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ASSIGNMENT
MID TERM

I.D # 13131

SUBJECT # DIFFERENTIAL

SUBMITTED TO # SIR LATIF

Q1 a) Define 2nd Order linear homogeneous differential equations along with two example.

2nd Order Linear Homogeneous Differential Equation:-

A second order differential involving y'', y', y with these multiple only function of x is called linear.

i.e; $a(x)y'' + b(x)y' + c(x)y = d(x)$

EXAMPLE 1:-

$$y'' + 36y = 0$$

$$t^2 + 25y = 0$$

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Solve the equation.

$t^2 = -36$

$t = \pm 6i$

$y = 0 \pm 5i$

$y = e^{0x}(C_1 \cos 5x + C_2 \sin 5x)$

$y = (C_1 \cos 5x + C_2 \sin 5x)$

EXAMPLE 2:

$y'' - 2y - 3y = e^{2t}$

$y'' - 2y - 3y = 0$

$t^2 - 2y - 3 = 0$

$t^2 - 2y - 3 = 0$

$t^2 - 3t + t - 3 = 0$

$t(t-3) + 1(t-3) = 0$

$(t+1)(t-3) = 0$

$t = -1, \quad t = 3$

$y = C_1 e^{-1t} + C_2 e^{3t}$

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b) Solve the following 2^{nd} order linear homogeneous / non homogeneous differential equation.

i) $16y'' + 2y' + 9y = 0$

SOLUTION:

$$16y'' + 2y' + 9y = 0$$

$$16r^2 + 24r + 9 = 0$$

$$r = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)}$$

$$= -3/4$$

$$y(t) = C_1 e^{rt} + C_2 t e^{rt}$$

with r double root of the characteristics equation.

$r = -3/4$ the general solution

$$y(t) = C_1 e^{-3t/4} + C_2 t e^{-3t/4}$$

$$y(t) = C_1 e^{-3t/4} + C_2 t e^{3t/4}$$

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b) $y'' - 4y' - 12y = 3e^{5x}$

SOLUTION :-

$y'' - 4y' - 12y = 3e^{5x}$

$y'' - 4y' - 12y = 0$

$y'' - 4y - 12y = 3e^{5x}$

$y = C_1 e^{6x} + C_2 e^{-2x} - \frac{3}{7} e^{5x}$

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Q2) Solve the following IVP for 2^{nd} order linear equations.

i) $2y'' + 5y' + 3y = 0 \quad y(0) = 3 \quad y'(0) = -4$

SOLUTION:-

$(2D^2 + 5D + 3)y = 0$ — eq (2)

Solution (1) is given by $y = CF$

For CF => AE = 0

$f(m) = 0$

$2m^2 + 5m + 3 = 0$

$2m^2 + 3m + 2m + 3 = 0$

$m(2m+3) + 1(2m+3) = 0$

$(m+1)(2m+3) = 0$

$m+1 = 0 \Rightarrow m = -1$

$2m + 3 = 0 \Rightarrow 2m = -3$

$m = -3/2$

$y = CF = C_1 e^{-x} + C_2 e^{-3/2 x}$ — (1)

$y(0) = 3$

$3 = C_1 + C_2$ — (2)

$y' = -C_1 e^{-x} - 3/2\, C_2 e^{-3/2 x}$

$y'(0) = -4$

$-4 = -C_1 - 3/2\, C_2$

$-4 = C_1 + 3/2\, C_2$ — (3)

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Solve (2) & (3) for C_1 & C_2

$C_2 = 2, \quad C_1 = 1$

$y = C_1 e^{-x} + C_2 e^{-3/2x}$

$y = e^{-x} + 2e^{-3/2x}$

ii) $2y'' + 5y' - 3y = 0 \quad y(0) = 3 \quad y'(0) = 4$

SOLUTION:-

$2y'' + 5y' - 3y = 0 \quad y(0) = 3, \; y'(0) = 4 \rightarrow eq(1)$

$(2D^2 + 5D - 3)y = 0$ — (2)

Solution (1) is given by $y = CF$

for CF $\Rightarrow Af() = 0$

$f() = 0$

$2m^2 + 6m - m - 3 = 0$

$2m(m+3) - 1(m+3) = 0$

$(m+3) = 0, \; (2m-1) = 0$

$m + 3 = 0, \quad 2m - 1 = 0$

$m = -3, \quad 2m = 1$

$m = 1/2$

$y = CF = C_1 e^{-x} + C_2 e^{1/2x}$

$y(0) = 3$

$3 = C_1 + C_2$ — (2)

$y' = -C_1 e^{-x} + 1/2\, C_2 e^{1/2x}$

$y' = 4$

$4 = -C_1 + 1/2\, C_2$ — (3)

Solve (2) & (3) for C_1 & C_2

$C_2 = \quad , \; C_1 = 1$

eq $y = C_1 e^{-x} + C_2 e^{1/2x}$

$y = e^{-x} + 2e^{1/2x}$

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iii) $y'' - 4y' + 9y = 0 \quad y(0) = 0, \; y'(0) = -8$

