Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: Sir Pir Meher **Total Marks:** 50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7** **CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks 7** **CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6** **CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks 6** **CLO 2**
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition H(0) = 1 and $\left\lvert H\left(\frac{\pi}{4}\right)\right\rvert^2 = \frac{1}{2}$.	**Marks 6** **CLO 3**

	(b)	Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.	**Marks** **6** **CLO** **3**
Q 4	(a)	A finite duration sequence of Length L is given as $x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$ Determine the N- point DFT of this sequence for N ≥ L	**Marks** **6** **CLO** **2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $x_1(n) = \left\{\underset{\uparrow}{2}, 1, 2, 1\right\}$ $x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$	**Marks** **6** **CLO** **2**

IQRA NATIONAL UNIVERSITY PESHAWAR

Final Term

Digital Signal Processing

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Q1:- (Part - a)

★ Solution:-

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \quad \text{Hence,}$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is;

$$y_p(n) = K(-1)^n u(n)$$

Substituting this solution into the difference equation, we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $K(1+4+4) = 2 \Rightarrow K = 2/9$

The total solution is;

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \tfrac{2}{9}(-1)^n\right] u(n)$$

From the initial conditions we obtain $y(0) = 1$, $y(1) = 2$ then, $C_1 + 2/9 = 1$

$$\Rightarrow C_1 = 7/9$$

$$2C_1 + 2C_2 - 2/9 = 2$$

$$\Rightarrow C_2 = 1/3$$

Q1:- (Part-b)

★ Solution:-

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

~~y(n) = 4y(n~~

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = 1/2, 1/5$ hence

~~$y_h(n) - 0.7y(0) =$~~

$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence, $C_1 + C_2 = 2$

& $\frac{1}{2}C_1 + \frac{1}{5} = 1.4 = 7/5$

$$\Rightarrow C_1 + \frac{2}{5}C_2 = 14/5$$

These equations yield

$$C_1 = \frac{10}{3}, \quad C_2 = -4/3$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

The step response is;

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}^{n} (2^{n+1} - 1\right) u(n) - \frac{1}{3} \left(\frac{1}{5}^{n} (5^{n+1} - 1\right) u(n)$$

Q 2 :- (Part - a)

*Solution:-

$$x(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})^2}$$

$$x(z) = \frac{1}{4}\,\frac{z^{-1}}{(1-2z^{-1})^2} + \frac{3}{4}\,\frac{1}{1-2z^{-1}} + \frac{1}{2}\,\frac{z^{-1}}{(1-z^{-1})^2}$$

By applying inverse transform we obtain

$$x(n) = \frac{1}{8}(-1)^n u(n) - \frac{3}{8}u(n) + \frac{1}{2}n\,u(n) =$$

$$\left[\frac{1}{8}(-1)^n + \frac{3}{8} + \frac{n}{2}\right]u(n)$$

Q2:- (Part-b)

* Solution:-

We have

$$x(n) = \frac{1}{2\pi j}\oint_c \frac{z^{n-1}}{1-az^{-1}}\,dz = \frac{1}{2\pi j}\oint_c \frac{z^n\,dz}{z-a}$$

where c is a circle at radius greater than |a|. We shall evaluate this integral with $f(z) = z^n$. We distinguish two cases

Case 1:-

If $n \geq 0$ has only zeros & hence no poles inside c. The only pole inside c is $z = a$. Hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

Case 2:-

If $n < 0$, $f(z) = z^n$ has n^{th} order pole at $z = 0$ which is also inside c. thus there are contributions from both poles. For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}\,dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n = -2$ we have,

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}\,dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing is the same way we can show that $x(n) = 0$ for $n < 0$ thus, $x(n) = a^n u(n)$

