

Q.1) Given Data:

$$F = 15067 \text{ N}, \quad E = 207,000 \text{ KPa},$$

$$E = 223 \times 10^6 \text{ KPa}, \quad \nu = 0$$

$$N = 1, \quad \text{FOS} = 1, \quad \ell = 15,$$

$$\text{Dimensions } \phi = 20 \text{ cm.}$$

Sol: The moment at section A

$$\text{is } M = 45000 \times 0.2 = 9000 \text{ Nm}$$

& the torque on the shaft is

$$T = 45000 \times 0.15 = 6750 \text{ Nm}$$

The normal stress due to M at A is

$$\sigma = -\frac{64Md}{\pi d^4} = -\frac{32M}{\pi d^3}$$

& the maximum shear stress due to T at A is

$$\tau = \frac{32Td}{\pi d^4} = \frac{16T}{\pi d^3}$$

The shear stress due to shear force F is zero at A.

$$\sigma_{1,3} = \frac{1}{2} \sigma \pm \frac{1}{2} (\sigma^2 + 4\tau^2)^{1/2}, \quad \sigma_2 = 0$$

i) Max shear stress theory

$$\begin{aligned} \tau_{\text{max}} &= \frac{1}{2} (\sigma_1 - \sigma_3) \\ &= \frac{1}{2} (\sigma^2 + 4\tau^2)^{1/2} \end{aligned}$$

$$= \frac{1}{2} \frac{32}{\pi d^3} (M^2 + T^2)^{1/2}$$

$$= \frac{16}{\pi d^3} (9000^2 + 6750^2)^{1/2}$$

$$= \frac{57295.8}{d^3} \text{ Pa}$$

F.O.S $N=2$ the value of τ_{max} becomes

$$N \tau_{max} = \frac{114591.6}{d^3} \text{ Pa}$$

$$\frac{1}{d^3} (114591.6) = \frac{\delta_y}{2} = \frac{207}{2} \times 10^6$$

$$\therefore d^3 = 1107 \times 10^{-6} \text{ m}^3$$

$$\text{or } d = 10.35 \times 10^{-2} \text{ m} = 10.4 \text{ cm}$$

ii) Octahedral Shear Stress Theory.

$$\tau_{oct} = \frac{1}{3} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

with $\sigma_2 = 0$

$$\tau_{oct} = \frac{1}{3} [2\sigma_1^2 + 2\sigma_3^2 - 2\sigma_1\sigma_3]^{1/2}$$

substituting & simplifying

$$\tau_{oct} = \frac{\sqrt{2}}{3} (\sigma^2 + 3\tau^2)^{1/2}$$

$$= \frac{\sqrt{2}}{3\pi d^3} [(32M)^2 + 3(16T)^2]^{1/2}$$

$$= \frac{16\sqrt{2}}{3\pi d^3} (4M^2 + 3T^2)^{1/2}$$

$$= \frac{16\sqrt{2}}{3\pi d^3} [4(9000)^2 + 3(6750)^2]^{1/2}$$

$$= \frac{\sqrt{2}}{3\pi d^3} \times 343418$$

Equating this to Octahedral shear stress at yielding of a uniaxial ~~stress~~ tension bar using FOS $n=2$.

$$\frac{\sqrt{2}}{3\pi d^3} \times 2 \times 343418 = \frac{\sqrt{2}}{3} \cdot \sigma_y$$

$$\text{or } 2 \times 343418 = \pi d^3 \sigma_y = \pi d^3 \times 207 \times 10^6$$

$$\therefore d^3 = 1.056 \times 10^{-3}$$

$$\text{or } d = 0.1018 \text{ m} = 10.18 \text{ cm}.$$

Q2:-

Given data:-

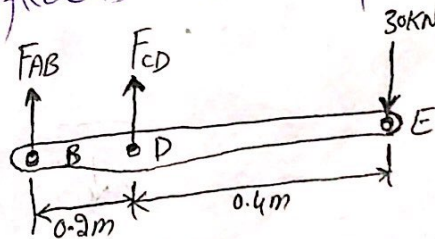
⇒ In figure the rigid bar BDE is supported by two links AB & CD.

⇒ Link AB is made of aluminum ($E = 70 \text{ GPa}$) and has a cross-sectional area of 5 mm^2 . ⇒ Link CD is made of steel ($E = 200 \text{ GPa}$) and has a cross-sectional area of (60 mm^2) .

⇒ For the first two digits of R - KN force. Determine the deflection a) of B, b) of D, and c) of E.

