Department of Electrical Engineering
Final Exam Assignment
Date: 28/09/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th

Instructor: **Total Marks:** 50

Student Details

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Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation ( ) ( ) ( ) () ( ) To the input () ( ) ( ) And the initial conditions are y (-1) = y (-2) = 0.	**Marks** **8** **CLO** **2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. () ( ) ( ) () ( )	**Marks** **7** **CLO** **2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform ( ) ( )( ) (Hint: Take inverse z-transform using partial fraction method)	**Marks** **8** **CLO** **2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step ( ) { } ( ) * +	**Marks** **7** **CLO** **2**
Q.3	(a)	A two- pole low pass filter has the system response	**Marks** **12**

		() ( ) Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\vert (\quad)\vert^2 =$.	**CLO 3**
	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in = at $\omega = 4\pi/9$.	**Marks 8** **CLO 3**

NAME NOOR UL WAHAB

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Q No 1

(a)

Sol: The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_p(n) = k(-1)^n u(n)$$

$$k(-1)^n u(n) - 4k(-1)^{n-1} y(n-1) + 4k(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$ $k(1+4+4) \Rightarrow 2 \Rightarrow k$

$= k = 2/9$ total solution is

$$y(n) = \left[C_1 2^n + C_2 n^{2n} + \frac{2}{9}(-1)^n\right] u(n)$$

For initial condation we have $y(0)$

$y(0) = 1$, $y = 2$ then

$$C_1 + 2/9 = 1$$

$$\Rightarrow C_1 = 7/9$$

$$2C_1 + 2C_2 - 2/9 = 2$$

$\Rightarrow C_2 = 1/3$

Q No 1
part (b)

Sol:- The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) = 0.7y(0) = 0 \Rightarrow (1) = 1.4$$

Hence $C_1 + C_2 = 2$ and

$$\frac{1}{2} C_1 + \frac{1}{5} C_2 = 1.4 = 7/5$$

$$\Rightarrow C_1 + 2/5\, C_2 = 14/5$$

The evaluation yield

$C_1 = 10/3$, $C_2 = -4/3$

$$h(n) = 10/3 \left(1/2\right)^n - 4/3 \left(1/5\right)^n \Big] u(n)$$

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The set response is

$$\theta(n) = \sum_{k=0}^{n} h(n-k)$$

$$\frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}^{n} \left(2^{n+1} - 1\right) u(n) - \frac{1}{3} \left(\frac{1}{5} \left(5^{n+1} - 1\right) \right. \right.$$

$$u(n).$$

QNo2

(a):-

Sol:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{4(1+2z^{-1})} + \frac{3}{4}\frac{1}{1-2z^{-1}}$$

$$+ \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

By applying inverse transform.

$$X(n) = 1/8\,(-1)^{x}\,u(x) - 3/8\,u(x) + 1/2\,n\,u(x)$$

$$= \left[1/8\,(-1)^{n} + 3/8 + n/2\right] u(x)$$

Q No 2(b)

Sol:- Circular convolution using circular convolution:

