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Paper: Signal and System.

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x ——————————— x

Q No: 1 (a)

Show with a help of an equation.-- –

Sol: Let $x(t)$ be a continuous time signal with a fourier Transform of $X(j\omega)$

i.e

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Differentiating both side with t.

$$\frac{d\,x(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,\frac{d}{dt}\left\{e^{j\omega t}\right\} d\omega.$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} X(j\omega)\left\{e^{j\omega t}\cdot j\omega\right\} d\omega\right.$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{j\omega X(j\omega)\right\} e^{j\omega t}\, d\omega$$

$$F\left\{\frac{d}{dt}x(t)\right\} = j\omega\, X(j\omega)$$

Result: we conclude that if a fun is differential in time domain it is Multiplied by $j\omega$ in frequency domain -

x———————————x

(2)

(b)

If $x(n) = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h(n) = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Produce $Y(z)$ and $Y(n)$.

Sol: $X(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + z^{-1} + 2z^{-2}$

Now $Y(z) = H(z) * X(z)$

$= (3 + z^{-1} + 2z^{-2}) * (2 - 4z^{-2} + 2z^{-3})$

$\cancel{= 2 - 4z}$

$Y(z) = 6 - 12z^{-2} + 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4}$

$+ 4z^{-2} - 8z^{-4} + 4z^{-5}$

$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

To find $Y[n]$ Use the delay Property.

$Y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3]$

$- 6\delta[n-4] + 4\delta[n-5]$.

X ———————— X

Q:2

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} f(x)\,dx + \frac{1}{2\pi}\int_{0}^{\pi} f(x)\,dx.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} -\pi/2\,dx + \frac{1}{2\pi}\int_{0}^{\pi} \pi/2\,dx$$

$$= \frac{-\pi}{4\pi}\left[x\Big|_{-\pi}^{0}\right] + \frac{\pi}{4\pi}\left[x\Big|_{0}^{\pi}\right]$$

$$= \frac{-\pi}{4\pi}\left[0-(-\pi)\right] + \frac{\pi}{4\pi}\left[\pi - 0\right]$$

$$= \frac{-\pi}{4\pi}(\pi) + \frac{\pi}{4\pi}(\pi)$$

$$a_0 = -\pi/4 + \pi/4$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \cos nx\,dx + \int_{0}^{\pi} \pi/2 \cos nx\,dx.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} \pi/2 \cos nx\,dx$$

$$= \frac{-\pi}{2\pi}\left[\int_{-\pi}^{0} -\cos nx\,dx\right] + \left[\frac{\pi}{2\pi}\int_{0}^{\pi} \cos nx\,dx\right]$$

$$= \frac{-\pi}{2\pi}\left[- \sin nx\Big|_{-\pi}^{0} + \frac{\pi}{2\pi}\left[\sin nx\Big|_{0}^{\pi}\right]\right]$$

$$= \frac{-\pi}{2\pi}\left(-\left[\sin 0 - \sin(-\pi)\right]\right) + \frac{\pi}{2\pi}\left[\sin\pi - \sin 0\right]$$

$$= \frac{-1}{2}\left[-(0-0) + \frac{1}{2}\left[0-0\right]\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\,dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} \pi/2 \sin nx\,dx.$$

$$= \frac{1}{\pi}\cdot\frac{-\pi}{2}\int_{-\pi}^{0} \sin nx\,dx + \frac{1}{\pi}\cdot\frac{\pi}{2}\int_{0}^{\pi} \sin nx\,dx$$

$$= \frac{1}{\pi}\cdot\frac{\pi}{2}\left[-\cos nx\Big|_{-\pi}^{0} + \frac{1}{\pi}\cdot\frac{\pi}{2}\left[\cos nx\Big|_{0}^{\pi}\right]\right].$$

$$= \frac{\pi}{2\pi}\left(-\left[\cos 0 - \cos(-\pi)\right] + \frac{\pi}{2\pi}\left[\cos\pi - \cos 0\right]\right)$$

$$= \frac{\pi}{2\pi}\left(-1(1+1)\right) + \frac{\pi}{2\pi}\left[-1-1\right]$$

4(b)

