Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 20

Student Details

Name: **Student ID:**

Q1.

(a) Determine the response y(n), n ≥ 0, of the system described by the second order difference equation — Marks 6

To the input

(b) Determine the impulse response and unit step response of the systems described by the difference equation.

Q2.

(a) Determine the causal signal x(n) having the z-transform — Marks 6

(Hint: Take inverse z-transform using partial fraction method)

(b) Determine the partial fraction expansion of the following proper function

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A two- pole low pass filter has the system response — Marks 4

Q.3 (a) Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and ^{2} = .

(b) Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in at ω = 4π/9.

Q 4

(c) A finite duration sequence of Length L is given as — Marks 4

Determine the N- point DFT of this sequence for N ≥ L

(d) Compute the DFT of the four-point sequence

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Sessional Assignment

Question 1

(a): Determin the responce
$y(n), n \geq 0$.
second order
$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$
to the input $x(n) = 4^n u(n)$.

Sol:

consider the equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \quad \text{--- (1)}$$

homogeneous equation system

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

the characteristic equation of system is

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

Determin ~~the~~ root of eq 2

$\lambda^2 - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda-4) + 1(\lambda-4) = 0$

$(\lambda-4)(\lambda+1) = 0$

$\lambda = -1, 4$

the homogenous eq is

$y_h(n) = C_1(-1)^n u(n) + C_2(4)^n u(n)$

since 4 is a root and the eq is

$u(n) = 4^n u(n)$

we assume particular solution of

$y_p(n) = Kn4^n u(n)$

then,

$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2)$
$= 4^n u(n) + 2(4)^{n-1} u(n-1)$

for $n = 2$

$K(32-12) = 4^2 + 8 = 24 \Rightarrow K = \frac{6}{5}$

the total solution is

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + C_1 4^n + C_2(-1)^n\right] u(n)$

To solve for C_1 and C_2

$y(-1) = y(-2) = 0$

$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9$

Hence,

$$C_1 + C_2 = 1$$

$$\frac{24}{5} + 4C_1 - C_2 = 9$$

$$4C_1 - C_2 = \frac{21}{5}$$

Therefore,

$$C_1 = \frac{26}{25} \quad \& \quad C_2 = \frac{-1}{25}$$

the total solution is

$$\boxed{y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right] u(n)}$$

(b): Impulse Response: (4)

the difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

Sol:

consider difference eq

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$$

To obtain homogenous eq

$$x(n) = 0$$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$$

Solution to homogenous eq

$$y_h(n) = \lambda^n$$

substitute the solution obtain in homogenous eq

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$(\lambda - 0.2)(\lambda - 0.4) = 0$$

roots are

$$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$$

The general form of solution to homogenous eq,

$$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$$y(n) = C_1(0.2)^n + C_2(0.4)^n \quad \ldots\ldots (1)$$

$\lambda = 0.2$, $\lambda = 0.4$ hence

$$y_h(n) = C_1 \frac{1}{5}^n + C_2 \frac{2}{5}^n$$

with $x(n) = \delta(n)$, the initial condition,

$$y(0) = 1$$

$$y(1) = 0.6y(0) = 0$$

$$y(1) = 0.6$$

Hence, $C_1 + C_2 = 1$ &

$$\frac{1}{5}C_1 + \frac{2}{5} = 0.6$$

$$\Rightarrow C_1 = -1, \quad C_2 = 3$$

therefore,

$$h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n)$$

the step response is

$$y(n) = \sum_{k=0}^{n} h(n-k), \quad n > 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k} \right]$$

$$\boxed{= \left[\frac{1}{0.12}\left(\frac{2}{5}^{n+1} - 1\right) - \frac{1}{0.16}\left(\frac{1}{5}^{n+1} - 1\right)\right] u(n)}$$

Question 2

(a): Causal signal

$$u(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Hint: Take inverse z-transform

Sol:

z-transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The expression is written as.

