Name: Shah Zeb
ID # 14798
Class/Sec B
Class Timing WED 8:00 to 11:00

Q1 There are total of "5" machines and "5" employments are to be relegated & the related cost network is as per the following locate the best possible task.

JOBS	Machines				
	A	B	C	D	E
1	6	12	3	11	15
2	4	2	7	1	10
3	8	11	10	7	11
4	16	19	122	23	21
5	9	5	7	6	10

Sol

Here we have "5" jobs and "5" Machines.

In this problem we have "5" numbers of Columns and "5" numbers of Rows so this one is balanced one.

Now we used the "Hungerian Method"

JOBS	Machines						Row Minimum
		A	B	C	D	E	3
	1	6	12	3	11	15	3
	2	4	2	7	1	10	1
	3	8	11	10	7	11	7
	4	16	19	122	23	21	16
	5	9	5	7	6	10	5

Now subtract the minimum value of each row from the entries of that row.

JOBS	Machines					
	1	A	B	C	D	E
	1	3	9	0	8	12
	2	3	1	6	0	9
	3	1	4	3	0	4
	4	0	3	106	7	5
	5	4	0	2	1	5

"Now column reduction".

JOBS	Machines					
		A	B	C	D	E
	1	3	9	0	8	12
	2	3	1	6	0	9
	3	1	4	3	0	4
	4	0	3	106	7	5
	5	4	0	2	1	5

Column Minimum

0 0 0 0 4

"Now Subtract the minimum value of each column from the entries of that column".

JOBS	Machines					
		A	B	C	D	E
	1	3	9	0	8	8
	2	3	1	6	0	5
	3	1	4	3	0	0
	4	0	3	106	7	1
	5	4	0	2	1	1

"Now Optimation of the problem"

Draw a minimum no of lines to cover all the zeros of the matrix.

(a) Row Scaning

JOBS	Machines				
	A	B	C	D	E
1	3	9	$\boxed{0}$	8	8
2	3	1	6	$\boxed{0}$	5
3	1	4	3	0	$\boxed{0}$
4	$\boxed{0}$	3	106	7	1
5	4	$\boxed{0}$	2	1	1

No of squaremarked Nof The rows of the Matrix

$5 = 5$

So it is "Optimal solution"

Job	Machines	Time
1	C	3
2	D	1
3	E	11
4	A	16
5	B	5
		36

Q2 Solve the following Linear Programming problem.

$$\min z = 2x_1 + 3x_2$$
$$\text{s.t } (1/2)x_1 + (1/4)x_2 \leq 4$$
$$x_1 + 3x_2 \geq 20$$
$$x_1 + x_2 = 10$$
$$x_1, x_2 \geq 0.$$

Sol

$$\min Z = 2x_1 + 3x_2$$
$$\text{s.t } (1/2)x_1 + (1/4)x_2 \leq 4$$
$$x_1 + 3x_2 \geq 20$$
$$x_1 + x_2 = 10$$
$$x_1, x_2 \geq 0$$

step#1 Add surplus variable & artifical variable

Now In Standard form

$$\min \; z - 2x_1 - 3x_2 = 0$$
$$\text{s.t } (1/2)x_1 + (1/4)x_2 + S_1 = 4$$
$$x_1 + 3x_2 - e_2 = 20$$
$$x_1 + x_2 = 10$$
$$x_1, x_2, S_1, e_2 \geq 0$$

"Then Add Artificial variables in Constraints 2 & 3"

$$\min \; z - 2x_1 - 3x_2 - Ma_2 - Ma_3 = 0$$
$$\text{s.t } (1/2)x_1 + (1/4)x_2 + S_1 = 4$$
$$x_1 + 3x_2 - e_2 + a_2 = 20$$
$$x_1 + x_2 + a_3 = 10$$
$$x_1, x_2, S_1, e_2, a_2, a_3 \geq 0$$

Tableau before "Clean-up":

z	x_1	x_2	S_1	e_2	a_2	a_3	R.H.S
1	-2	-3	0	0	-M	-M	0
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

First tableau (after "clean-up"):

z	x_1	x_2	S_1	e_2	a_2	a_3	R.H.S
1	2M-2	4M-3	0	-M	0	0	30M
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

Now find out point column to find point row, and then point value.

z	x_1	x_2	S_1	S_2	a_1	a_2	
1	2M-2	4M-3	0	-M	0	0	30M
0	1/2	1/4	1	0	0	0	$4 : \frac{4}{1/4} \; 8$
0	1	3	0	-1	1	0	$20 \div \frac{20}{3}$
0	1	1	0	0	0	1	$10 \; \frac{10}{1} = 10$

Now perform row operation to make pivot equal to 1 and the remaining elements on the pivot column equal to Zero $(\frac{1}{3}R_3)$

