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" 13032 "

" Dept : Electrical Engineering "

" Assignment : Differential Equations "

Question # 01

Solve & graph the solution. Show the detail of your work.

(13) $x^2y'' - 4xy' + 6y = 0, \quad y(1) = 0.4 \ . \ y'(1) = 0$

Solve:-

Let's Substitute

$y = x^n \ , \quad y' = mx^{n-1}, \ y'' = m(m-1)x^{n-2}$

equation becomes

$x^2 m(m-1)x^{n-2} - 4xmx^{n-4} + 6x^n = 0$

$x^2 m(m-1)x^n . x^{-2} - 4xmx^n x^{-1} + 6x^n = 0$

x^n is a common factor

$x^m \{m(m-1) - 4m + 6\} = 0$

$m(m-1) - 4m + 6 = 0$

$\Rightarrow m^2 - 5m + 6 = 0$

So $y = x^n$ is a solution of given ODE if m is a root of equation

To find root of Equation

$$m^2 - 5m + 6 = 0$$

$$m = \frac{5 \pm \sqrt{(-5)^2 + (4)(6)}}{2}$$

$$m = \frac{5 \pm 1}{2}$$

Roots are distinct & Real

$m_1 = 3$, $m_2 = 2$

Real different root gives two real solutions

$$y_1 = x^{m_1} = x^3$$

$$y_2 = x^{m_2} = x^2$$

As

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1 x^3 + c_2 x^2$$

$$y = 3c_1 x^2 + 2c_2 x$$

we have to determine c_1 & c_2, from IVP

$$\Rightarrow \begin{cases} 0.4 = y(1) = c_1 \cdot 1^3 + c_2 \cdot 1^2 \\ 0 = y(1) = 3c_1 \cdot 1^2 + 2c_2 \cdot 1 \end{cases}$$

$$\Rightarrow \begin{cases} 0.4 = c_1 + c_2 & \text{——— (1)} \\ 0 = 3c_1 + 2c_2 & \text{——— (2)} \end{cases}$$

$0.4 - C_2 = C_1$ Put in eq 2

$0 = 3(0.4 - C_2) + 2C_2$

$0 = 1.2 - C_2$

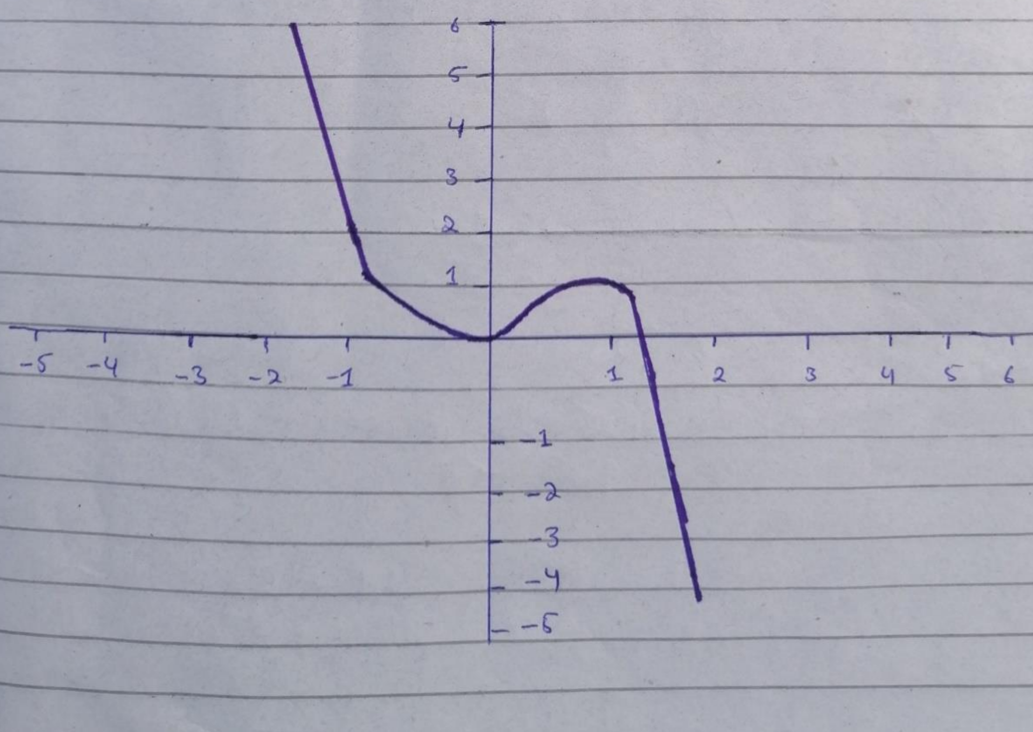
$$\boxed{C_2 = 1.2}$$

Put in eq 1

$$\boxed{C_1 = -0.8}$$

Particular Solution of IVP is

$$y = -0.8x^3 + 1.2x^2$$



(13) $x^2y'' + 3xy' + 0.75y = 0$

$y(1) = 0 \quad y'(1) = -1.5$

Solution:-

Let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

The equation becomes

$x^2 m(m-1)x^{m-2} + 3x m x^{m-1} + 0.75x^m = 0$

$\cancel{x^2} m(m-1)x^n . \cancel{x^{-2}} + 3\cancel{x} m x^m . \cancel{x^{-1}} + 0.75x^m = 0$

