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Q=1 (a) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$ identify the (3,2) entry of AB.

Sol:- $row_3(A) \cdot Col_2(B)$

$= \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} =$

$= 0 \times 4 + 1 \times (-1) + (-2) \times 2$

$= 0 - 1 - 4$

$= \boxed{-5}$

Q=1 (b) Label the quadratic Polynomial That Interpolate The Points (1,3),(2,4),(3,4)

Sol:- As

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2 \quad \text{— ①}$$
$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now $(x_1, y_1) = (1,3)$ $(x_2, y_2) = (2,4)$ $(x_3, y_3) = (3,4)$

Put in above equations

$$a_2 + a_1 + a_0 = 3$$
$$4a_2 + 2a_1 + a_0 = 4$$
$$9a_2 + 3a_1 + a_0 = 4$$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 4 \end{array}\right]$$

$$\underset{\sim}{R} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 9 & 3 & 1 & 4 \end{array}\right] \quad R_2 - 4R_1$$

$$\underset{\sim}{R} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -23 \end{array}\right] \quad R_3 - 9R_1$$

③

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_3 - 3R_2$$

So
$$a_2 + a_1 + a_0 = 3 \quad \text{—— (1)}$$
$$-2a_1 - 3a_0 = -8 \quad \text{—— (2)}$$

Here $\boxed{a_0 = 1}$

put the value of a_0 in eq (2)

$$-2a_1 - 3a_0 = -8$$
$$-2a_1 - 3(1) = -8$$
$$-2a_1 = -8 + 3$$
$$\boxed{a_1 = \frac{5}{2}} \quad \text{or} \quad \boxed{a_1 = 2.5}$$

put the value of a_1 and a_0 in eq 1

$$a_2 + \frac{5}{2} + 1 = 3$$
$$a_2 = 3 - \frac{5}{2} - 1$$
$$\boxed{a_2 = -0.5}$$

So $\boxed{y = -0.5x^2 + 2.5x + 1}$

Q=2 (a) if A & B are $n \times n$ matrices where $|A| = 2$
& $|B| = -3$ Calculate $|A^{-1}B^{T}|$.

Sol:- Since $|A^{-1}B^{t}| = |A^{-1}||B^{t}|$

$= \frac{1}{|A|} \cdot |B|$ become $|B^{t}| = |B|$

So $|A^{-1}B^{t}| = \frac{1}{|A|} \cdot |B|$

$= \frac{1}{2} \cdot 3 = \boxed{3/2}$

(5)

Q=2 (b) Estimate the linear System of equation

$$x+y+2z=1$$
$$x-2y+z=-5$$
$$3x+y+z=3$$

Sol:-

$$A=\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 3 & 1 & 1 & 3 \end{array}\right] \quad R_2-R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \quad R_3-3R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & 0 & -13 & 12 \end{array}\right] \quad 3R_3-2R_2$$

$$x+y+2z=1 \quad \text{——} \quad (1)$$
$$-3y-z=-6 \quad \text{——} \quad (2)$$
$$-13z=12 \quad \text{——} \quad (3)$$

$$\frac{\cancel{-13}z}{\cancel{-13}}=\frac{12}{-13} \qquad \boxed{z=-0.92}$$

Put the value of z in eq (2)

$$-3y-(-0.92)=-6$$
$$-3y+0.92=-6$$
$$-3y=-6-0.92$$
$$\frac{\cancel{-3}y}{\cancel{-3}}=\frac{+6.92}{+3} \qquad \boxed{y=2.30}$$

Put The Value of $y^{(6)}$ & Z in eq (1)

$$x + 2.30 + 2(-0.92) = 1$$

$$x + 2.30 - 1.84 = 1$$

$$x = 1 - 2.30 + 1.84$$

$$\boxed{x = 0.54}$$

Q=3 Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Sol:-

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$= 3\begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-4-6) + 2(-15-2) + 1(0-6)$$

$$|A| = -94$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

Now $A_{11} = (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & \underline{1} \\ 6 & 2 & -28 \end{bmatrix}$$