Solution:- * Apply a free-body analysis to the bar BDE to find the forces exerted by links AB & DC.
* Evaluate the deformation of links AB and DC or the displacements of B and D.
* Work out the geometry to find the deflection at E given the deflections at B & D.

FREE-BODY:- Bar BDE



$$\sum M_B = 0$$

$$0 = (30 \text{ kN} \times 0.6 \text{ m}) + F_{CD} \times 0.2 \text{ m}$$

$$F_{CD} = +90 \text{ kN Tension}$$

$$\sum M_D = 0$$

$$0 = -(30 \text{ kN} \times 0.4 \text{ m}) - F_{AB} \times 0.2 \text{ m}$$

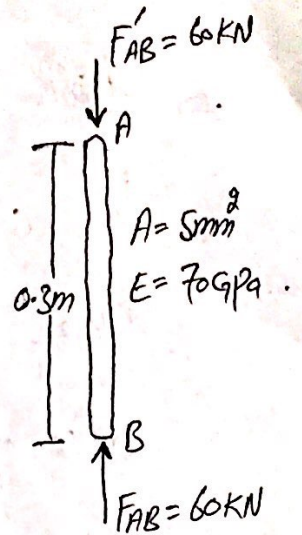
$$F_{AB} = -60 \text{ kN (compression)}$$

Displacement of B:-

$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60 \times 10^3 \text{ N})(0.3 \text{ m})}{(5 \times 10^{-6} \text{ m}^2)(70 \times 10^9 \text{ Pa})}$$

$$\delta_B = 0.514 \text{ mm} \uparrow$$



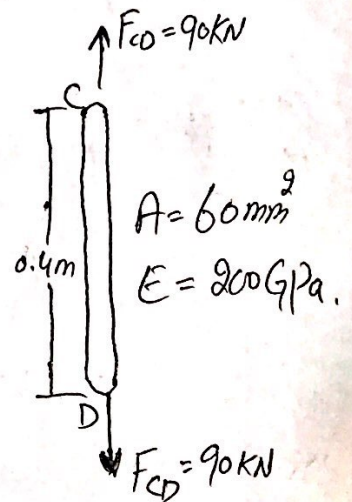
Displacement of D:-

$$\delta_D = \frac{PL}{AE}$$

$$= \frac{(90 \times 10^3 \text{ N})(0.4 \text{ m})}{(600 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})}$$

$$\delta_D = 300 \times 10^{-6} \text{ m}$$

$$\delta_D = 0.300 \text{ mm} \downarrow$$



$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

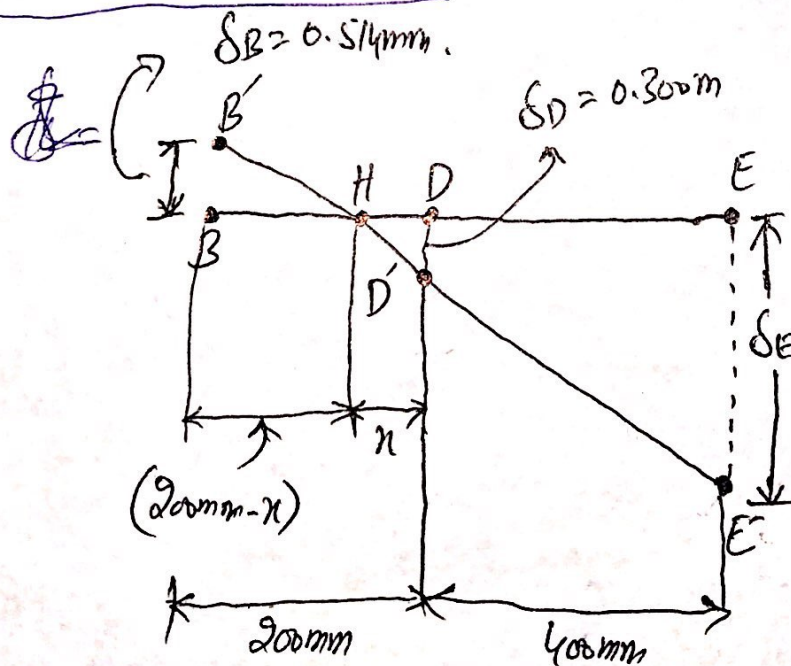
$$\frac{0.514\text{mm}}{0.300\text{mm}} = \frac{(200\text{mm}) - x}{x}$$

$$x = 73.7\text{mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.300\text{mm}} = \frac{(400 + 73.7)\text{mm}}{73.7\text{mm}}$$