$m(m-1)x^m + 3mx^m + 0.75x^m = 0$

$x^m[(m-1)m + 3m + 0.75] = 0$

$m(m-1) + 3m + 0.75 = 0$

$m^2 + 2m + 0.75 = 0$

$m = \dfrac{-2 \pm \sqrt{4-(4)(0.75)}}{2}$

$m = \dfrac{-2 \pm 1}{2}$

$m_1 = -1/2 \quad , \quad m_2 = -3/2$

$\Rightarrow$ Real differential root m_1 & m_2 provide two real solutions.

$y_1 = x^{m_1} \Rightarrow x^{-1/2} \Rightarrow x^{-0.5}$

$y_2 = x^{m_2} \Rightarrow x^{-3/2} \Rightarrow x^{-1.5}$

So the general solution is

$y = c_1y_1 + c_2y_2$

$= c_1x^{-0.5} + c_2x^{-1.5}$

$\Rightarrow y' = -0.5c_1x^{-0.5} - 1.5c_2x^{-1.5}$

we have to determine C_1 & C_2 from IVP:

$$y_1 = 1 = C_1 \cdot 1^{-0.5} + C_2 \cdot 1^{-1.5}$$

$$1 = C_1 + C_2$$

$$-1.5 = y'(1) = -0.5C_1 \cdot 1^{-1.5} - 1.5\, C_2 \cdot 1^{-1.5}$$

$$-1.5 = 0.5C_1 - 1.5C_2$$

$$C_1 + C_2 = 1 \longrightarrow (1)$$

$$3 = C_1 + 3C_2 \longrightarrow (2)$$

From (1) $C_1 = 1 - C_2$ put in (2)

$$\cancel{3 = 1 - 1C_2 + AC_2}$$

$$3 = 1 - C_2 + 3C_2$$

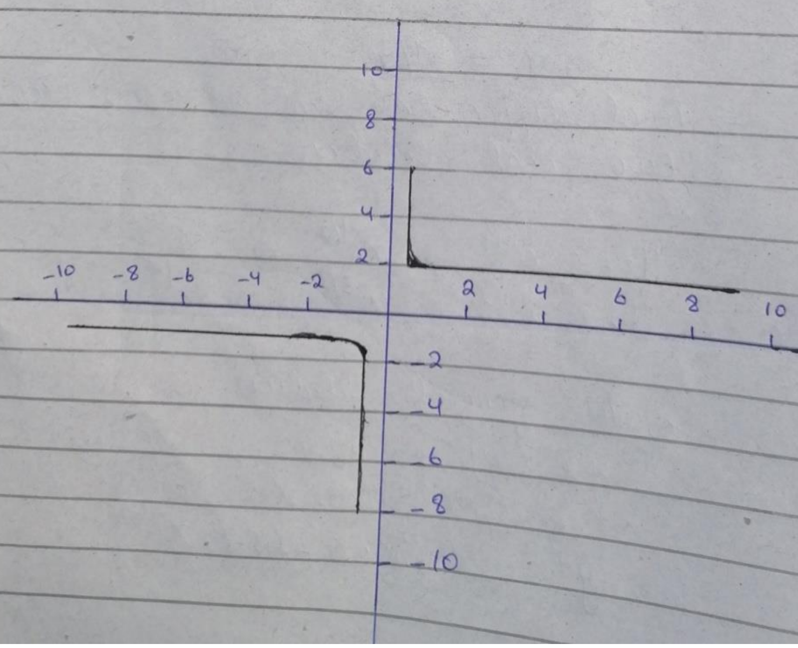
$$2 = 2C_2$$

$$\boxed{C_2 = 1} \quad \text{put in (1)}$$

$$\boxed{C_1 = 0}$$

Particular Solution of IVP is:

$$\boxed{y = x^{-1.5}}$$



(14) $x^2y'' + xy' + 9y = 0, \quad y(1) = 0, \quad y'(1) = 2.5$

Solution:-

let $y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)\,x^{m-2}$

Equation becomes

$x^2 m(m-1)\,x^{m-2} + mx^{m-1}.x + 9x^m = 0$

$\cancel{x^2}\, m\,(m-1)\,x^m.\cancel{x^{-2}} + mx^m.\cancel{x^{-1}}\cancel{x} + 9x^m = 0$