$x_1(n) = \{1, 2, 3, 4\}$

and $x_2(n) = \{1, 2, 1, 2\}$

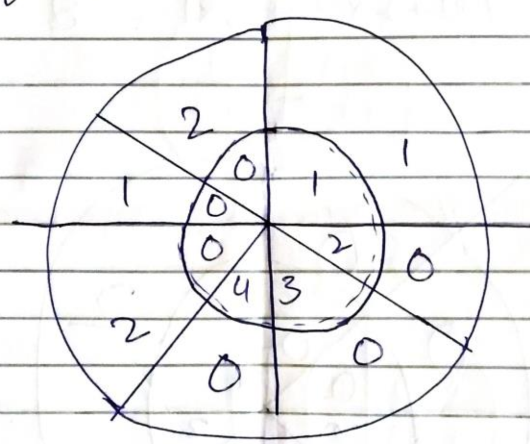
$L = 4$, $M = 4$

length of $y(n) = L + M - 1 = 4 + 4 - 1 = 7$

$x_1(n) = \{1, 2, 3, 4, 0, 0, 0\}$

and $x_2(n) = \{1, 2, 1, 2, 0, 0, 0\}$

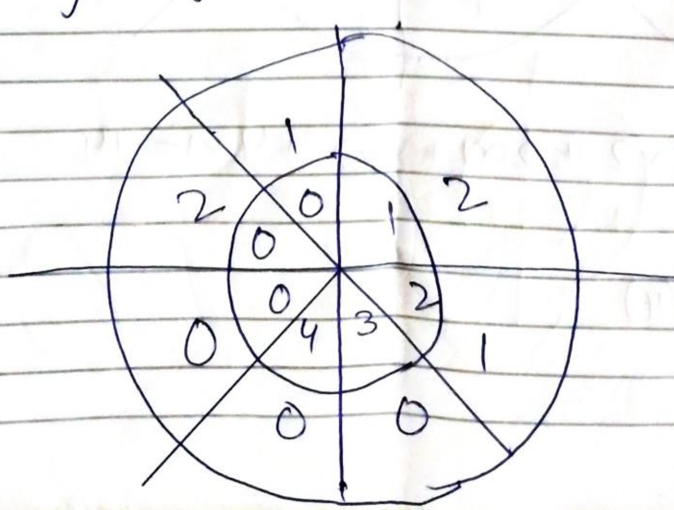
for $y(0)$



$n = 0$

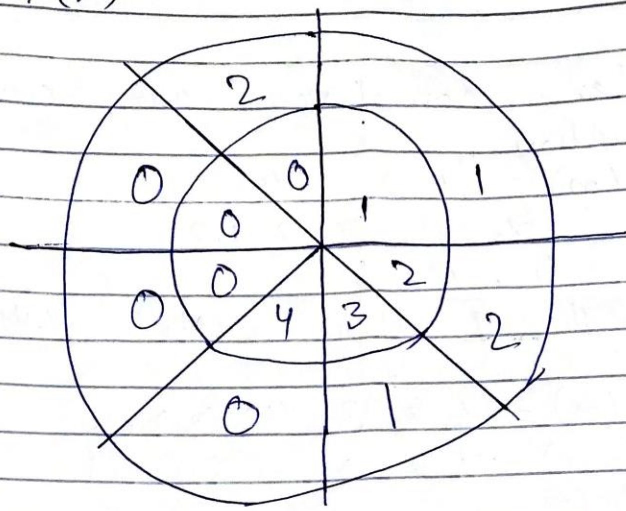
$y(0) = 1 \times 1 = 1$

for $y(1)$