$$x(z) = \frac{1}{\left(1-\frac{2}{z}\right)\left(1-\frac{1}{z}\right)^2}$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \quad \text{———} \quad (1)$$

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$X(z)$ has a simple pole at $p_1 = 2$ & double $p_2 = p_3 = 1$.
The partial fraction is,

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}$$

to find $A_1, A_2,$ & A_3, we will proceed as in the case of distinct pole to find A_1. we will multiply both side by $(z-2)$ and result $z=-2$

$$(z-2)X(z) = A_1 + \frac{z-2}{z-1}A_2 + \frac{z-2}{(z-1)^2}A_3$$

$$A_1 = \left.\frac{(z-2)X(z)}{z}\right|_{z=2}$$

$$\boxed{A_1 = 4}$$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$$\boxed{A_2 = -3}$$

$$A_3 = A_1 + \frac{z-2}{z-1}A_2$$

$$\boxed{A_3 = -1}$$

Hence,

$$\boxed{x(n) = [4(2)^n - 3 - n]\,u(n)}$$

(b): <u>Partial fraction:</u>

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$

<u>Sol</u>: first we will eliminate negative power by multiplying z^2,

$$X(z) = \frac{z^2}{z^2-1.5z+0.5}$$

Poles of $X(z)$ are $P_1=1$ & $P_2=0.5$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

to find A_1 & A_2 we will multiply the eq $(z-1)(z-0.5)$

$$z = (z-0.5)A_1 + (z-1)A_2 \quad \text{--- (1)}$$

Now, if we set $z = P_1 = 1$ in eq (1)

$$1 = (1-0.5)A_1$$

thus we obtain the result $A_1 = 2$, next we return to eq (1) & $z = P_2 = 0.5$ & eliminate A_1, So, we get

$$0.5 = (0.5-1)A_2$$

$$A_2 = -1$$

hence, $A_2 = -1$, ~~the~~ result will be

$$\boxed{\frac{X(z)}{z} = \frac{z}{z-1} - \frac{1}{z-0.5}}$$

Question No 3

(a):

Sol: At $\omega = 0$, we have

$$H(0) = \frac{b^0}{(1-p)^2} = 1$$

hence,

$$b_0 = (1-p)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp\sqrt{2})^2}$$

Hence,

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 - p^2/\sqrt{2}]^2} = \frac{1}{2}$$

or equivalently,

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

(12)

the value of $P = 0.32$ satisfies this eq consequently, the system function desired filter is.

$$\boxed{H(z) = \frac{0.46}{(1-0.32z^{-1})^2}}$$

(b):