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even.} \\ 2/n\pi & \text{if } n \text{ is odd} \end{cases}$$

$$f(x) = \frac{2}{1\pi} \sin x + \frac{2}{3\pi} \sin 3x + \frac{2}{5\pi} \sin 5x$$

$$+ \frac{2}{7\pi} \sin 7x \text{ - - - - - -}$$

x ———————————————— x

Q No: 3

If $X(z) = \frac{2z^2+2z}{z^2+2z-3}$

Retrive $x[n]$ Using Inverse Z-Transform

Sol: $X(z) = \frac{2z^2+2z}{z^2+2z-3}$

$X(z) = \frac{2z(z+1)}{z^2+3z-z-3}$

$X(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$

$\frac{X(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$

or $\frac{2(z+1)}{z^2+2z-3} = \frac{A}{(z+3)\cancel{(z-1)}} + \frac{B}{(z-1)}$

Or $\Rightarrow$ Put

$2(z+1) = A(z-1) + B(z+3) \Rightarrow (1)$

Put $z = 1$ in eq (1).

$2(1+1) = B(1+3)$

$4 = 4B$

$\boxed{B = 1}$

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Put $z = -3$ in eq ①.

$$2(-3+1) = A(-3-1)$$

$$-4 = -4A$$

$$\boxed{A = 1}$$

Now put A, and B in eq ①

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{(z-1)}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

Inverse z - Transform.

$$X[k] = u[k] + 1(-1)^k.$$

X ———— X

Q No: 4

Express the Transfor f/n Using given Data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

$$G(s) = C\left[SI - A\right]^{-1} B + D$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s(s+2)+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \left\{ \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix} \right.$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$$

[num, den] = ss2tf(A, B, C, D)

[A, B, C, D] = tf2ss[num, den]

Q No: 5

Apply Fourier Transform on the signal

$x(t) = e^{-a|t|} u(t)$ . . . . . .

$X(j\omega) = ?$

Sol: The fourier Transform of the given fun $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

Note:
$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left. \frac{e^{(a-j\omega)t}}{a-j\omega} \right|_{-\infty}^{0} + \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

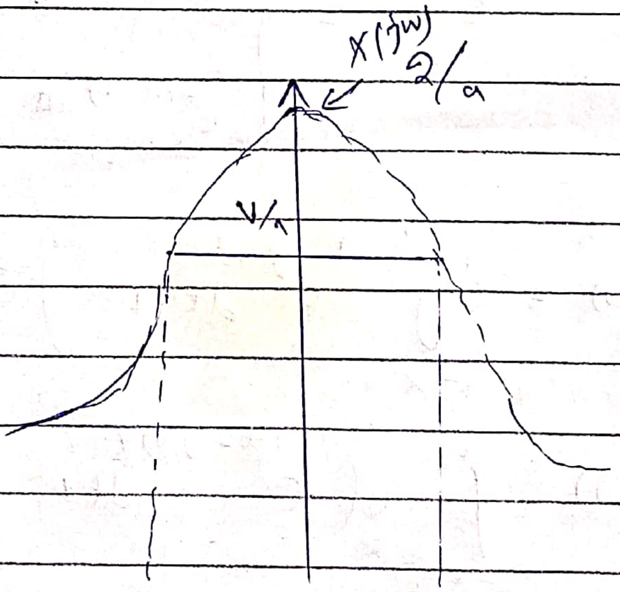
$$= \frac{1}{(a-j\omega)} \left[e^{0} - e^{-\infty} \right] - \frac{1}{(a+j\omega)} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{1}{(a-jw)}\,[1-0] - \frac{1}{(a+jw)}\,[0-1]$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw}$$

$$= \frac{a+\cancel{jw}+a-\cancel{jw}}{a^2-(jw)^2}$$

$$X(jw) = \frac{2a}{a^2+w^2}$$



X ———————— X

The end.