$x^m\,[m(m-1) + m + 9] = 0$

$m^2 + 9 = 0$

$m^2 = -9$

$\sqrt{m^2} = \sqrt{-9}$

$m = \pm 3i$

$m_1 = 3i, \quad m_2 = -3i$

As,

$x = e^{\ln x}$

$x^{m_1} = x^{3i} = (e^{\ln x})^{3i} = e^{3i\ln x}$

$x^{m_2} = x^{-3i} = (e^{\ln x})^{-3i} = e^{-3i\ln x}$

As we know that

$e^z = e^{a+ib}$

$= e^a(\cos b + i\sin b)$ ZEC

So, $e^{3i\ln x} = e^0\,[\cos(3\ln x) + i\sin(3\ln x)]$

$= \cos(3\ln x) + i\sin(3\ln x)$

&

$e^{-3i\ln x} = e^0\,[\cos(3\ln x) - i\sin(3\ln x)]$

$= \cos(3\ln x) - i\sin(3\ln x)$

This gives

$x^{m_1} = \cos(3\ln x) + i\sin(3\ln x)$ ——(1)

$x^{m_2} = \cos(3\ln x) - i\sin(3\ln x)$ ——(2)

Adding (1) & (2) & dividing by 2i

$$\frac{x^{m_1} - x^{m_2}}{2i} = \cos 3\ln x + i\sin(3\ln x) - \cos(3\ln x) + i\sin(3\ln x)$$

$$\Rightarrow \frac{\cancel{2i}\sin(3\ln x)}{\cancel{2i}}$$

$$y_1 = \cos(3\ln x)$$

$$y_2 = \sin(3\ln x)$$

so general solution is

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1 \cos(3\ln x) + c_2 \sin(3\ln x)$$

$$y' = -\frac{3c_1}{x}\sin(3\ln x) + \frac{3c_2}{x}\cos(3\ln x)$$

To determine c_1 & c_2

$$y(1) = 0 = c_1 \cos(3\ln 1) + c_2 \sin(3\ln 1)$$

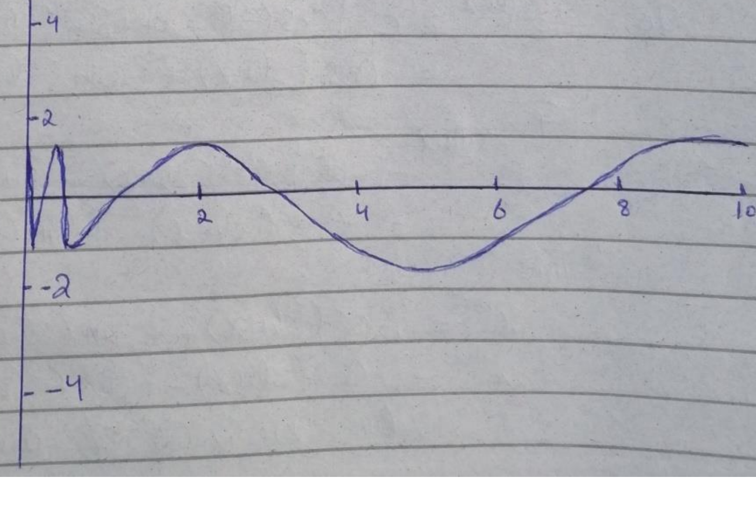
$$\boxed{c_1 = 0}$$

$$2.5 = y'(1) = -3c_1 \sin(3\ln x) + 3c_2 \cos(3\ln x)$$

$$\boxed{c_2 = 5/6}$$

particular solution of the IVP is:

$$y = 5/6 \sin(3\ln x)$$



15. $x^2y'' + 3xy' + y = 0 \qquad y(1) = 3.6,\ y'(1) = 0.4$

Solution:- Let

$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)\,x^{m-2}$$

equation becomes

$$x^2 m(m-1)\,x^m \cdot x^{-2} + 3m x^m + x^m = 0$$
$$x^m\{m(m-1) + 3m + x^m\} = 0$$
$$m^2 + 2m + 1 = 0$$
$$(m+1)^2 = 0$$

To find second linearly Independent solution y_2 by using method of reduction of order

ODE of standard form

$$y'' + 3/x\ y' + 1/x^2\ y = 0$$
$$P(x) = 3 \cdot \frac{1}{x} \Rightarrow \int P\,dx = 3\ln|x|$$

$$v = \int U\,dx \ \ \& \ \ U = \frac{1}{y_1^2}\, e^{-\int P\,dt}$$

$$e^{-\int P\,dt} = e^{-3\ln x} = \left(e^{\ln|x|}\right)^{-3} = x^{-3}$$

$$U = x^{-3} \cdot \frac{1}{x^2} = \frac{1}{x}$$

$$y_2 = \frac{1}{x}\ln x$$

So the general solution is:

$$y = C_1 y_1 + C_2 y_2$$

(9)