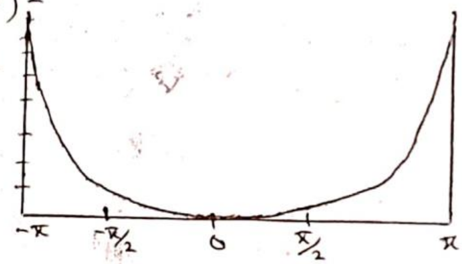
Q 3:- (Part – a)

★ Solution:-

At $\omega = 0$ we have

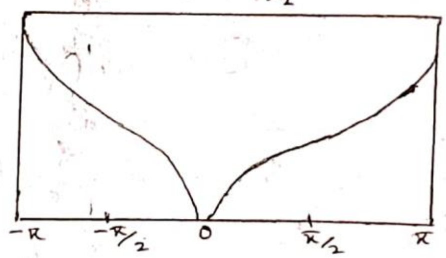
$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

Hence $b_0 = (1-P)^2$



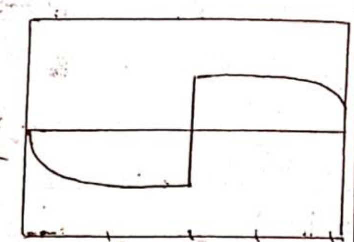
At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$



$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+(jP\sin(\pi/4)))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}\; jP/\sqrt{2})^2}$$



Hence $\dfrac{(1-P)^4}{[(1-P/\sqrt{2})^2+P^2/2]^2} = 1/2$

Or equivalently

$$\sqrt{2}\,(1-P)^2 = 1+P^2-\sqrt{2}P$$

The value of $P = 0.32$ satisfies this equation. Consequently, the system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principles can be applied for the design of bandpass filters.

Q 3:- (Part - b):-

★ **Solution:-**

Clearly, the filter must have poles at

$$P_{1,2} = re^{\pm j\pi/2}$$

& zeros at $z=1$ & $z=-1$

Consequently the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$

Thus we have

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

~~The~~

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

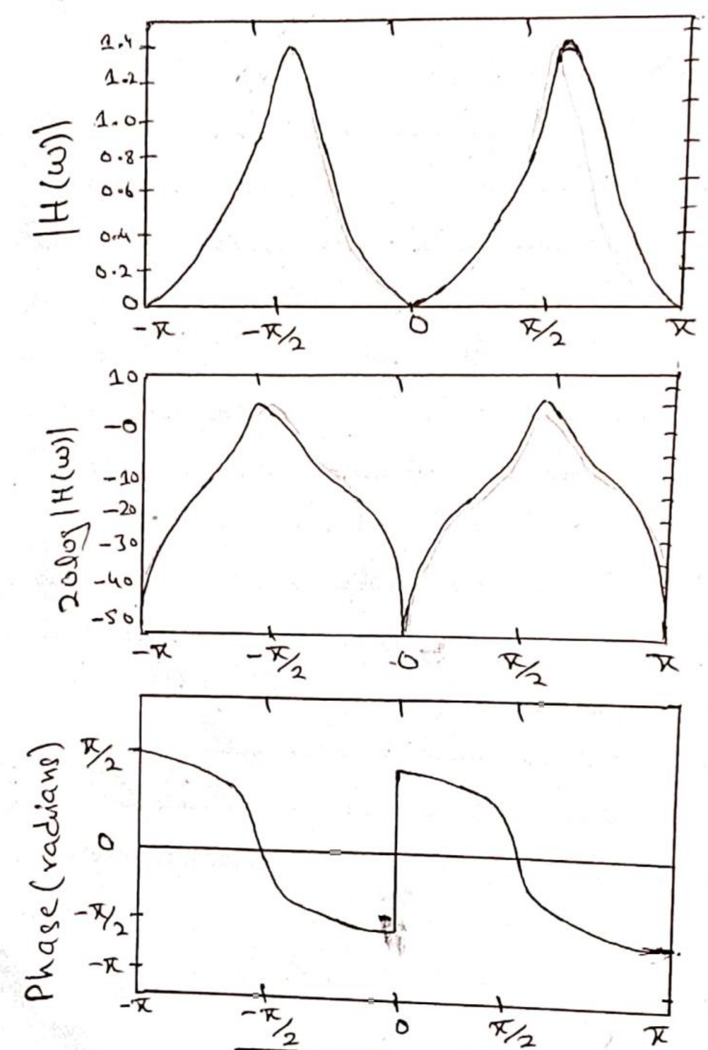
or equivalently,

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore, the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its frequency response is illustrated in below figure.



