Q3

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol:- Augmented matrix.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 2 & 2 & 4 & 18 \\ 3 & 2 & -3 & 14 \end{array}\right] \quad R_2 \leftrightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 0 & -7 & -9 & -25 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & 1 & 0 & 2 \\ 0 & 7/9 & 1 & 25/9 \end{array}\right] \quad \begin{array}{l} -1/4\, R_2 \\ -1/9\, R_3 \end{array}$$

~~echelon form~~

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 7/9 & 1 & 25/9 \end{array}\right] \quad R_1 - 3R_2$$

$$|A| = 3\begin{vmatrix} -7 & 7 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 7 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$|A| = 3(-7+14) - 4(14-35) + 5(-4+5$$

$$|A| = 3(7) - 4(-21) + 5(1$$

$$= 21 + 84 + 5$$

$$= 110$$

inverse of A^{-1}

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A.$$

$$A^{-1} = \frac{1}{110}\begin{pmatrix} 7 & -21 & 1 \\ -38 & -4 & 26 \\ 33 & 18 & -11 \end{pmatrix}$$

A^{-1}

A_{23} $\begin{pmatrix} 3 & 5 \\ 2 & 7 \end{pmatrix} = 28 - 10 = 18.$

$A_{31} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4 + 5 = 1$

$A_{32} = \begin{pmatrix} 3 & 4 \\ 5 & -2 \end{pmatrix} = -6 - 20 = -26$

$A_{33} = \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix} = -3 - 8 = -11$

$$A = \begin{bmatrix} 7 & 38 & 33 \\ 21 & -4 & 18 \\ 1 & -26 & -11 \end{bmatrix}$$

Cofactor of matrix $= \begin{bmatrix} 7 & -38 & 33 \\ -21 & -4 & -18 \\ 1 & 26 & -11 \end{bmatrix}$

adj A $\begin{pmatrix} 7 & -21 & 1 \\ -38 & -4 & 26 \\ 33 & 18 & -11 \end{pmatrix}$

Q.2.

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 7 \\ 5 & -2 & 7 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}.$$

$$\Rightarrow A_{11} \Rightarrow \begin{vmatrix} -1 & 7 \\ -2 & 7 \end{vmatrix} = -7 + 14 = 7$$

$$\Rightarrow A_{12} = \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = 28 + 10 = 38$$

$$\Rightarrow A_{13} = \begin{vmatrix} 4 & 5 \\ -1 & 7 \end{vmatrix} = 28 + 5 = 33.$$

$$\Rightarrow A_{21} = \begin{vmatrix} 2 & 7 \\ 5 & 7 \end{vmatrix} = 14 - 35 = 21$$

$$\Rightarrow A_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = 21 - 25 = -4.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right] \quad R_3 - \frac{7}{9} R_2$$

$$\frac{25}{9} - \frac{7}{9}(2)$$

$$\frac{25}{9} - \frac{14}{9}$$

$$\frac{11}{9}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right] \quad R_1 - 2R_3$$

So

$$\boxed{x = 41/9}$$

$$\boxed{y = 2}$$

$$\boxed{z = 11/9}$$

Q₇

B-2L

Q(1).

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 5 & -5 & 10 \end{array}\right]$$

The maxtrix is consistant.

$(-2)^{(-7)} = (-4)^{2}$

$$\begin{bmatrix} 4 & & \\ -5 & 2 & -2 \\ -2 & 3 & 2 \\ & 4 & 1 \end{bmatrix}$$

3
3
9
39
14

18

$Q = 6$

1

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

-7
9

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 6 \\ 1 & 3 & 4 & 0 \end{bmatrix} \quad 3R_1 - R_2$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \quad R_2 \longleftrightarrow R_3$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \frac{1}{6} R_3$$

2

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot R_1 - R_2$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} 1^2 & 0 \\ 0 & 0 \end{bmatrix}.$$

$$\boxed{i^2 = 3}.$$

Q5 ①

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda-4 & -2 & 2 \\ -5 & \lambda-3 & 2 \\ +2 & 4 & \lambda-1 \end{bmatrix}$$

$$= +5 \begin{vmatrix} -2 & 2 \\ 4 & \lambda-1 \end{vmatrix} + (\lambda-3) \begin{vmatrix} \lambda-4 & 2 \\ 2 & \lambda-1 \end{vmatrix}$$

$$-2 \begin{pmatrix} \lambda-4 & -2 \\ 2 & 4 \end{pmatrix}$$

$$= 5\left(-2(\cancel{\lambda}-\lambda)-8\right) + (\lambda-3)\left((\lambda-4)(\lambda-1) - 4\right)$$

$$-2\left((\lambda-4)(4)+4\right)$$

$$= 5(-14-2\lambda-8) + (\lambda-3)(\lambda^2-\lambda-4\lambda+4 - 4)$$

$$-2(4\lambda-16+4)$$

$$\Rightarrow 5(-2\lambda-22) + (\lambda-3)(\lambda^2-5\lambda + \cancel{4} - \cancel{4})$$

$$-2(4\lambda-12)$$

2

$-10\lambda - 110 + \lambda^3 - 5\lambda^2 + 3\lambda^2 - 15\lambda$

$-8\lambda + 24$

$-86 - 33\lambda + \lambda^3 - 2\lambda^2$

$\lambda^3 - 2\lambda^2 - 33\lambda - 86$

$\lambda^2(\lambda - 2 - 33) - 86.$

$\lambda - 2 - 33 = 86$

$\lambda - 2.$

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

$$\begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 5 \\ -25 \\ 1 \end{pmatrix} x_2 + \begin{pmatrix} -4 \\ 4 \\ -8 \end{pmatrix} x_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

As the system has zero on right side so it called trival sol.