

Haseeb - Ullah

ID: 16314

Paper = Linear Algebra,

1) Ques 5)
1)
(Question 2)

(Part a)

$$\text{let } A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

identify the (3,2) entry of AB.

Solution

Row₃(A) and Col₂(B)

$$= [0, 1, -2] \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} = -5$$

==x ==x ==x ==x ==x ==x ==x ==x

Question (3) :-

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix} \text{ find } A^{-1}$$

Solution

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$|A| = 3 \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2 \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$|A| = 3(-4-6) + 2(-15-2) + 1(0-6)$$

$$|A| = -94$$

Now

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & -3 & -1 & 1 & -4 \\ 0 & -2 & -5 & 1 & 6 \end{bmatrix} \begin{array}{l} \\ R_2 - 3R_1 \\ \end{array}$$

$$R = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & -4 & 1 & 10 \\ 0 & 0 & -13 & 1 & 26 \end{bmatrix} \begin{array}{l} R_3 - R_2 \\ R_3 + 2R_2 \\ \end{array}$$

$$R = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & -4 & 1 & 10 \\ 0 & 0 & 1 & 1 & -2 \end{bmatrix} \begin{array}{l} \\ \\ \frac{1}{13} R_3 \end{array}$$

$$x + y + 2z = 1 \quad \text{--- (1)}$$

$$0x + 1y - 4z = 10 \quad \text{--- (2)}$$

$$0x + 0y + 1z = -2 \quad \text{--- (3)}$$

$$z = -2$$

$$\boxed{z = -2}$$

Put $(z = -2)$ in eq (2)

$$+y - 4z = 10$$

$$+y - 4(-2) = 10$$

$$+y + 8 = 10$$

$$y = 10 - 8$$

$$\boxed{y = 2}$$

Put these value in eq ①

$$x + y + 2z = 1$$

$$x - 2 + 2(-2) = 1$$

$$x - 2 - 4 = 1$$

$$x - 6 = 1$$

$$x = 1 + 6$$

$$x = 7$$

x = x = x = x = x = x = x = x = x = x

(Question 2)

Part a :- If A and B are $n \times n$ matrices $|A| = 2$

and $|B| = -3$ Calculate $|A^{-1}B^T|$

Solution

Since $|A^{-1}B^T| = |A^{-1}||B^T|$

$$= \frac{1}{|A|} |B|$$

So $|A^{-1}B^T| = \frac{1}{|A|} |B|$

$$= \frac{1}{2} \cdot 3$$

$$= \frac{3}{2} \quad \checkmark$$

(Question 1)

(Part b :-)

Label the quadratic polynomial the interpolate the point $(1,3), (2,4), (3,4)$

Solution

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$

$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$

$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now $(x_1, y_1) = (1, 3)$

$(x_2, y_2) = (2, 4)$

$(x_3, y_3) = (3, 4)$

$$a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 4$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 4 \end{array} \right] R_2 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 4 & 3 & 1 & 4 \end{array} \right] R_3 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -1 & -3 & 8 \end{array} \right] R_3 + 1R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & -2 & -5 \end{array} \right]$$

$$a_2 + a_1 + a_0 = 3 \quad \text{--- (1)}$$

$$-2a_1 - 3a_0 = -8 \quad \text{--- (2)}$$

$$\frac{-2a_0}{-2} = \frac{-5}{-2}$$

$$a_0 = 2.5$$

$$-2a_1 - 3(2.5) = -8$$

$$-2a_1 - 7.5 = -8$$

$$-2a_1 = -8 + 7.5$$

$$\frac{-2a_1}{-2} = \frac{-0.5}{-2}$$

$$a_1 = 0.25$$

$$a_2 + a_1 + a_0 = 3$$

$$a_2 + 0.25 + 2.5 = 3$$

$$a_2 + 2.75 = 3$$

$$a_2 = 3 - 2.75$$

$$a_2 = 0.25$$