Q 4:- (Part-a)

Solution:-

The Fourier transform of this sequence ~~For~~ is;

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}}$$

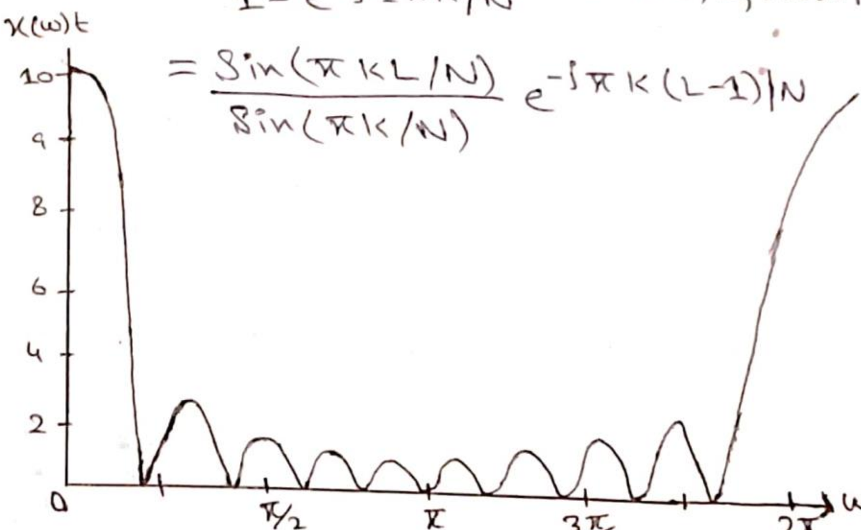
$$= \frac{\sin(wL/2)}{\sin(w/2)} e^{-jw(L-1)/2}$$

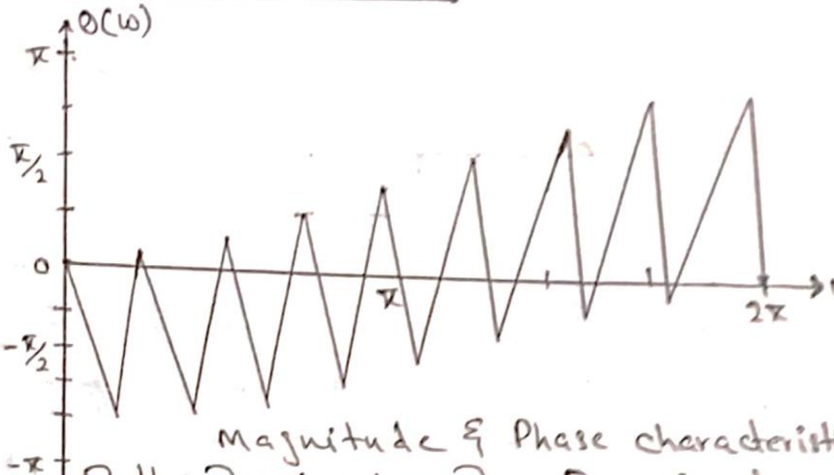
The magnitude & Phase of $X(w)$ are illustrated in below Figure for $L=10$. The N-Point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequencies

$W_K = 2\pi K/N$, $K = 0, 1, \ldots\ldots N-1$ Hence

$$X(K) = \frac{1-e^{-j2\pi KL/N}}{1-e^{-j2\pi K/N}} \quad K = 0, 1, \ldots\ldots N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





Magnitude & Phase characteristics of the Fourier transform for signal.