$$= \frac{1}{x} = (C_1 + C_2 \ln x)$$

$$y' = \frac{1}{x^2}(-C_1 - C_2 \ln x + C_2)$$

To determine C_1 & C_2

$$3.6 = y(1) \frac{1}{1}(C_1 + C_2 \ln x)$$

$$\boxed{C_1 = 3.6}$$

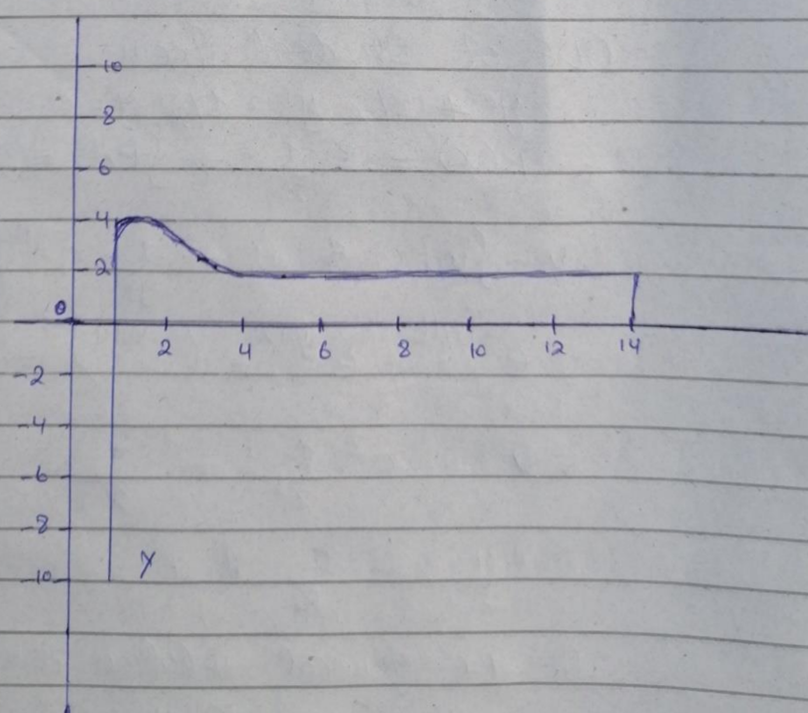
$$0.4 = y'(1) = \frac{1}{1}(-C_1 - C_2 \ln x + C_2)$$

$$0.4 = -C_1 + C_2$$

$$\boxed{C_2 = 4}$$

Particular solution of IVP is:

$$y = (3.6 + 4 \ln x)\frac{1}{x}$$



16. $(x^2D^2 - 3xD + 4I)y = 0, \quad y(1) = -\pi, \; y'(1) = 2\pi$

Solution :- $x^2D^2y - 3xDy + 4Iy$

$= x^2D(Dy) - 3xDy + 4y$

$= x^2y'' - 3xy' + 4y$ ——— (1)

Let

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)\,x^{m-2}$

Eq (1) $\Rightarrow$

$\cancel{x^2}\,m(m-1)\,x^m . x^{-2} - 3\cancel{x}m(x^m)\cancel{x^{-1}} + 4x^m$

$x^m(m(m-1) - 3m + 4) = 0$

$m^2 - 4m + 4 = 0$

$(m - 2)^2 = 0$

It has real double root

$m = 2$

Real double root provide real Solution

$y_1 = x^m = x^2$

To find y_2 we use method of reduction of order

ODE in standard form

$y'' - 3/x \cdot y' + 4/x^2 \cdot y = 0$

$P(x) = -3 \cdot 1/x \Rightarrow \int P dx = -3\ln|x|$

put $y_2 = uy_1$

$u\int U dx \quad , \quad U = \frac{1}{y_1^2}e^{-\int P dx}$

Let's find U

$= e^{-\int P dx} = e^{3\ln|x|} = (e^{\ln|x|})^3 = x^3$

$= U = x^3 . \frac{1}{(x^2)^2} = \frac{1}{x}$

By Integrating

$$u = \int \frac{dx}{x} = \ln|x|$$

$$y_2 = uy_1 = y_1 \ln x = x^2 \ln x$$

So, the general solution is :

$$y = C_1 y_1 + C_2 y_2$$
$$= C_1 x^2 + x^2 \ln x$$
$$\boxed{= x^2 (C_1 + C_2 \ln x)}$$

$$y' = (x^2)'(C_1 + C_2 \ln x) + x^2 (C_1 + C_2 \ln x)'$$
$$= 2x(C_1 + C_2 \ln x) + C_2 x^2 . 1/x$$
$$= 2C_1 x + 2C_2 x \ln x + C_2 x$$
$$= 2C_1 x + C_2 x (2\ln x + 1)$$

Now we have to determine C_1 & C_2

$$-\pi = y(1) = 1^2 (C_1 + C_2 \ln x)$$
$$-\pi = C_1 \quad \text{——— (1)}$$
$$2\pi = y'(1) = 2C_1 + C_2(2\ln \cancel{x} 1 + 1)$$
$$2\pi = 2C_1 + C_2 \quad \text{—— (2)}$$

put (1) in (2)

$$2\pi = 2(-\pi) + C_2$$

$$\boxed{C_2 = 4\pi}$$

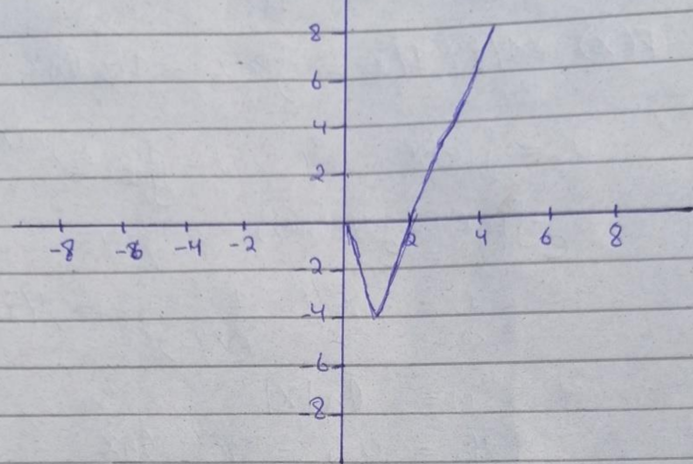
Particular Solution of IVP is

$$y = x^2 (-\pi + 4\pi \ln x)$$

(12)