SOLUTION:-

$$m^2 - 4m + 9 = 0$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(9)}}{2}$$

$$= \frac{4 \pm \sqrt{16 - 36}}{2}$$

$$= \frac{4 \pm i2\sqrt{5}}{2}$$

$$m = 2 \pm i\sqrt{5}$$

hence the general solution is

$$y = e^{2x}(A \cos \sqrt{5}x + B \sin \sqrt{5}x)$$

$$y(0) = 0$$

$$\Rightarrow 0 = 1[A(1) + B(0)]$$

$$\Rightarrow A = 0$$

$$y = e^{2x}(B \sin \sqrt{5}x)$$

$$y'(0) = -8$$

$$y' = 2e^{x}(B \sin \sqrt{5}x) + \sqrt{5}e^{x}(B \cos \sqrt{5}x)$$

$$-8 = 0 + \sqrt{5}(B)$$

$$B = -8/\sqrt{5}$$

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the Solution is

$$y = -e^{2x}\left(\frac{8}{\sqrt{5}} \sin\sqrt{5}x\right)$$

Q3) Define leplace transform along with two examples?

LEPLACE TRANSFORM:-

The leplace transforms, named after its inventor Pierre - Simon leplace, is an integral transform that converts a function of a real variable (often time) to a function of a complex variable

EXAMPLE:-

$$y'' + y = \sin(2t)$$

$$y = (1/3)\,6\cos(t) + 5(\sin t) - \sin(2t))$$

$$y'' = (1/3)(-6\cos(t) - 5\sin t) + 4\sin(2t))$$

$$y'' + y = \sin(2t$$

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A.) Find the Laplace transforms of the given functions.

1) $f(t) = 6e^{\wedge}(-5t) + e^{\wedge}3t + 5(t^{\wedge}3) - 9$

SOLUTION:-

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3}{s^{3}+1} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{90}{s^{4}} - \frac{9}{s}$$

2) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

SOLUTION:-

$$G(s) = 4\frac{s}{s^{2}+(4)^{2}} - 9\frac{4}{s^{2}+(4)^{2}} + 2\frac{s}{s^{2}+(10)^{2}}$$

$$= \frac{4s}{s^{2}+16} - \frac{36}{s^{2}+16} + \frac{2s}{s^{2}+100}$$

3) $h(t) = e^{\wedge}3t + \cos(6t) - e^{\wedge}(3t)\cos(6t)$

SOLUTION:-

$$G(s) = \frac{1}{s-3} + \frac{s}{s^{2}+(6)^{2}} - \frac{s-3}{(s-3)^{2}+(6)^{2}}$$

$$= \frac{1}{s-3} + \frac{s}{s^{2}+36} - \frac{s-3}{(s-3)^{2}+36}$$

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Q4) Solve the following IVP using Laplace Transform.

i) $y'' - 4y' = e^{n}(3t)$ $y(0) = 0, y'(0) = 0$

SOLUTION:-

$y(t) \to Y(s)$

$y'(t) \to sY(s) - y(0)$

$y''(t) \to s^2Y(s) - sy(0) - y'(0)$

$s^2y(s) - sy(0) - y'(0) - 4sY(s) + 4y(0) = 1/s-3$

$s^2y(s) - s\times 0 - 0 - 4sY(s) + 4\times 0 = 1/s-3$

$s^2y(s) - 4sY(s) = 1/s-3$

$Y(s)(s^2 - 4s) = 1/s-3$

$$Y(s) = \frac{1}{s(s-4)(s-3)}$$

Hence,

$$\frac{1}{s(s-4)(s-3)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s-3}$$

$1 = A(s-4)(s-3) + B(s)(s-3) + C(s)(s-4)$

Take $s = 0$

$1 = A(-4)(-3)$

$A = 1/12$

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Take $S = 4$

$1 = B \times 4 \times 1$

$B = 1/4$

Take $S = 3$

$1 = C \times 3 \times -1$

$C = -1/3$

Hence

$$Y(s) = \frac{1}{12}\frac{1}{S} + \frac{1}{4}\frac{1}{S-4} - \frac{1}{3}\frac{1}{S-3}$$

ii) $y'' + 3y' + 2y = e^{\wedge}(-t)$, $y(0) = 0$ $y'(0) = 0$

SOLUTION:-

$L[y''] + 3L[y'] + 2L[y] = L[e^{-t}]$

$S^2 Y(s) - SY(0) - y'(0) + 3[Sy(s) - y(0)] + 2yS = 1/S+1$

Put initial value

$S^2 Y(s) + 3[sY(s)] + 2Y(s) = \frac{1}{S+1}$

$Y(s)[S^2 + 3S + 2] = 1/S+1$

$y(s) = 1/S+1[S^2 + 3s + 2]$

$= 1/(s+1)^2(s+2)$

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Now, find Partial Function transformation.

$$y(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}$$

Since, we have to terms of s+1.
So, take one which + multiple

$$y(t) = e^{-t} + te^{-t} + e^{-2t}$$