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Section	A
Subject	Differential equation
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Q No 1)

(i) The order of matrix

AB is $m \times n$

(ii) The number of non zero rows in an Echelon from is ONE

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = \underline{8}$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= -2i^2 - i^2 \qquad \because i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$= \underline{3}$

(v) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scalar Matrix.

The given Matrix A is a Scalar matrix because the diagonal elements are some and nondiagnal are zero.

(vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol $\frac{dy}{dx} + 2xy = y$

$$\frac{dy}{dx} = y - 2xy$$

(y Taking common)

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{dx} = (1-2x)\,dx$$

Taking integration

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$\ln y = e^{x-x^2+c}$$

$$e^{\ln y} = e^{x-x^2+c}$$

$$\boxed{y = e^{x(1-x)+c}}$$

(vii) The order and degree of differential equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^3} \text{ is}$$

order = $\underline{1}$

Degree = $\underline{3}$

(viii) The order and degree of differential equation

$$\frac{d^2y}{d^2x} - 4xy = \sin\left(\frac{d^2y}{d^2x}\right) \text{ is}$$

order = $\underline{\text{TWO}}$

Degree = $\underline{\text{ONE}}$

(xii) The differential equation $2dy/dx + x^2y = 2x+3$

$y=(0)=5$ is ______

Sol $2y' + x^2y = x^2+3,\ y(0)=5$

$$y' + (x^2/2)y = \frac{x^2+3}{2}$$

$$y' + (x^2/2)y = 1/2\,(x^2+3)$$

$$\mu = x^2/2$$

$$e^{\int \frac{x^2}{2}dx} = e^{x^3/6}$$

$$e^{x^3/6}y' + e^{x^3/6}\left(\frac{x^2}{2}\right)y = 1/2\, e^{x^3/6}(x^2+3)$$

$$y(x) = e^{x^3/6}\left(\frac{x^2}{2}\right)y = 1/2\, e^{x^3/6}(x^2+3)$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + 3/2$$

Q No 2 part "A"

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factor which are linear in a,b,c

Sol

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^2cb^3 - a^3b^2c$$

Common abc

$$abc(bc^2 - b^2c - ac^3 - a^2c + ab^2 - a^2b)$$

$$abc[bc(c-b) - ac(c-a) + ab(b-a)]$$

Ans

QNo2 Part B

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic equation

$$|A - \lambda I| = 0 \rightarrow (A)$$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Expand by R_1

$$2-\lambda \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} 0 - (-1) \begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

$$-1 \begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} = 0 \longrightarrow (B)$$

Again

$$\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$(3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big)+1\big((-1)(2-\lambda)-(-1)(-1)\big)$$

$$-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$=(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)=$$

$$(-1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda 3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \rightarrow (a)$$

$$\Rightarrow -1 \begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow -1 \begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix}-(-1)\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix}+0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$=$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow (c)$$

put (a), (b) and (c) in (B)

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2$$
$$+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3$$
$$+18\lambda^2-8\lambda$$

$$= \lambda^4-2\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda$$
$$-8\lambda+6\lambda+6\lambda+16-16$$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By Synthetic division

we get.

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization

Method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)(\lambda-4) = 0$

$\lambda = 4, \quad \lambda = 4$

$$\boxed{\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4}$$

Q.no 3

$$(x^2+3y^2)\,dx-2xy\,dy=0$$

$$x=2\ ,\ \ y=6$$

$$(x^2+3y^2)\,dx-2xy\,dx=0$$

$$(x^2+3y^2)\,dx=2xy\,dy$$

Dividing both sides by $2xydx$ we get.

$$\frac{dy}{dx}=\frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx}=\frac{x^{\cancel{2}}}{2\cancel{x}y}+\frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{x}{y}+\frac{3y}{x}\right]\longrightarrow (*)$$

Let $y=vx$

Diff :

$$dy = v\,dx + x\,dv$$

dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a)$$

put (a) in (*)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{xv} + 3\frac{vx}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multipy Both sides by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiply Both sides By "~~x~~" $\frac{dv}{dx}$

We get:-

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiply Both sides By $\frac{v}{x(1+v^2)}$

we get:-

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on Both sides

$$\int\frac{2v}{1+v^2}\,dv = \int\frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on Both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$1 + v^2 = xc$

put $v = y/x$

$1 + (y/x)^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$x^2 + y^2 = x^3 c \longrightarrow (**)$

put $x = 2$, $y = 6$, in $(**)$

$(4) + (36) = 8c$

$c = \frac{40}{8}$

$\boxed{c = 5} \longrightarrow$ put in $(**)$

So

$x^2 + y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x - 1)$

Taking "$\sqrt{}$" on Both Sides

(15)

$$\boxed{y = + x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}}$$

OR

$$\boxed{y = \pm x\sqrt{5x-1}}$$