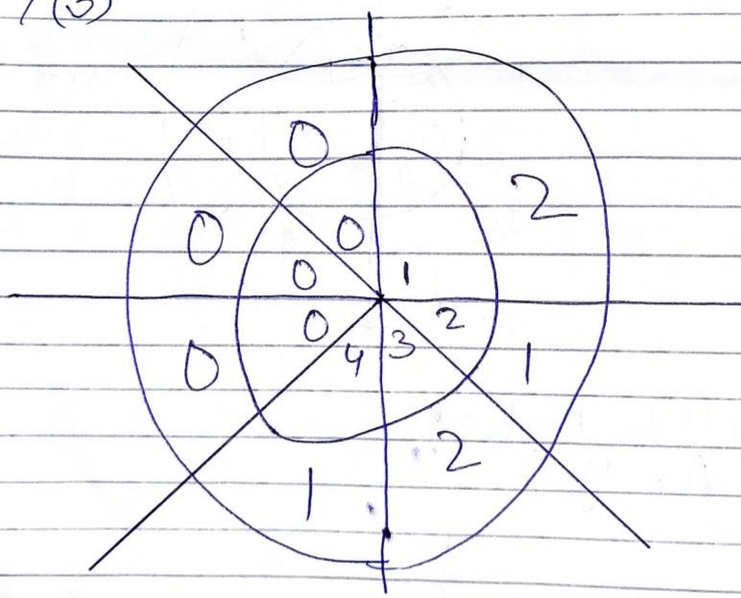
$y(1) = 2 \times 1 + 1 \times 2$
$= 4$

For $y(2)$



$$y(2) = 1\times1+2\times2+3\times1 = 8$$

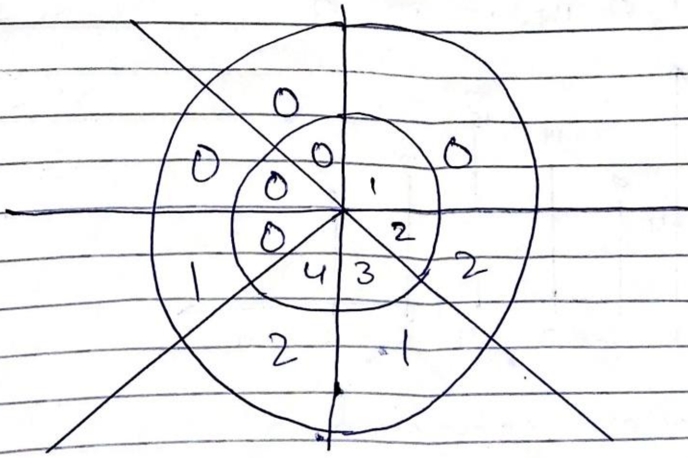
For $y(3)$



$$y(3) = 1\times2+2\times1+3\times2+4\times1 = 14$$

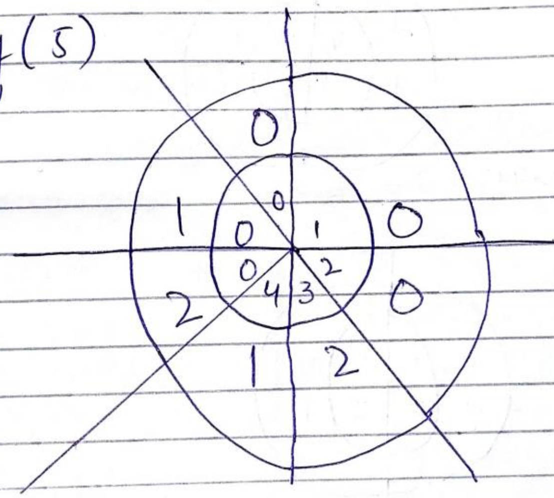
For $y(4)$

P# 7



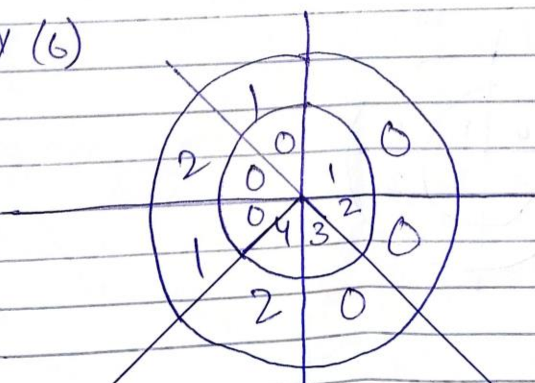
$Y(4) = 4\times2+3\times1+2\times2 = 15$

