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$$Q. \int_0^1 \frac{4t^3 - 2t^2 + 3t - 1}{2t^2 + 1} dt$$

Soln.

$$\int_0^1 \frac{4t^3 - 2t^2 + 3t - 1}{2t^2 + 1} dt$$

$$= \int_0^1 \frac{4t^3 + 3t - 2t^2 - 1}{2t^2 + 1} dt$$

$$= \int_0^1 \frac{t(4t^2 + 3) - (2t^2 + 1)}{2t^2 + 1} dt$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1} dt - \int_0^1 \frac{2t^2 + 1}{2t^2 + 1} dt$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1} dt - \int_0^1 1 dt$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1} dt - [1 - 0]$$

$$= \int_0^1 \frac{t(4t^2 + 3)}{2t^2 + 1} - 1 \rightarrow \textcircled{1}$$

Now

$$\Rightarrow \text{let } 2t^2 + 1 = y \Rightarrow 2t^2 + 1 = y$$

$$\Rightarrow \text{As } t \rightarrow 1 \text{ i.e. } y = 3$$

$$\Rightarrow t \rightarrow 0 \text{ i.e. } y = 1$$

$$2t^2 = y - 1$$

$$4t^2 = 2y - 2$$

$$4t^2 + 3 = 2y - 2 + 3$$

$$4t^2 + 3 = 2y + 1$$

Now Diff

$$\Rightarrow 4t = \frac{dy}{dt}$$

$$\Rightarrow dt = \frac{dy}{4t}$$

$$= \int_1^3 \frac{t(2y + 1)}{y} \cdot \frac{dy}{4t} - 1$$

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$$= \int_1^3 \frac{2y+1}{4y} dy - 1$$

$$= \frac{1}{4} \left[\int_1^3 \frac{2y}{y} dy + \int_1^3 \frac{1}{y} dy \right] - 1$$

$$= \frac{1}{4} \left[\int_1^3 2 dy + \int_1^3 \frac{1}{y} dy \right] - 1$$

$$= \frac{1}{4} \left[2y \Big|_1^3 + \ln y \Big|_1^3 \right] - 1$$

$$= \frac{1}{4} \left[2(3) - 2(1) + \ln(3) - \ln(1) \right] - 1$$

$$= \frac{1}{4} \left[6 - 2 + 1.0986 \right] - 1$$

$$= \frac{1}{4} \left[5.0986 \right] - 1$$

$$= 1.27465 - 1$$

$$= 0.2746$$

Ans

(Q3) Find $\int_2^3 t \sin t^2 dt$

Sol:

Let $t^2 = u$

Then $2t = \frac{du}{dt} \Rightarrow t dt = \frac{du}{2}$

As $t \rightarrow 2$ Then $u \rightarrow 4$

$t \rightarrow 3$ Then $u \rightarrow 9$

So the given question will transform as,

$$\int_2^3 t \sin t^2 dt = \int_4^9 \sin u \frac{du}{2}$$

$$= \frac{1}{2} \int_4^9 \sin u du$$

$$= \frac{1}{2} [-\cos u]_4^9$$

$$= \frac{1}{2} (-\cos 9 - (-\cos 4))$$

$$= \frac{1}{2} (\cos 4 - \cos 9)$$

$$= \frac{1}{2} (0.998 - 0.987)$$

$$= 0.0055$$

Ans