Q No (1)

Solve the following objectives type question.

(i)

Ans:- $m \times n$

(ii)

(Ans): One.

(iii) $\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = a \times 1 - 2 \times 4$

$= a - 8 = 0$

$\Rightarrow a = 8$

(iv)

(Ans): $|A| = 2i(-i) - i(i)$

$= 2i^2 - i^2$

$= -2(-1) - (-1)$

$= 2 + 1$

$= 3$

(v) (Ans):.

Scalar matrix

(vi) (Ans):

$$\frac{dy}{dx} + 2xy = y$$

Separting variables

$$\Rightarrow \frac{dy}{dx} = y - 2xy$$

$$\Rightarrow \frac{dy}{dx} = y(1-2x)$$

$$\Rightarrow \frac{dy}{y} = (1-2x)dx$$

$$\Rightarrow \int \frac{1}{y} dy = \int 1 dx - \int 2x dx$$

$$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$\Rightarrow \ln y = x - x^2 + C$$ Ans

(vii)

(Ans):-

Order = 1

Degree = 3

(viii)

(Ans):-

Order = 2

Degree = 1.

(ix)

(Ans):-

2

(x)

(Ans):-

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= bc^2 - cb^2 - a(c^2 - b^2) + a^2(c-b)$$

$$= bc(c-b) - a(c-b) + a^2(c-b)$$

$$= (c-b)(bc - a + a^2)$$

Que 2

(i) express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c

Sol:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

expand by R1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & b^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2)$$
$$+ c(a^2b^3 - a^3b^2)$$

$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$

Common abc

$\Rightarrow abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$\Rightarrow abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right]$

Ans

Q NO 2

(ii):

find the Eigen value.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equ $\rightarrow |A-\lambda I| = 0 \rightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determinant

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \to (B)$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[(3-\lambda)(2-\lambda) - (-1)(-1)\right) + 1((-1)(2-\lambda) - (-1)(-1)) - 1$$
$$((-1)(-1) - (-1)(3-\lambda))\big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \to (a)$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1\,(6-3\lambda-2\lambda+\lambda^2-1) + 1\,(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C1

$$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -[-(-2+\lambda-1) + 1\,(6-3\lambda-2\lambda+\lambda^2-1)$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

Put (a), (b) & (c) in (R)

$(2-\lambda)[-\lambda^3+8\lambda^2-12\lambda+8]-\lambda^3+6\lambda-8-\lambda^2$
$+6\lambda-8$

$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$
$+6\lambda-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$
$+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

we get <u>By synthetic division</u>

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$(\lambda=0)$

$\lambda-2=0 \Rightarrow \lambda=2$

$\lambda^2-8\lambda+16=0$

<u>By factorization method</u>

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)$

$\lambda-4=0, \quad \lambda-4=0$

$\Rightarrow \lambda=4, \quad \lambda=4$

$\boxed{\lambda_1=0, \quad \lambda_2=2, \quad \lambda_3=4, \quad \lambda_4=4.}$ Ans

6

Que #3

The rate of change in the form of differential equation is given by.

$(x^2+3y^2)dx - 2xdy = 0$, find the general solution at $x=2$ & $y=6$

$$(x^2+3y^2)dx - 2xdy = 0$$

$$x=2, \quad y=6$$

Solution:

$$(x^2+3y^2)dx - 2xydy = 0$$

$$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$$

$\div$ing b/s by $2xy\,dx$.

we get.

$$\frac{dy}{d\cancel{x}} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2x\cancel{y}} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \to 1$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \to 1$$

let $y = v\cancel{x}$

di~~ff~~

$$dy = v\,dx + x\,dv$$

$\div$ing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \to a$$

put a in eq 1

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{x}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying b/s by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v.$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

xing b/s by $\frac{dx}{dv}$.

we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

xing b/s by $\frac{v}{x(1+v^2)}$.

we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx.$$

Take "$\int$" on b/s

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Tak 'e' on b/s

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

put $v = y/x$.

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \rightarrow (2)$$

put $x=2$, $y=6$ in eq (2)

$$(4)+36 = 8c$$

$$c = \frac{40}{8} \; 5$$

$$\boxed{c=5} \rightarrow \text{put in eq (2)}$$

so. $x^2+y^2 = 5x^3$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

take $\sqrt{\ }$ on b.t.s

$$\sqrt{y^2} = \sqrt{x^2(5x-1)}$$

$$y = +x\sqrt{5x-1},\quad y = -x\sqrt{5x-1} \quad \text{or}$$

$$y = \pm x\sqrt{5x-1} \quad \text{Ans.}$$

x ———————— x

NAME = M. Atif Khan

ID = 7901

Sec = A.

Subject = Differential equation

Miss Shumaila Paper

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