$$\delta_E = 1.928\text{mm}$$



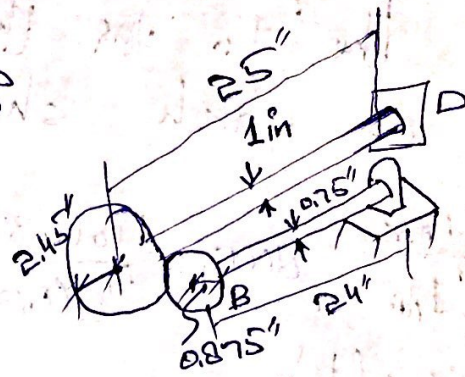
Q3) Two solid steel shaft are connected by gears.

Given Data

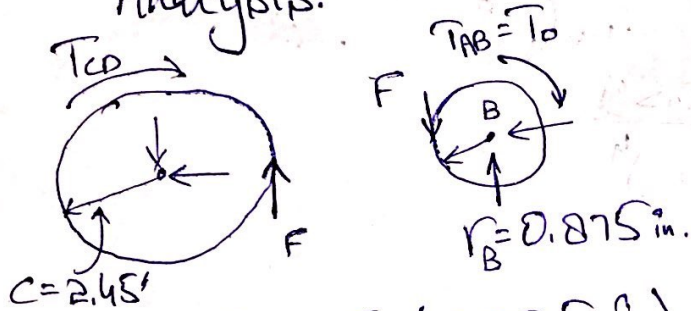
$G = 15 \times 10^6 \text{ psi}$, $\sigma_{all} = 10 \text{ ksi}$

Dimensions $\lambda = 15 + 10 = 25$

Required $T_0 = ?$, $L = ?$



Sol Static Equilibrium Analysis.

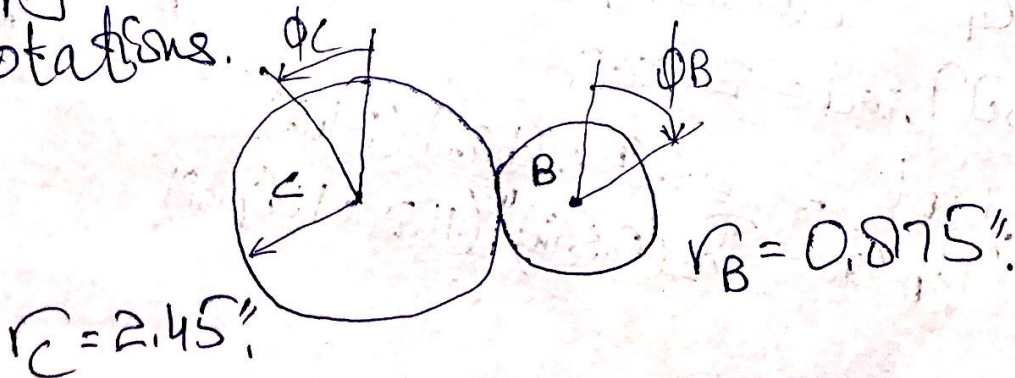


$\sum M_B = 0 = F(0.875 \text{ in}) - T_0$

$\sum M_C = 0 = F(2.45 \text{ in}) - T_{CD}$

$T_{CD} = 2.8 T_0$

Apply Kinematic Analysis to relate angular rotations.



$$r_B \phi_B = r_C \phi_C$$

$$\phi_B = \frac{r_C}{r_B} \phi_C = \frac{2.45 \text{ in}}{0.875 \text{ in}} \phi_C$$

$$\phi_B = 2.8 \phi_C$$

Find the T_0 for the maximum allowable torque on each shaft.

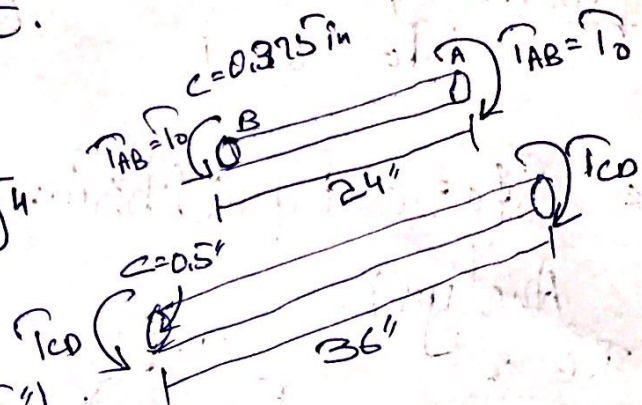
$$T_{max} = \frac{T_{ABC}}{J_{AB}} \cdot 8000 \text{ psi} = \frac{T_0 (0.375 \text{ in})}{\frac{\pi}{2} (0.375 \text{ in})^4}$$

