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Assignment no 1

First, we need to apply the given operator to the given function:

$$\begin{aligned} 9x^2D^2y + 3xDy + Iy &= 9x^2D(Dy) + 3xDy + y \\ &= 9x^2y'' + 3xy' + y \end{aligned}$$

Let's solve the equation:

$$9x^2y'' + 3xy' + y = 0$$

Let's substitute:

$$y = x^m, \quad y' = mx^{m-1}, \quad y'' = m(m-1)x^{m-2}$$

into the given ODE. This gives:

$$9x^2m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$$

$$9\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} + 3\cancel{x}mx^m\cancel{x^{-1}} + x^m = 0$$

We can see that x^m is a common factor, dropping it gives:

$$9m(m-1)+3m+1 = 0 \iff 9m^2-9m+3m+1 = 0 \iff 9m^2-6m+1 = 0 \quad (*)$$

So, $y = x^m$ is a solution of the given ODE if m is a root of the equation $(*)$

Let's find the roots of the equation $(*)$

$$m^2 - 4m + 4 = 0 \iff (m-2)^2 = 0$$

$$\begin{aligned} 9m^2 - 6m + 1 = 0 &\iff m_{1/2} = \frac{6 \pm \sqrt{6^2 - 4 \cdot 9}}{18} \\ &\iff m_{1/2} = \frac{6}{18} \\ &\iff m_{1/2} = \frac{1}{3} \end{aligned}$$

So, it has the real double root:

$$m = \frac{1}{3}$$

Real double root m provides a real solution:

$$y_1 = x^m = x^{\frac{1}{3}}$$

First, we need to apply the given operator to the given function:

$$\begin{aligned} x^2D^2y - 3xDy + 4Iy &= x^2D(Dy) - 3xDy + 4y \\ &= x^2y'' - 3xy' + 4y \end{aligned}$$

Let's solve the equation:

$$x^2y'' - 3xy' + 4y = 0$$

Let's substitute:

$$y = x^m, \quad y' = mx^{m-1}, \quad y'' = m(m-1)x^{m-2}$$

into the given ODE. This gives:

$$x^2m(m-1)x^{m-2} - 3xmx^{m-1} + 4x^m = 0$$

$$\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} - 3\cancel{x}mx^m\cancel{x^{-1}} + 4x^m = 0$$

We can see that x^m is a common factor, dropping it gives:

$$m(m-1) - 3m + 4 = 0 \iff m^2 - 4m + 4 = 0 \quad (*)$$

So, $y = x^m$ is a solution of the given ODE if m is a root of the equation $(*)$

Let's find the roots of the equation $(*)$

$$m^2 - 4m + 4 = 0 \iff (m-2)^2 = 0$$

So, it has the real double root:

$$m = 2$$

Real double root m provides a real solution:

$$y_1 = x^m = x^2$$

$x^2y'' + 3xy' + 0.75y = 0$
putting $y = x^m$ we get
$m^2 + 2m + 0.75 = 0$
$m = -0.5, -1.5$
General solution is given as
$y = c_1x^{-0.5} + c_2x^{-1.5}$
$y' = -0.5c_1x^{-1.5} - 1.5c_2x^{0.5}$
Applying the boundary conditions:
$c_1 = 3,\ c_2 = -2$

Complete solution is given as
$y = 3x^{-0.5} + -2x^{-1.5}$

First, we need to apply the given operator to the given function:

$$\begin{aligned} x^2D^2y + xDy + Iy &= x^2D(Dy) + xDy + y \\ &= x^2y'' + xy' + y \end{aligned}$$

Let's solve the equation:

$$x^2y'' + xy' + y = 0$$

Let's substitute:

$$y = x^m, \quad y' = mx^{m-1}, \quad y'' = m(m-1)x^{m-2}$$

into the given ODE. This gives:

$$x^2m(m-1)x^{m-2} + xmx^{m-1} + x^m = 0$$

$$\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} + \cancel{x}mx^m\cancel{x^{-1}} + x^m = 0$$

We can see that x^m is a common factor, dropping it gives:

$$m(m-1) + m + 1 = 0 \iff m^2 - \cancel{m} + \cancel{m} + 1 = 0 \iff m^2 + 1 = 0 \quad (*)$$

So, $y = x^m$ is a solution of the given ODE if m is a root of the equation $(*)$

Let's find the roots of the equation $(*)$

$$m^2 + 1 = 0 \iff m^2 - i^2 = 0 \iff (m-i)(m+i) = 0$$

So, it has the complex conjugate roots:

$$m_1 = i \quad \wedge \quad m_2 = -i$$

Let's substitute:

$$y = x^m, \quad y' = mx^{m-1}, \quad y'' = m(m-1)x^{m-2}$$

into the given ODE. This gives:

$$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0$$

$$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{-2}} - 4\cancel{x}mx^m\cancel{x^{-1}} + 6x^m = 0$$

We can see that x^m is a common factor, dropping it gives:

$$m(m-1) - 4m + 6 = 0 \iff m^2 - 5m + 6 = 0 \quad (*)$$

So, $y = x^m$ is a solution of the given ODE if m is a root of the equation $(*)$

Let's find the roots of the equation $(*)$.

$$\begin{aligned} m^2 - 5m + 6 = 0 &\iff m_{1/2} = \frac{5 \pm \sqrt{(-5)^2 + 4 \cdot 6}}{2} \\ &\iff m_{1/2} = \frac{5 \pm 1}{2} \end{aligned}$$

So, it has the distinct real roots:

$$m_1 = 3 \quad \wedge \quad m_2 = 2$$

Real different roots m_1 and m_2 provide two real solutions:

$$y_1 = x^{m_1} = x^3 \quad \wedge \quad y_2 = x^{m_2} = x^2$$

Their quotient is not constant, so the solutions y_1 and y_2 are linearly independent and constitute a basis of solutions for the given ODE, for all x for which $y_1, y_2 \in \mathbb{R}$.

So, the general solution is:

$$\begin{aligned} y &= c_1 y_1 + c_2 y_2 \\ &= \boxed{c_1 x^3 + c_2 x^2} \end{aligned}$$

$$\implies y' = 3c_1 x^2 + 2c_2 x$$