 9 \qquad 248 = 648C \qquad \Rightarrow C = 31/81$

$s = 2 \qquad 45 = -14A + \frac{4345}{81} \Rightarrow A = 50/81$

Plugging in the constant gives

$$Y(s) = \frac{50}{81} + \frac{5/9}{s^2} + \frac{31/81}{s-9} - \frac{2}{s-1}$$

By taking the inverse transform the solution is

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{2t} - 2e^{t}.$$

Answer