$$T_0 = 663 \text{ lb.in.}$$

$$T_{max} = \frac{T_{CD}}{J_{CD}} \cdot 8000 \text{ psi} = \frac{2.8 T_0 (0.5 \text{ in})}{\frac{\pi}{2} (0.5 \text{ in})^4}$$

$$T_0 = 561 \text{ lb.in.}$$

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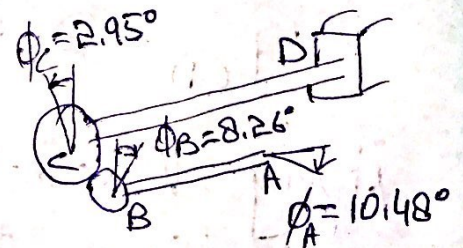


Find the corresponding angle of twist for each shaft & the net angular rotation of end A.

$$\phi_{A/B} = \frac{T_{AB} L}{J_{AB} G} = \frac{(561 \text{ lb.in.}) (24 \text{ in})}{\frac{\pi}{2} (0.375 \text{ in})^4 (11.2 \times 10^6 \text{ psi})}$$

$$= 0.387 \text{ rad} = 2.22^\circ$$

$$\phi_C = \frac{T_{CD} L}{J_{CD} G} = \frac{2.8 (561 \text{ lb.in.}) (24 \text{ in})}{\frac{\pi}{2} (0.5 \text{ in})^4 (11.2 \times 10^6 \text{ psi})}$$



$$= 0.514 \text{ rad} = 2.95^\circ$$

$$\phi_B = 2.0 \phi_C = 2.0 (2.95^\circ) = 5.90^\circ$$

$$\phi_A = \phi_B + \phi_{A/B} = 5.90^\circ + 4.58^\circ$$

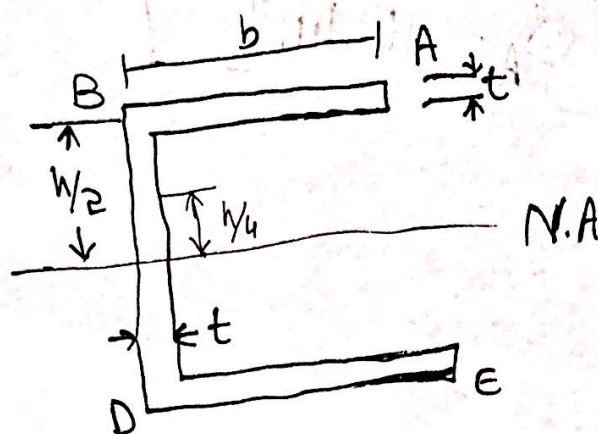
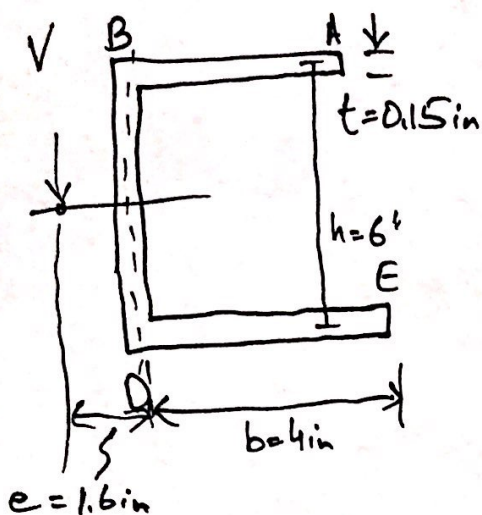
$$\boxed{\phi_A = 10.48^\circ}$$

Q4) Determine the location for the shear center of the channel section.

Given Data:

$b = 4 \text{ in}, h = 6 \text{ in}, t = 0.15 \text{ in}$

$V = 159 \text{ kips.}$



Sol's Shear stress distribution for $V = 159 \text{ k}$.

$$\tau = \frac{q}{t} = \frac{VQ}{It}$$

Shearing stresses in the flanges

$$\tau = \frac{VQ}{It} = \frac{V \cdot (bt) \cdot \frac{h}{2}}{It} = \frac{Vh}{2I}$$

$$\tau_B = \frac{Vhb}{2 \left(\frac{1}{2} th^2 \right) (6b+h)} = \frac{6Vb}{th(6b+h)}$$

$$= \frac{6(2.5 \text{ kips})(4 \text{ in})}{(0.15 \text{ in})(6 \text{ in})(6 \times 4 + 6 \text{ in})}$$

$$\tau_B = 2.22 \text{ ksi}$$

Shearing stress in the web.