Particular Solution of IVP

$y = x^2(-\pi + 4\pi \ln x)$



(17) $(x^2D^2 + xD + I)\,y = 0 \quad , \quad y(1) = 1 \quad , \quad y'(1) = 1$

Solution:- By applying operator

$$x^2D(Dy) + xDy + y$$
$$= x^2y'' + xy' + y \quad \text{——— (1)}$$

Let

$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)\,x^{m-2}$$

eq (1) becomes

$$x^2 m(m-1)\,x^{m-2} + xmx^{m-1} + x^m = 0$$
$$x^m\,(m(m-1) + xm + 1) = 0$$
$$m(m-1) + m + 1 = 0$$
$$m^2 - m + m + 1 = 0$$
$$m^2 + 1 = 0$$
$$\sqrt{m^2} = \sqrt{-1}$$

$m = \pm i$

$m_1 = i \quad , \quad m_2 = -i$

As,

$x = e^{\ln x}$

$x^{m_1} = x^{i} = (e^{\ln x})^{i} = e^{i\ln x}$

$x^{m_2} = x^{-i} = (e^{\ln x})^{-i} = e^{-i\ln x}$

$e^{z} = e^{a+ib} = e^{a}(\cos b + i\sin b) \quad z \in \mathbb{C}$

So we have

$e^{i\ln x} = e[\cos(\ln x) + i\sin(\ln x)]$

$= \cos(\ln x) + i\sin(\ln x)$ ——— (i)

&

$e^{-i\ln x} = \cos(\ln x) - i\sin(\ln x)$ ——— (ii)

Adding (i) & (ii) & $\div$ing by 2

$x^{m_1} + x^{m_2} = \cos(\ln x) + i\sin(\ln x) + \cos(\ln x) - i\sin(\ln x)$

$= \frac{x\cos(\ln x)}{x} = \cos \ln x$

Now subtract (1) & (2) & $\div$ by $2i$

$x^{m_1} - x^{m_2} = \cancel{\cos(\ln x)} + i\sin(\ln x) \mp \cancel{\cos(\ln x)} + i\sin(\ln x)$

$= \frac{2i \sin(\ln x)}{2i} = \sin(\ln x)$

$y_1 = \cos(\ln x)$ & $y_2 = \sin(\ln x)$

So general solution is

$y = C_1 y_1 + C_2 y_2$

$= C_1 \cos(\ln x) + C_2 \sin(\ln x)$

$y' = -\frac{C_1}{x} \sin(\ln x) + \frac{C_2}{x} \cancel{\sin}\cos(\ln x)$

we have to determine C_1 & C_2

$1 = y(1) = C_1\cos(\ln 1) + C_2 \sin(\ln 1)$

$\boxed{C_1 = 0}$

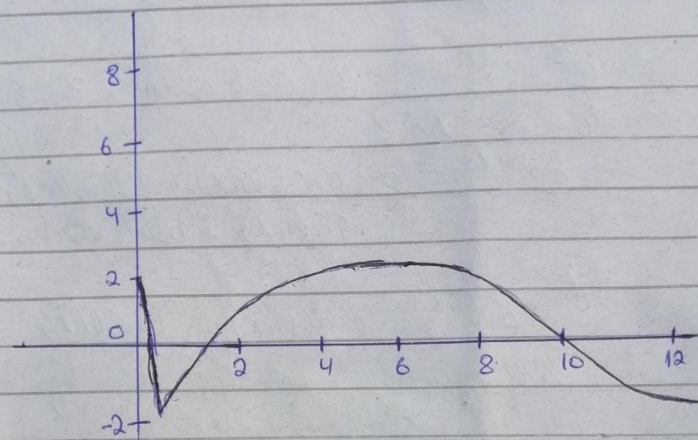
(14)

$$1 = y'(1) = -C_1 \sin(\ln 1) + 3C_2 \cos(\ln 1)$$

$$\boxed{C_2 = 1}$$

Particular Solution of IVP is:

$$y = \sin(\ln x) + C_2 (\ln x)$$



(18) $(9x^2D^2) + 3xD + Iy = 0, \; y(1) = 1, \; y'(1) = 0$

Solution: By applying operator

$$= 9x^2y'' + 3xy' + y$$

Let $y = x^m \Rightarrow y' = mx^{m-1}$

$$y'' = m(m-1)\,x^{m-2}$$

$$9x^2 m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$$

$$x^m(9m(m-1) + 3m + 1 = 0$$

$$9m^2 - 6m + 1 = 0$$

$$m = \frac{6 \pm \sqrt{6^2 - (4)(9)}}{18}$$

$$m_1 = \frac{6}{18} \Rightarrow m_1 = 1/3$$

It has real double root

To find y_2 we use method of reduction of order

$$y'' = \frac{1}{3x} = y' + \frac{1}{9x^2} \cdot y = 0$$

$$P(x) = 1/3 \cdot 1/x \Rightarrow \int P dx = 1/3 \ln x$$

$$u = \int v dx \quad \& \quad U = \frac{1}{y_1^2} e^{-\int P dx}$$

$$e^{-\int P dx} = e^{-1/3 \ln(x)} = x^{-1/3}$$

$$U = x^{-1/3} \cdot \frac{1}{(x^{1/3})^2} = 1/x$$

$$U = \ln|x|$$

$$y_2 = Uy_1 = x^{1/3} \ln x$$

General Solution is

$$y = C_1 y_1 + C_2 y_2$$

$$= x^{1/3}(c_1 + C_2 \ln x)$$

$$y' = 1/3\, x^{-2/3}(C_1 + C_2 \ln x) + x^{1/3} C_2$$

we have to determine C_1 & C_2

$$1 = y(1) = 1^{1/3}(C_1 + C_2 \ln 1)$$

$$\boxed{C_1 = 1}$$

$$U = y'(1) = 1/3 \cdot 1^{-2/3}(C_1 + C_2 \ln 1) + 1^{1/3} C_2$$

$$\boxed{C_2 = -1/3}$$

Particular Solution is

$$\boxed{y = x^{1/3}\left(1 - \frac{1}{3} \ln x\right)}$$

7. $(x^2 D^2 - xD - 15I)y = 0$

$y(1) = 0.1 \quad , \quad y'(1) = -4.5$

Solve:-

By applying operator

$x^2 D(Dy) - xDy - 15Iy = 0$

$= x^2 y'' - xy' - 15y$

Let $y = x^m$

$y' = mx^{m-1} \Rightarrow y'' \; m(m-1)x^{m-2}$

eq becomes

$\cancel{x^2} m(m-1) x^m \cancel{x^{-2}} - \cancel{x} m x^m \cancel{x^{-1}} - 15x^m = 0$

$x^m [m(m-1) - m - 15] = 0$

$m^2 - 2m - 15 = 0$

$m = \frac{2 \pm 8}{2}$

$m_1 = 5 \quad , \quad m_2 = -3$

Real different roots m_1 & m_2 provide two real solutions.

$y_1 = x^{m_1} = x^5$

$y_2 = x^{m_2} = x^{-3}$

So the general solution is

$$y = c_1 y_1 + c_2 y_2$$
$$= c_1 x^5 + c_2 x^{-3}$$

$$y' = 5c_1 x^4 - 3c_2 x^{-4}$$

we have to find c_1 & c_2

$$0.1 = y(1)$$
$$\Rightarrow c_1 \cdot 1^5 + c_2 \cdot 1^{-3}$$
$$0.1 = c_1 + c_2 \quad \text{——} \quad (1)$$

$$-4.5 = y'(1)$$
$$\Rightarrow 5c_1 \cdot 1^4 - 3c_2 \cdot 1^{-4}$$
$$-4.5 = 5c_1 - 3c_2 \quad \text{——} \quad (2)$$

By solving (1) & (2) simultaneously

$0.1 - c_2 = c_1$ put in (2)

$$\boxed{\begin{aligned} c_2 &= 0.625 \\ c_1 &= -0.525 \end{aligned}}$$

particular solution is

$$y = -0.525x^5 + 0.625x^{-3}$$

Question # 02

1. use the method of separation of variables to find general solutions to the following differential equations.