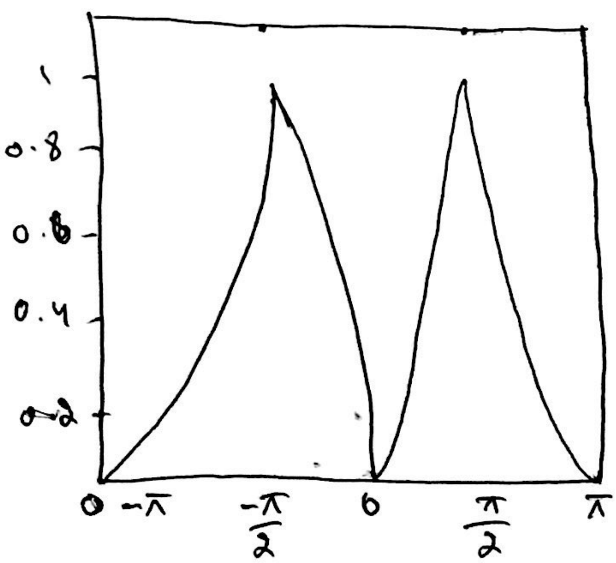
Sol: the filter must have poles at,

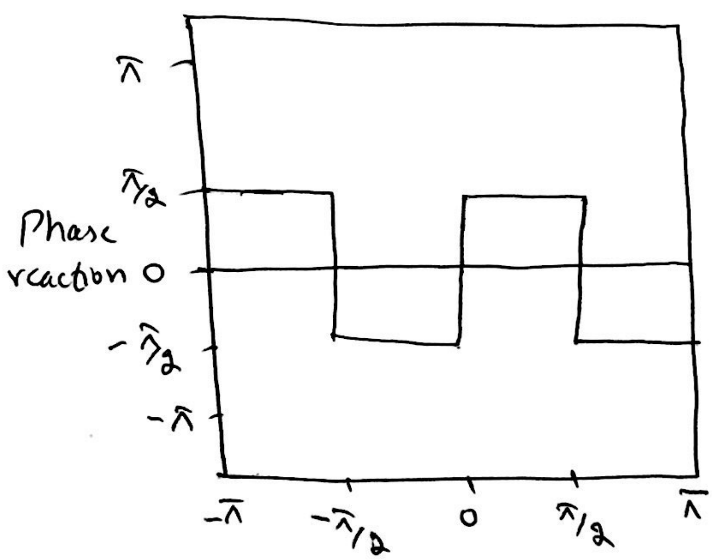
$P_{1,2} = re^{\pm j\pi/2}$

and zeros at $z=1$, & $z=-1$.

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\,\frac{z^2-1}{z^2+r^2}$$





The freaquency response $H(\omega)$ of the filter at $\omega = \pi/2$,

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determine by evaluating $H(\omega)$ at $\omega = 4\pi/9$,

$$H\left(\frac{4\pi}{9}\right)^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(r^2\pi/9)}{1+r^4+2r^2\cos(r\pi/9)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

the value of $r^2 = 0.7$, satisfies the eq, therefore, the system function for the desired filter is,

$$\boxed{H(z) = 0.5 \frac{+z^{-2}}{1+0.7z^{-2}}}$$

Question 4

(a): the fourier transform is,

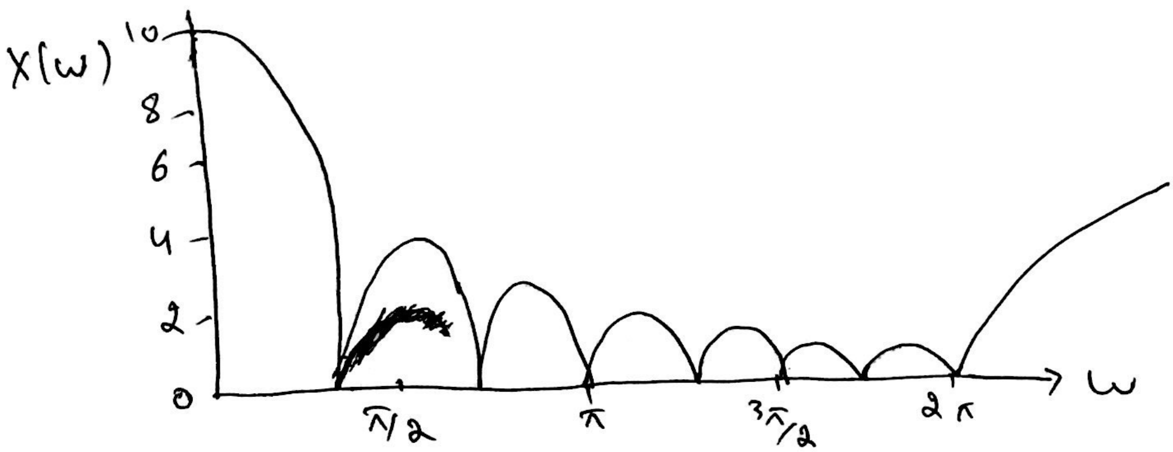
$$X(\omega) = \sum_{n=0}^{i-1} x(n) e^{-j\omega n}$$