If N is selected such that $N=L$ then the DFT becomes

$$X(K)\begin{cases} L, & K=0 \\ 0, & K=1,2,\ldots L-1 \end{cases}$$

Thus there is only Non-zero value in DFT. This is apparent from observation of $X(\omega)$, Since $X(\omega)=0$ at the frequencies $\omega_K=2\pi K/L$ $K\neq 0$. The reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT.

Q4:- (Part-b)

★ Solution:- Each sequence consists of four non-zero points. For the purposes of illustrating the operations involved in circular convolution it is desirable to graph each sequence as points on a circle.

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as Begining with $m=0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded & graphed on a circle as illustrated below.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$, point by point. This sequence is also illustrated. Figs-8(b)

Finally, we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

It is easily verified that $x_2((1-4))_4$ is simply the sequence $x_2((-n))_4$ rotated counterclockwise by one unit in time as illustrated in Fig. 5-8(c).

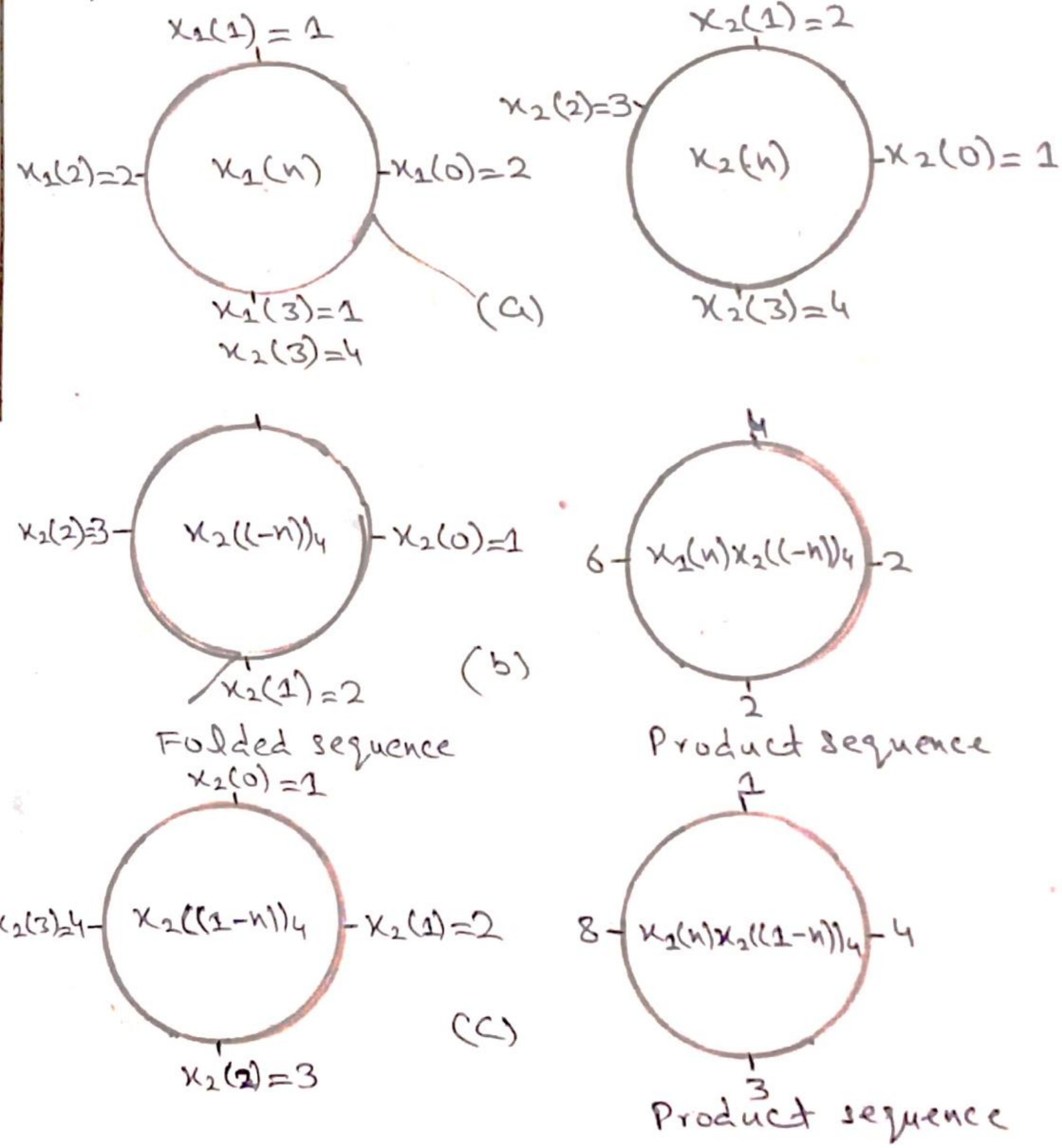
Finally we sum the values in the product sequence to obtain $x_3(1)$