(a) $x' = \sqrt{x}$

Sol:-
$$\frac{dx}{dy} = \sqrt{x}$$

$$\frac{1}{\sqrt{x}}\, dx = dy$$

$$\int \frac{1}{\sqrt{x}}\, dx = \int (1)\, dy$$

$$\int (x)^{-1/2}\, dx = \int (1)\, dy$$

$$\frac{x^{-1/2+1}}{\frac{-1}{2}+1} + C = y$$

$$y = \frac{x^{-1+3/2}}{\frac{-1+2}{2}} + C$$

$$y = \frac{x^{1/2}}{1/2} + C$$

$$\boxed{y = 1/2\ x^{1/2} + C}$$

(2) $x' = e^{-2x}$

Sol :-

$$\frac{dx}{dy} = e^{-2x}$$

$$\frac{1}{e^{-2x}} \, dx = (1).dy$$

$$\int \frac{1}{e^{-2x}} \, dx = \int 1.dy$$

$$\int e^{2x} \, dx = \int 1.dy$$

$$y = e^{2x} . x^2 + C$$

$$x(0) = 1$$

$$y = e^{2(0)} . (0)^2 + C$$

$$y = 1 \times (0) + C$$

$$y = C$$

$$y = e^{2(0)} . (0)^2 + C$$

$$\boxed{y = 0}$$

(c) $y' = 1 + y^2$

Sol :-

$$\frac{dy}{dx} = 1 + y^2$$

$$\frac{1}{1+y^2} \cdot dy = (1)\, dx$$

$$\int \frac{1}{(1+y^2)'} \cdot dy = \int (1)\, dx$$

$$\int \frac{1}{(1+y^2)^{+1}}\, dy = \int (1) \cdot dx$$

$$\tan^{-1} y + C = x$$

$$x + C = \tan^{-1} y$$
$$\tan x + C = y$$
$$y = \tan x + C$$

(d) $u' = \frac{1}{5-2u}$

Sol :-

$$\cancel{dy}\ \frac{du}{dx} = \frac{1}{5-2u}$$

$$(5-2u) \cdot du = (1)\, dx$$

$$\int (5-2u) \cdot du = \int 1 \cdot dx$$

$$5\int (1)\, du - 2\int u \cdot du = \int 1 \cdot dx$$

$$\Rightarrow 5u - 2\frac{u^{1+1}}{1+1} = x$$

$$5u - \cancel{2}\frac{u^2}{\cancel{2}} = x + C$$

$$\boxed{5u - u^2 = x + C}$$

(e) $x' = au + b, \quad a, b > 0$

<u>Sol:-</u> $\frac{dx}{du} = au + b$

$$(1)\ dx = (au + b)\,du$$

$$\int 1 \cdot dx = a\int u \cdot du + b\int (1) \cdot du$$

$$C + x = a \cdot \frac{u^{1+1}}{1+1} + bu$$

$$C + x = a \cdot \frac{u^2}{2} + bu$$

$$\boxed{\frac{1}{2}au^2 + bu = x + C}$$

(f) $Q' = \dfrac{Q}{4+Q^2}$

Sol :-

$$\frac{dQ}{dx} = \frac{Q}{4+Q^2}$$

$$\frac{4+Q^2}{Q} \cdot dQ = (1)\, dx$$

$$\left(\frac{4}{Q} + \frac{Q^2}{Q}\right) \cdot dQ = (1)\, dx$$

$$\left(\frac{4}{Q} + Q\right) \cdot dQ = (1)\, dx$$

$$4\int \frac{1}{Q}\, dQ + \int Q\, dQ = \int 1 \cdot dx$$

$$\boxed{4 \ln|Q| + \frac{Q^2}{2} = x + C}$$

(g) $x' = e^{x^2}$

$$\frac{dx}{dy} = e^{x^2}$$

$$\frac{1}{e^{x^2}} \cdot dx = (1) \cdot dy$$

$$\int e^{-x^2} \cdot dx = \int (1) \cdot dy$$

$$e^{-x^2} \cdot \int (-x)^2 \, dx = y$$

$$y = -e^{-x^2} \cdot \frac{x^{2+1}}{2+1} + C$$

$$y = -e^{-x^2} \cdot \frac{x^3}{3} + C$$

$$\boxed{y = -\frac{1}{3} e^{-x^2} \cdot x^3 + C}$$

(h) $y' = r(a-y)$

$$\frac{dy}{dr} = r(a-y) \Rightarrow \frac{dy}{dr} = ra - ry$$

$$\frac{dy}{dr} = r(a-y) \Rightarrow \frac{1}{a-y} \, dy = r \cdot dr$$

$$\int \frac{1}{a-y} \cdot dy = \int r \cdot dr$$

$$-\int \frac{-1}{a-y} \cdot dy = \frac{r^2}{2} + C$$

$$\boxed{-\ln |a-y| = \frac{r^2}{2} + C}$$

2. Solve $y' = r(a-y)$ where r & a are constants.

Solve:- $\frac{dy}{dx} = ra - ry$

$$\int \frac{1}{ra-ry} \cdot dy = \int (1) \cdot dx$$

$$\frac{1}{r} \int \frac{1}{a-y} \, dy = x$$

Multiply by $(-)$

$$\frac{1}{r} \int \frac{-1}{a-y} \cdot dy = -x$$

$$\frac{1}{r} \ln |a-y| = -x + C$$

$$\frac{1}{r} \ln |a-y| = -x + C$$

3. In Exercises 1(a)-(b) find the solution to the resulting IVP when $x(0) = 1$

Sol:- (a) $\frac{dx}{dy} = \sqrt{x} \Rightarrow \frac{1}{\sqrt{x}} \cdot dx = dy$

$$\int \frac{1}{\sqrt{x}} \cdot dx = \int (1) \, dy$$

$$\frac{(x)^{-1/2+1}}{\frac{-1}{2}+1} + C = y \Rightarrow y = \frac{1}{2} x^{1/2} + C$$