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$2M-2$	$4M-3$	0	$-M$	0	0	$30M$
0	$\frac{1}{2}$	$1/4$	1	0	0	0	4
0	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$20/3$
0	1	1	0	0	0	1	10

$(-1R_3 + R_4)$

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$2M-2$	$4m-3$	0	$-M$	0	0	$30M$
0	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	4
0	$1/3$	1	0	$-\frac{1}{3}$	$1/3$	0	$20/3$
0	$2/3$	0	0	$\frac{1}{3}$	$-1/3$	1	$10/3$

$((-4M+3)R_3 + R_1)$

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$(\frac{2}{3})M-1$	0	0	$(1/3)M-1)$	$(1-\frac{4}{3}M)$	0	$20+\frac{10}{3}M$
0	$5/12$	0	1	$1/12$	$-1/12$	0	$\frac{7}{3}$
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	$2/3$	0	0	$1/3$	$-1/3$	1	$10/3$

Repeat process again find pivot column pivot row, & pivot value

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	$(\frac{2}{3})M-1$	0	0	$(\frac{1}{3})M-1$	$1-\frac{4}{3}M$	0	$20+\frac{10}{3}M$
0	$5/12$	0	1	$1/12$	$1/-12$	0	$7/3 = 5.6$
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3 = 20$
0	$2/3$	0	0	$1/3$	$-1/3$	1	$10/3 = 3$

Now again perform operation to make pivot value equal to "1" and remaining elements in the pivot column equal to "0".

$((\frac{3}{2})R_4)$

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	$\frac{2}{3}M-1$	0	0	$\frac{1}{3}M-1$	$1-\frac{4}{3}M$	0	$20+\frac{10}{3}M$
0	$\frac{15}{2}$	0	1	$1/12$	$-1/12$	0	$7/3$
0	$\frac{1}{3}$	1	0	$-1/3$	$\frac{1}{3}$	0	$20/3$
0	1	0	0	$1/2$	$-1/2$	$\frac{3}{2}$	5

$(-\frac{2}{3}M+1)R_4+R_1$

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	0	0	0	$-1/2$	$\frac{1}{2}-M$	$\frac{3}{2}-M$	25
0	$5/12$	0	1	$1/12$	$-\frac{1}{12}$	0	$7/3$
0	$1/3$	1	0	$-1/3$	$\frac{1}{3}$	0	$20/3$
0	1	0	0	$1/2$	$-\frac{1}{2}$	$\frac{3}{2}$	5

$((-5/12) R_4 + R_2)$

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	0	0	0	$-1/2$	$(1/2 - M)$	$3/2 - M$	25
0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	1	0	0	$1/2$	$-1/2$	$3/2$	5

$(-\frac{1}{3} R_4 + R_3)$

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	0	0	0	$-1/2$	$(1/2 - M)$	$3/2 - M$	25
0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
0	0	1	0	$-1/2$	$1/2$	$-1/2$	5
0	1	0	0	$1/2$	$-1/2$	$3/2$	5

Minimum $Z = 25$

	Z	x_1	x_2	S_1	S_2	a_1	a_2	
Z	1	0	0	0	$-1/2$	$(1/2 - M)$	$(3/2 - M)$	25
S_1	0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
x_2	0	0	1	0	$-1/2$	$1/2$	$-1/2$	5
x_1	0	1	0	0	$1/2$	$-1/2$	$3/2$	5

$\text{Min } Z = 25$ $\quad x_1 = 5$ $\quad S_2 = 0$

$S_1 = 1/4$ $\quad x_2 = 5$ $\quad a_1 = 0$

$a_2 = 0$

Q3 Use Vogel's Approximation Method to obtain the initial feasible solution of.

Sol

Origin	1	2	3	4	Supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

Here Demand = Supply
So, Balanced Transportation Problem.

	1	2	3	4	Supply	Row Diff			
1	X 20	[40] 22	17	[80] 4	~~120~~ ~~80~~ 0	13	13	–	–
2	[10] 24	X 37	[30] 9	[30] 4	~~70~~ ~~40~~ 0	2	2	2	17
3	[50] 32	X 37	X 20	X 7	~~50~~ 0	5	5	5	17
Demand	~~60~~ ~~50~~ 0	~~40~~ 0	~~30~~ 0	~~110~~ ~~30~~ 0	240 / 240				
Column Diff	4	15	8	3					
	4	–	8	3					
	8	–	11	8					
	8	–	–	8					

$(40\times22)+(80\times4)+(10\times24)+(30\times9)+(30\times7)$
$+(50\times32) = 3520$

$880+320+240+270+210+1600 = \boxed{3520}$ Ans