For $y(5)$



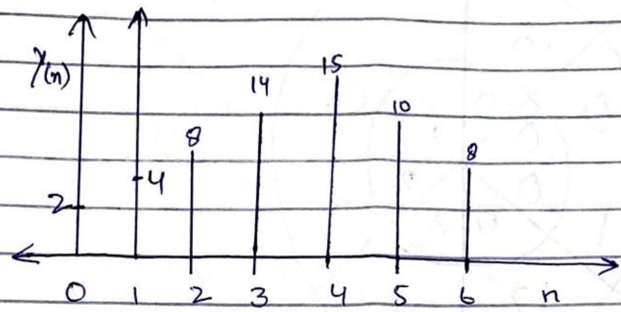
$Y(5) = 4\times1+3\times2 = 10$

For $Y(6)$



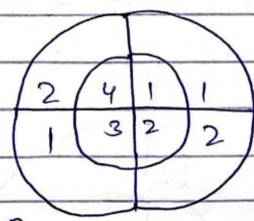
$Y(6) = 4\times2 = 8$

$Y(n) = \{1, 4, 8, 14, 15, 10, 8\}$



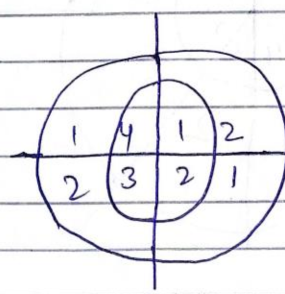
Linear circular convolution

For $y(0)$



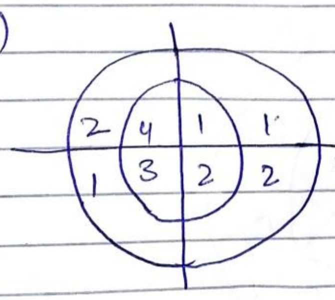
$Y_0(0) = 1+4+3+8 = 16$

For $y(1) =$

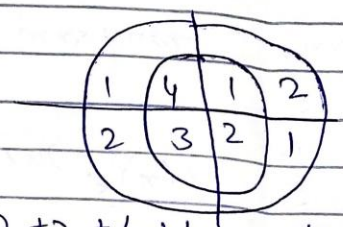


$Y(1) = 2+2+6+4 = 14$

For $Y(2)$

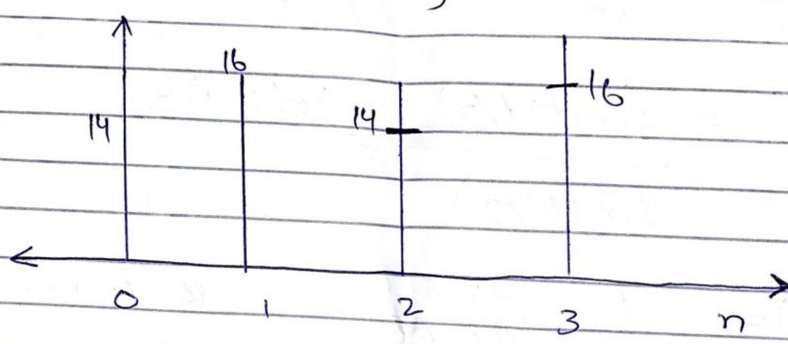


For $y(3)$ P # 9



$y(3) = 2+2+6+4 = 14$

$Y_n = \{16, 14, 16, 14\}$



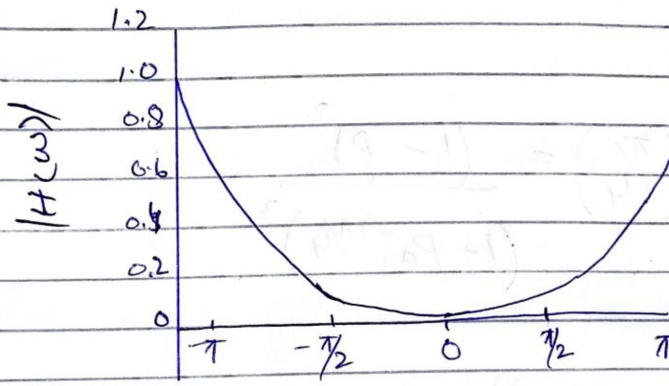
Result: $Y(n) = \{14, 16, 14, 16\}$

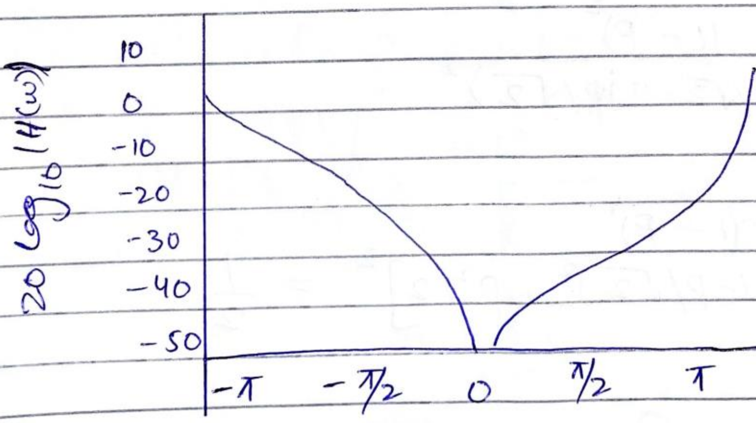
QNo3 (a):-

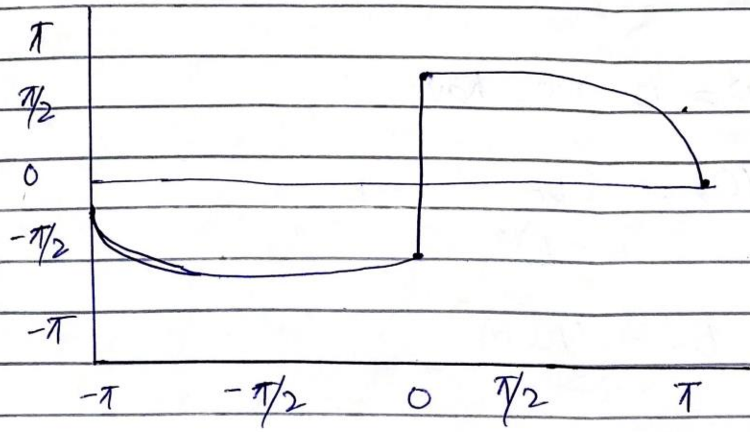
Sol:- At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$







At $\omega = \pi/4$

$$H\left(\pi/4\right) = \frac{(1-p)^2}{\left(1-pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-p)^2}{1-p(\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

$$= \frac{(1-p)^4}{\left[(1-p/\sqrt{2})^2 + p^2/2\right]^2} = \frac{1}{2}$$

Q No 3

Part (b):-

Sol: By the filter requirments:-

Poles $P_{1,2} = re^{\pm j\pi/2}$ pass band center

Zeroes $z_{1,2} = \pm 1$ Stopband Canter.

$$\Rightarrow H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)} = G\frac{z^2-1}{z^2+r^2}$$

By the filter requirement

$$H(\pi/2) = G\frac{-2}{-1+r^2} = 1$$

$$= G\frac{1-r^2}{2}$$

To set r use $H(4\pi/9) = 1/\sqrt{2}$

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

Evaluating gives $r^2 = 0.7$ therefore

$$H(z) = 0.15\frac{1-z^{-2}}{1+0.7z^{-2}}$$

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