$$x(0) = 1 \Rightarrow y = \frac{1}{2} (0)^{1/2} + C$$

$$\boxed{y = C}$$

(b) $x' = e^{-2x}$

$$\frac{dy}{dx} = e^{-2x} \Rightarrow \frac{1}{e^{-2x}}\, dx = (1)\, dy$$

$$\int \frac{1}{e^{-2x}} \cdot dx = \int 1 \cdot dy \Rightarrow \int e^{2x}\, dx = \int 1 \cdot dy$$

$$y = e^{2x} \cdot x^2 + C \Rightarrow x(0) = 1 \Rightarrow y = e^{2(0)} \cdot (0)^2 + C$$

$$y = 1 \times 0 + C \Rightarrow y = C$$

$$y = e^{2(0)} \cdot (0)^2 + C \Rightarrow \boxed{y = 0}$$

4. Find General Solution.

(a) $x' = \dfrac{2x}{t+1}$

$$\frac{dx}{dt} = \frac{2x}{t+1}$$

$$\int \frac{1}{2x}\, dx = \int \frac{1}{1+t}\, dt$$

$$\frac{1}{2} \int \left(\frac{1}{2}\right) \cdot dx = \int \frac{1}{1+t} \cdot dt$$

$$\frac{1}{2} \ln|x| + C = \ln|1+t|$$

$$\left[\frac{1}{2} \ln|x|\right] + C = 1+t$$

$$\ln|1+t| = \frac{1}{2}\left\{\ln|x|\right\} + C$$

(b) Q $\theta' = t\sqrt{t^2+1}\ \sec\theta$

Solve :-

$$\frac{d\theta}{dt} = t\sqrt{t^2+1}\cdot\sec\theta$$

$$\frac{1}{\sec\theta}\,d\theta = t\sqrt{t^2+1}\,.\,dt$$

$$\int\frac{1}{\sec\theta}\cdot d\theta = \int\sqrt{t^2+1}\cdot dt$$

$$\int\frac{1}{1/\cos\theta}\cdot d\theta = \int\sqrt{1+t^2}\,.\,dt$$

$$\int\cos\theta\,.\,d\theta = \int\sqrt{1+t^2}\,.\,dt$$

$$\sin\theta + C = \frac{(1+t^2)^{1/2+1}}{\frac{1}{2}+1}\cdot\int(1+t^2)\,.\,dt$$

$$\sin\theta + C = \frac{(1+t^2)^{3/2}}{3/2}\int 1\cdot dt\int t^2\cdot dt$$

$$\sin\theta + C = \frac{2}{3}(1+t^2)^{3/2}\cdot t\cdot\frac{t^3}{3}$$

$$\sin\theta + C = \frac{2}{9}(1+t^2)^{3/2} + 4$$

(c) $(2u+1)\,u' - (t+1) = 0$

Solve:-

$$(2u+1)\,u' = (t+1)$$

$$u' = \frac{t+1}{2u+1}$$

$$\frac{du}{dt} = \frac{t+1}{2u+1}$$

$$\int (2u+1) \cdot du = \int (t+1)\, dt$$

$$2\int u \cdot du + \int 1 \cdot du = \int t\,dt + \int 1 \cdot dt$$

$$\frac{\cancel{2}u^2}{\cancel{2}} + u = \frac{t^2}{2} + t$$

$$u^2 + u = \frac{t^2}{2} + t$$

$$u^2 + u - \frac{t^2}{2} = t$$

$$\boxed{t = u^2 + u - \frac{t^2}{2}}$$

(d) $R' = (t+1)(R^2+1)$

Sol:-

$$\frac{dr}{dt} = (t+1)(R^2+1)$$

$$\frac{1}{R^2+1} \cdot dr = (t+1)\, dt$$

$$\int \frac{1}{R^2+1}\, dr = \int t \cdot dt + \int 1 \cdot dt$$

$$\tan^{-1} R = \frac{t^2}{2} + t + C$$

$$\boxed{R = \tan\left[\frac{t^2}{2} + t\right] + C}$$

(e) $y' + y + \frac{1}{y} = 0$

Sol:-

$$y' = -y - \frac{1}{y}$$

$$\frac{dy}{dx} = -y - \frac{1}{y}$$

$$\left(y + \frac{1}{y}\right) dy = (1) \cdot dx$$

$$\int y\,dy + \int \frac{1}{y}\, dy = \int 1 \cdot dx$$

$$\frac{y^2}{2} + \ln|y| = x$$

$$\boxed{\frac{1}{2} y^2 + \ln|y| + C = x}$$

(f) $(t+1)x' + x^2 = 0$

Sol :-

$$\frac{\cancel{(t+1)}\,x'}{\cancel{(t+1)}} + x^2 = 0$$

$$\frac{\cancel{(t+1)}\,x'}{\cancel{(t+1)}} = \frac{x^2}{(t+1)}$$

$$\frac{dx}{dt} = \frac{x^2}{(t+1)}$$

$$\int \frac{1}{x^2}\,dx = \int \frac{1}{1+t}\,dt$$

$$\int \cancel{1}\,x^{-2}\,dx = \int \frac{1}{1+t}\,dt$$

$$-x^{-1} = \ln|t| + c$$

$$\boxed{-x^{-1} = +\ln|t| + c}$$