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Subject # Differential Equation

Q.No:1

Solution:

QUESTION 2

Solution:-

$$\Rightarrow \frac{dy}{dt} = e^{y-t} \sec y (1+t^2)$$

$$\Rightarrow \frac{dy}{dt} = \frac{e^y}{e^t} \frac{1}{\cos y} (1+t^2)$$

or $e^{-y} \cos y \, dy = e^{-t}(1+t^2)\, dt$

Integrating B/s.

$$\int e^{-y} \cos y \, dy = \int e^{-t} (1+t)^2 \, dt \quad -(i)$$

By parts.

$$\cos y \cdot \frac{e^{-y}}{-1} - \int \sin y \cdot \frac{e^{-y}}{-1} \, dy$$

$$= \int e^{-t} (1+t^2) \, dt.$$

$\Rightarrow -e^{-y} . \cos y - [-\sin y . e^{-y} + \int \cos y .$
$e^{-y} dy] = \int e^{-t} (1+t^2) dt$

$\Rightarrow -e^{-y} . \cos y + e^{-y} \sin y - \int \cos y .$
$e^{-y} dy = \int e^{-t} (1+t)^2 dt .$

$e^{-y} (\sin y - \cos y) - \int \cos y . e^{-y} dy =$
$\int e^{-t} (1+t)^2 dt .$

from Equation (i)

$\int e^{-t} (1+t)^2 dt = \int e^{-y} \cos y \, dy$

So $e^{-y} (\sin y - \cos y) - \int e^{-t} (1+t^2)$
$dt = \int e^{t} (1+t^2) dt .$

$\Rightarrow e^{-y} (\sin y - \cos y) = \int e^{-t} (1+t^2)$
$dt + \int e^{-t} (1+t)^2 dt .$

(3)

$$\Rightarrow e^{-y}(\sin y - \cos y) = 2\int e^{-t}(1+t^2)\,dt$$

$$= 2\left(1+t^2\right)\frac{e^{-t}}{-1} + \int 2t \cdot e^{-t}\,dt.$$

$$\Rightarrow e^{-y}(\sin y - \cos y) = 2\left[e^{-t}(1+t^2) + 2\left(\frac{e^{-t}}{-1} + \int e^{-t}\,dt\right)\right]$$

$$'' = 2\left[e^{-t}(1+t^2) + 2\left[te^{-t} - e^{-t}\right]\right] + c$$

$$\Rightarrow e^{-y}(\sin y - \cos y) = 2\left[-3e^{-t} - t^2 e^{-t} - 2t \cdot e^{-t}\right] + c.$$

$$\Rightarrow e^{-y}(\sin y - \cos y) = -2e^{-t}\left(t^2 + 2t + 3\right] + c$$

Ans.

Q.No:2

Solution:

Q: $(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{x+y}-\sqrt{x-y})dy=0$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}\longrightarrow (1)$$

This is Homogeneous Differential eq in x and y to solve this put $y=vx$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq (1) becomes

$$v+x\cdot\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v+x\frac{dv}{dx}=\frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$v+x\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}}{v}-v$$

$$x\frac{dv}{dx}=\frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\text{[illegible]}}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking Integral on b/s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\cancel{\ln}(1+\sqrt{1-v^2}) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x + \sqrt{x^2 - y^2} = \frac{1}{c}$$

$$x + \sqrt{x^2 - y^2} = c_1 \quad \because \frac{1}{c} = c_1$$

Which is a Required Solution.

Q.No:3

Solution:

Q 3 Solution:-

$(D^4 + D^2)y = 3x^2 + 4 \cdot \sin x - 2\cos x.$

Sol:-

$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

$\Rightarrow f(D)\, y = f(x)$

As it is a non-homogenous linear equation.

So sol:, will be.

$y = y_c + y_p \quad -(i)$

Complementary solution y_c.

$D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0.$

either $D^2 = 0 \Rightarrow D = 0.$

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(2).

$D^2+1=0 \Rightarrow D^2=-1.$

$D=\sqrt{-1} \Rightarrow \boxed{D=i}$ or $D=\boxed{0+i}$

Roots ξ real ξ complex.

$y_c = C_1 e^{0x} + C_2 e^{0x}(C_2 \cos x + C_3 \sin x)$

$y_c = C_1 + C_2 \cos x + C_3 \sin x.$

$y_P = \frac{1}{f(D)} f(x).$

$y_P = \frac{1}{D^4+D^2}(3x^2+4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4+D^2$

at $D=0 \Rightarrow f(D)=0.$

So, $f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow f'(D)=0$.

Again differentiating;

$$f''(D) = 12D+2.$$

So for $D=0$

$$f''(0) = 12(0)+2 = 2.$$

So, replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$y_P = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

So, Putting $D=0$ in all,

$$\Rightarrow y_P = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2}$$

$$\Rightarrow y_P = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= 3/2\, x^4 + 2x^2 \sin x - x^2 \cos x$$

So, putting in equation (1).

$$y = C_1 + C_2 \cos x + C_3 \cos x + 3/2\, x^4 + 2x^2 \sin x - x^2 \cos x.$$

$$y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + 3/2\, x^4$$

Ans