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SUBJECT: DIFFERENTIAL EQUATIONS
EXAM: MID-TERM (SPRING - 2020)

Q. No. 1:- Solution of Objective type Questions.

(i) The order of matrix $A$ is $m \times P$ & the order of matrix $B$ is $P \times n$ then the order of matrix $AB$ is?

Ans:-

Order of matrix $AB$ = rows of matrix $A$ × column of matrix $B$

order of matrix $AB$ $= m \times n$

(ii) The number of non-zero rows in an Echelon Form?

Ans:-

The number of non-zelo rows in an Echelon form are called rank of the matrix.

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Ans:- we know $|B| = 0$

$$\Rightarrow \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$$

$$\Rightarrow a - 8 = 0$$

$$\boxed{a = 8}$$

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(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Ans:-

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = -2i^2 - i^2$$

$$|A| = -2(-1) - (-1) \qquad \because i^2 = -1$$

$$\boxed{|A| = 3}$$

(v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Ans:-

A is a scalar matrix.

(vi) Solution of $\frac{dy}{dx} + 2xy = ?$

Ans:-

$$\frac{dy}{dx} = y - 2xy \Rightarrow dy/dx = y(1-2x)$$

$$dy/y = (1-2x)dx$$

$$\int dy/y = \int (1-2x)\,dx$$

$$\ln y = (x - x^2) \cancel{x} + c_1$$

$$e^{\ln y} = e^{(x-x^2)+c_1}$$

$$\boxed{y = e^{x-x^2} \cdot e^{c_1}}$$

(vii) The order & degree of Diff Eq $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(dy/dx\right)^2}$ is ?

Ans:- . Taking Square on Both sides of D. Eq

$$\left(\frac{dy}{dx}\right)^6 = 1 + \left(dy/dx\right)^2$$

So Degree = 6 & order = 1.

(viii) The order & Degree of Diff Eq $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ ?

Ans:- Order = 2

& Degree is undefined.

(ix) The Diff Eq $2\frac{dy}{dx} + x^2 y = 2x+3$ , $y(0)=5$ is ?

Ans:-

$\Rightarrow y' + (x^2/2)y = 1/2(x^2+3) \Rightarrow \mu = x^2/2$

$e^{\int x^2/2\, dx} = e^{x^3/6} \Rightarrow e^{x^3/6} y' + e^{x^3/6}(x^2/2)y = \frac{1}{2}e^{x^3/6}(x^3+3)$

$y(0) = 3/2$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + 3/2$$

(x) If $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 1(bc^2 - b^2c) - 1(ac^2 - a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$$

$$= a^2c - a^2b + ab^2 - b^2c + bc^2 - ac^2$$

Q.No.2 (i):-

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$ as the product of factors which are linear in a, b, c.

Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$\Rightarrow a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

$$\Rightarrow a^3bc^2 - a^3b^2c + a^2b^3c - a^2bc^3 + ab^2c^3 - ab^3c^2$$

$$\Rightarrow abc\,(a^2c - a^2b + ab^2 - ac^2 + bc^2 - b^2c)$$

Q. No. 2 - (ii) Find the Eigen values

Sol.

Let $A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$ & '$\lambda$' is the eigen value of $A$.

So

$$\det[A - \lambda I] = 0$$

$$\Rightarrow \det \left\{ \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \right\} = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda) \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\left[(3-\lambda)\{(3-\lambda)(2-\lambda)-1\} + 1\{(-1)(2-\lambda)-1\} - 1\{1-(-1)(3-\lambda)\}\right]$$

$$+ \left[-1\{(3-\lambda)(2-\lambda)-1\} + 1\{(-1)(2-\lambda)-0\} - 1\{(-1)(-1)-0\}\right]$$

$$- \left[-1\{(-1)(2-\lambda)-1\} - (3-\lambda)\{(-1)(2-\lambda)-0\} - 1\{(-1)(-1)-0\}\right] = 0$$

$$\Rightarrow (2-\lambda)\{(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (\lambda-3) - (4-\lambda)\} + \{-1(6-3\lambda-2\lambda+\lambda^2-1) + (\lambda-2) - 1\}$$

$$-1\{-1(\lambda-3) - (3-\lambda)(\lambda-2) - 1\} = 0$$

$$\Rightarrow (2-\lambda)(3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda+2\lambda-7)+(2\lambda^2+6\lambda-8)-(3\lambda-3+6-\lambda^2-2\lambda)=0$$

$$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow -2\lambda^3+16\lambda^2+36\lambda+\cancel{16}+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-2\lambda^2+12\lambda-\cancel{16}=0$$

$$\Rightarrow \lambda^4+10\lambda^3+32\lambda^2-32\lambda=0$$

$$\lambda(\lambda^3-10\lambda^2+32\lambda-32)=0$$

$$\Rightarrow \lambda=0,\quad \lambda^3-10\lambda^2+32\lambda-32=0$$

Now $\lambda^3-10\lambda^2+32\lambda-32=0$

at $\lambda=2$

$$(2)^3-10(2)^2+32(2)-32=0$$

$$8-40+64-32=0$$

$$\cancel{72}-\cancel{72}=0$$

Using Synthetic Division

$$\begin{array}{c|cccc} & 1 & -10 & 32 & -32 \\ 2 & \downarrow & 2 & -16 & 32 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

So, $\lambda^2-8\lambda+16=0$

$$\Rightarrow \lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)=0$$

$$\Rightarrow (\lambda-4)=0,\quad (\lambda-4)=0$$

$$\lambda=4,\quad \lambda=4$$

So the required Eigen values are 0, 2, 4, 4.

Q.No.3:- Find the General Solution at given values of

$(x^2+3y^2)dx - 2xydy = 0$ , $x = 2$ & $y = 6$

Sol.

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

Let $u = y/x$

$$\Rightarrow y = ux$$

$$\frac{dy}{dx} = \frac{d\,ux}{dx}$$

$$\frac{dy}{dx} = u(1) + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{x^2+3(ux)^2}{2x(ux)}$$

$$u + x\frac{du}{dx} = \frac{x^2+3u^2x^2}{2ux^2}$$

$$u + x\frac{du}{dx} = \frac{\cancel{x^2}(1+3u^2)}{2u\cancel{x^2}}$$

$$u + x\frac{du}{dx} = \frac{1+3u^2}{2u}$$

$$x\frac{du}{dx} = \frac{1+3u^2}{2u} - u$$

$$x\frac{du}{dx} = \frac{1+3u^2-2u^2}{2u}$$

$$x\frac{du}{dx} = \frac{1+u^2}{2u}$$

$$x\,du = \frac{1+u^2}{2u}\,dx$$

$$\Rightarrow \frac{2u}{1+u^2}\,du = \frac{dx}{x}$$

integrating $\Rightarrow \int \frac{2u}{1+u^2}\,du = \int \frac{dx}{x}$

$$\Rightarrow \ln(1+u^2) = \ln x + \ln c$$

$$1+u^2 = cx$$

$$\Rightarrow \frac{x^2+y^2}{x^2} = cx$$

Now putting $x=2$ & $y=6$ in the above equation

$$\frac{4+36}{4} = 2c$$

$$\Rightarrow 2c = 40/4$$

$$\Rightarrow \boxed{c=5}$$

So the general solution is;

$$\frac{x^2+y^2}{x^2} = 5x$$

$$\Rightarrow \boxed{x^2+y^2 = 5x^3}$$