

Date

ASSIGNMENT:

DIFFERENTIAL
EQUATION

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PARTIAL DIFFERENTIAL EQUATION

EQUATION

- 1: It is derived as an equation involving two or more independent variables like x, y, \dots , a dependent variable like u and its partial differentiation derivatives.
- 2: Partial differential equation can be formed either by elimination of arbitrary constants or by the elimination of arbitrary functions from a relation involving three or more variables.

APPLICATIONS

TANGENT PLANES AND LINEAR APPROXIMATIONS :

Suppose a surface "s" has equations $z = f(x, y)$ where f has continuous first partial derivatives and set $P(x_0, y_0, z_0)$ be a

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point on "S". Let C_1 and C_2 be the two curves obtained by the intersection of the vertical planes $y = y_0$. Thus point "P" lies on both C_1 and C_2 . Let T_1 and T_2 be the tangent lines to the curves C_1 and C_2 at the point "P". Then the tangent plane to the surface S at the point P is defined to be the plane that contains both tangent lines T_1 and T_2 .

- Suppose a surface S has equation $z = F(x, y)$ where F has continuous first partial differential derivatives.
- Let $P(x_0, y_0, z_0)$ be a point on S .
- $P(x_0, y_0, z_0)$ has an equation of the form:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

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2. LINEAR APPROXIMATIONS AND LINEARIZATION

The linearization of F at (a, b) is the linear function whose graph is the tangent plane

- $F(x, y) = F(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$

- The approximation :

$$f(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

- is called the linear approximation of f at (a, b) .

- Recall that Δx and Δy are increments of x and y respectively. If $z = f(x, y)$ is a function of two variables the Δz - the increment of z is defined to be

- $\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$

- If $z = f(x, y)$ the f is differentiable at (a, b) . If Δz can be

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expressed in the form.

- where ϵ_1 and $\epsilon_2 \rightarrow 0$ as $(\Delta x, \Delta y) \rightarrow (0, 0)$.

THEOREM

$$dz = f_x(a, b) dx + f_y(a, b) dy = \frac{dz}{dx} dx + \frac{dz}{dy} dy$$

TAYLOR EXPANSION :

Let $f(x)$ be given as the sum of power series in the convergence interval of the power series.

$$f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

- Then such power series is unique and its coefficient are given by formula.

$$a_n = \frac{f^{(n)}(x_0)}{n!}$$

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- If function $f(x)$ has derivatives of all orders at x_0 then we can formally write the corresponding Taylor series as:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!} (x - x_0) + \frac{f''(x_0)}{2!} (x - x_0)^2 + \dots$$

- The power series when created in this way is then called the TAYLOR SERIES of function $f(x)$.
- These are function $f(x)$.

TAYLOR SERIES OF SOME FUNCTION:

- $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

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MAXIMA AND MINIMA

- The least and the greatest.
- Many problems that arises in mathematics call for finding the largest and smallest values that a differentiable function can assume on a particular domain.

There is a strategy for solving these applied problems.

STRATEGY FOR MAX-MIN PROBLEMS

Draw Picture .

label the parts that are important for the problem.

Keep track of what variable represent

Use a known formula for the quantity to be maximized or minimized.

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- Write an equation. Try to express the quantity that is to be maximized or minimized as a function of single variable so $y = f(x)$.

LAGRANGE METHOD:

Many time a stationary value of the function of several variables which are not all independent but connected by some relationship is needed to be known. Generally we do convert the given functions to the one having least number of independent variables with the help of these relation then it solved.

- Lagrange method proved to be very convenient, which is explained on the ongoing lines.