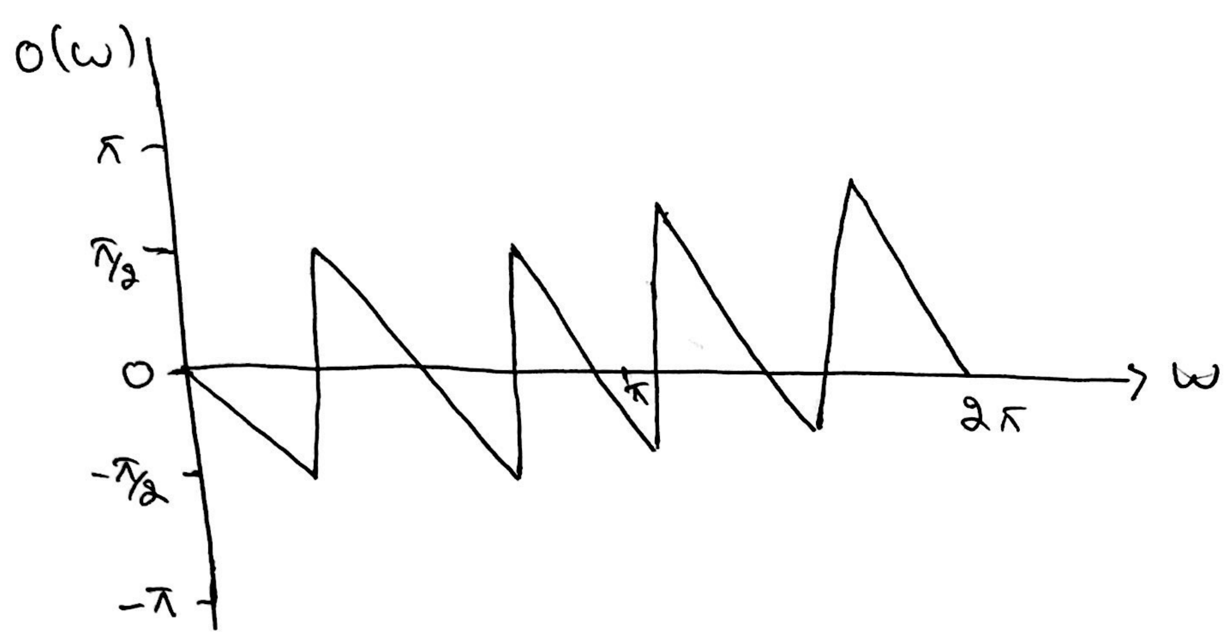
$$= \sum_{n=0}^{i-1} e^{-j\omega n} = \frac{1-e^{-j\omega n}}{1-e^{-j\omega}} = \frac{\sin(\omega/2)}{\sin(\omega/2)} e^{-j\omega(n-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustration for $i = 10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally speed frequencies $\omega_k = 2\pi K/N$, $K = 0, 1 \cdots N-1$,

$$X(k) = \frac{1-e^{-j2\pi k_2/N}}{1-e^{-j2\pi k/N}}, \quad K = 0, 1, \cdots, N-1$$

$$= \frac{\sin(\pi k_2/N)}{\sin(\pi k/N)} e^{-j\pi k(i-1)/N}$$





If N is selected such that $N = L$, then the DFT become,

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2, \cdots -2, -1 \end{cases}$$

Thus, there is only one non-zero value in DFT. This is apparent from observation of $X(\omega)$ since $X(\omega) = 0$, at the frequencies $\omega_k = 2\pi k/2$ $k \neq 0$. The reader should verify that $x(n)$ can be recovered from $X(k)$ by performing an L-point DFT.

(b): first we will find matrix W_4,

$$W_N^{k+N/2} = -W_N^k$$

~~the~~ matrix W_4 may be exposed as,

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

then,

$$Y_4 = W_4 V_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of Y_4 may be determined by conjugating ~~the~~ element in W_4 to obtain W_4' and then applying the formula.