$$\tau_{max} = \frac{VQ}{It} = \frac{V \left(\frac{1}{8} ht \right) (4b+h)}{\frac{1}{12} th^2 (6b+h)t}$$

$$= \frac{3V(4b+h)}{2th(6b+h)}$$

$$= \frac{3(159k)(4 \times 4 \text{ in} + 6 \text{ in})}{2(0.15 \text{ in})(6 \text{ in})(6 \times 6 \text{ in} + 6 \text{ in})}$$

$$= \neq 4.06 \text{ ksi}$$

Q5: Given data:-

\Rightarrow grade of steel used $\sigma_{all} = 15 \text{ ksi} + 4 \text{ ksi} = 19 \text{ ksi}$

\Rightarrow and

$\tau_{all} = 15 \text{ ksi} + 1 \text{ ksi} = 16 \text{ ksi}$

\Rightarrow select the wide-flange beam which should be used.

Selection:- Requirement

Solution:- * Determine reactions at "A" & "D".

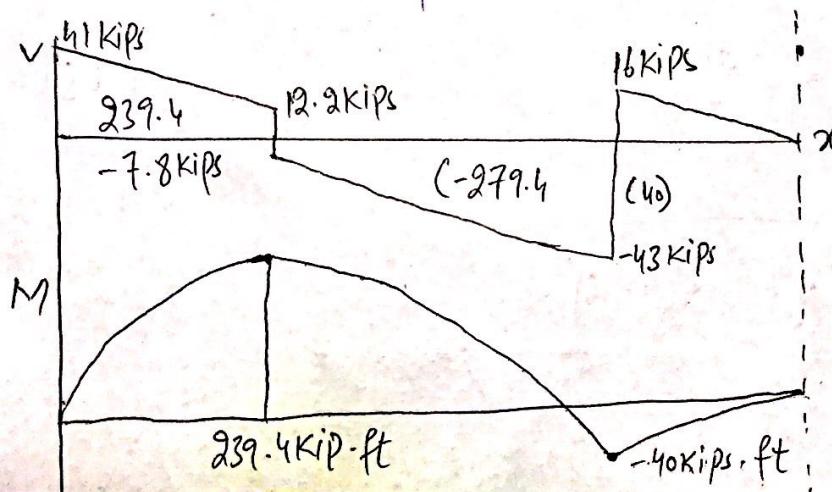
$$\sum M_A = 0 \rightarrow R_D = 59 \text{ Kips}$$

$$\sum M_D = 0 \rightarrow R_A = 41 \text{ Kips}$$

* Determine maximum shear and bending moment from shear and bending moment diagrams.

$$|M|_{\max} = 239.4 \text{ kip}\cdot\text{in} \quad \text{with} \quad V = 12.2 \text{ Kips}$$

$$|V|_{\max} = 43 \text{ Kips}$$



* Calculate required section modulus and select appropriate beam section.

$$S_{min} = \frac{(M)_{max}}{\sigma_{all}} = \frac{24 \text{ kips} \cdot \text{in}}{24 \text{ ksi}} = 119.7 \text{ in}^3$$

select $W_{21} \times 62$ beam section

* Find maximum shearing stress, Assuming uniform shearing stress in web,

$$\tau_{max} = \frac{V_{max}}{A_{web}} = \frac{43 \text{ kips}}{8.40 \text{ in}^2} = 5.12 \text{ ksi} < 14.5 \text{ ksi}$$

* Find maximum normal stress.

$$\sigma_a = \frac{M_{max}}{S} = 2873 \frac{60 \text{ kip} \cdot \text{in}}{127 \text{ in}^3} = 22.6 \text{ ksi}$$

$$\sigma_b = \sigma_a \frac{y_b}{c} = (22.6 \text{ ksi}) \frac{9.88}{10.5} = 21.3 \text{ ksi}$$

$$\tau_b = \frac{V}{A_{web}} = \frac{12.2 \text{ kips}}{8.40 \text{ in}^2} = 1.45 \text{ ksi}$$

$$\sigma_{max} = \frac{21.3 \text{ ksi}}{2} + \sqrt{\left(\frac{21.3 \text{ ksi}}{2}\right)^2 + (1.45 \text{ ksi})^2}$$

$$\sigma_{max} = 21.4 \text{ ksi} < 24 \text{ ksi}$$