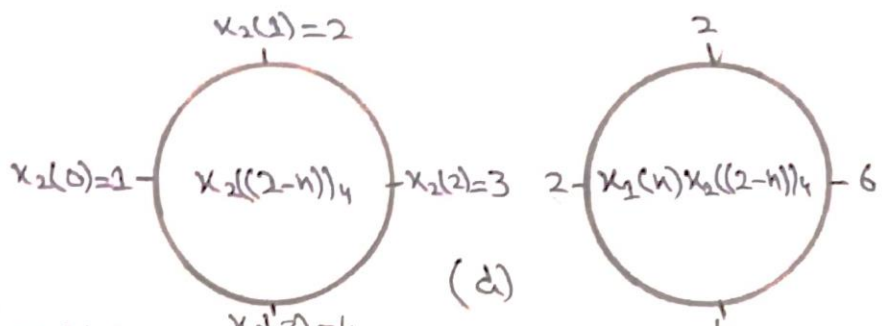
Thus, $x_3(1) = 16$

For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

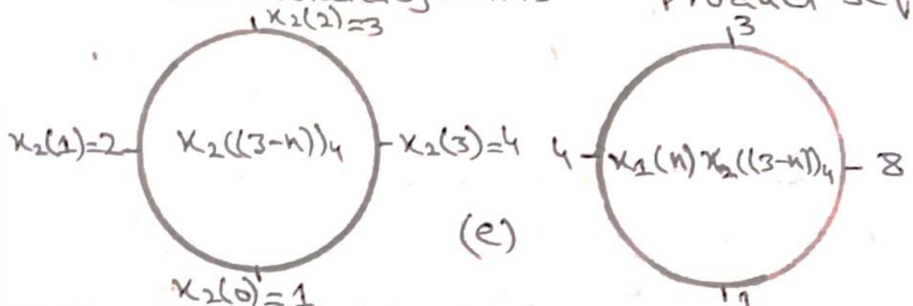
Now $x_2((2-n))_4$ is the folded sequence in Fig (b) rotated two units of time in the counter clockwise direction. The resultant sequence is illustrated in Fig. 5-8(d)





(d)

Folded sequence rotated by two times Product sequence



(e)

Folded sequence rotated by 3 times Product sequence

Along with the product sequence $x_1(n)x_2((2-n))_4$

By summing the four terms in the product sequence we obtain,

$$x_3(2) = 14$$

For $m=3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The folded sequence $x_2((-n))_4$ is now rotated by three units in time to yeild the product sequence as illustrated in Fig 5.8(e). The sum of the values in the product sequence is

$$x_3(3) = 16$$

we observe that if the computation above is continued beyond $m=3$, we simply repeat the sequence of four values obtained above. Therefore, the circular convolution of the two sequences $x_1(n)$ & $x_2(n)$ yields the sequence

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$