Subject : Electro Magnetic Field

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Module : 4^{th}

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Q 1 (a)

Transform the vector $B = yi(x+z)j$ located at Point $(-2, 6, 3)$ into cylindrical coordinates

Ans:

$$B = yi(x+z)j$$

given Points are $(-2, 6, 3)$

$$B = yi(xj + zj)$$

$$B = yxij + yzij$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{(-2)^2 + (6)^2}$$

$$\rho = 6.32$$

As we know that

$$Z = z$$

So

$$z = 3$$

As we know that

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \tan^{-1}\left(\frac{6}{-2}\right)$$

$$\phi = \tan^{-1}(-3)$$

$$\phi = -71.56$$

So

$$B = 6.32, -71.56, 3$$ Ans

Q1 (b)

Convert the Point (3, 4, 5) from Cartesian to spherical Coordinates.

Ans: Solve

$P(3, 4, 5)$

$x = 3, y = 4, z = 5$

In spherical coordinate System

r, θ, ϕ

$$r = \sqrt{x^2+y^2+z^2}$$

$$r = \sqrt{(3)^2+(4)^2+(5)^2}$$

$$r = \sqrt{9+16+25}$$

$$r = \sqrt{50}$$

$$r = 7.07$$

As

$$\theta = \tan^{-1}(y/x)$$

$$\theta = \tan^{-1}(4/3)$$

$$\theta = \tan^{-1}(1.33)$$

$$\theta = 53.1^\circ$$

As $$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{(3)^2+(4)^2}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$$

$$\phi = \tan^{-1}\left(\frac{5}{5}\right)$$

$$\phi = \tan^{-1}(1)$$

$$\phi = 45$$

$$\gamma = 7.67 \quad , \quad \theta = 53.1^\circ \quad , \quad \phi = 45$$

Q1 (c)

Find the spherical Coordinates of $A(2, 3, -1)$

γ, θ, ϕ

Ans: Solve

$$\gamma = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$r = \sqrt{4 + 9 + 1}$$

$$r = \sqrt{14}$$

$$r = 3.74$$

$$\theta = \tan^{-1}(y/x)$$

$$\theta = \tan^{-1}(3/2)$$

$$\theta = \tan^{-1}(1.5)$$

$$\theta = 56.3^\circ$$

As

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{(2)^2+(3)^2}}{-1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{4+9}}{-1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{13}}{-1}\right)$$

$\phi = \tan^{-1}(-3.60)$

$\phi = 74.4$

$r = 3.74$, $\theta = 56.3^\circ$, $\phi = 74.4$

Q 1 (d)

Find the cartesian coordinates of $B(4, 25, 120)$

Ans: The Point $B(4, 25, 120)$ is given in spherical (r, θ, ϕ)

So we have to find (x, y, z)

Now

$x = r \sin\theta \cos\phi$

$x = 4 \sin(25) \cos(120)$

$x = 4(0.42)(-0.5)$

$x = -0.84$

As $y = r \sin\theta \sin\phi$

$y = 4 \sin(25) . \sin(120)$

$y = 4 (0.42)(0.86)$

$y = 1.45$

As

$z = r \cos\theta$

$z = 4 \cos(25)$

$z = 4 (0.90)$

$z = 3.62$

$(x, y, z) = (-0.84, 1.45, 3.62)$ Ans

Q 1 (e)

Find the force between two charges when they are brought in contact and separated by 4 cm apart charges are 2 nC and -1 nC, in μN.

Ans :

Given data

$q_1 = 2nC$, $q_2 = -1nC$

$d = 4cm$

Required

$F = ?$

where $F = K \frac{q_1 q_2}{r^2}$

As $K = \frac{1}{4\pi\epsilon_0}$

$$F = \frac{2\times10^{-9} \times -1\times10^{-9}}{4(3.14)\times8.85\times10^{-12}\times(4\times10^{-2})^2}$$

$F = -1.124\times10^{-5}$

$F = -11.24 \mu N$

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Q 1 (f)

Find the electric field intensity of two charges -2C and -1C separated by a distance 1m in air.

Ans: Given data

$q_1 = -2C$, $q_2 = -1C$

$d = 1m$

Required $E = ?$

$K = 9 \times 10^9$

$$E_1 = \frac{K q_1}{d^2}$$

$$E_1 = \frac{9 \times 10^9 \times -2}{(1)^2}$$

$$E_1 = -18 \times 10^9 \ V/m$$

Now

$$E_2 = \frac{K q_2}{d^2}$$

$$E_2 = \frac{9\times10^9 \times (-1)}{(1)^2}$$

$$E_2 = -9\times10^9 \ V/m$$

As

$$E_T = E_1 + E_2$$

$$E_T = -18\times10^9 + (-9\times10^9)$$

$$E_T = -18\times10^9 - 9\times10^9$$

$$E_T = -27\times10^9 \ V/m$$

Q 1 (g)

Determine the charge that Produce an electric field streght of 40 V/cm at a distance of 30cm in vaccum in (10^{-8}C)

Ans: Given data

$E = 40 \ V/cm$, $d = 30cm$

Required

$$Q = ?$$

$$E = K\frac{Q}{d^2}$$

$$Ed^2 = KQ$$

$$\frac{Ed^2}{K} = Q$$

Put values in this equation

$$Q = \frac{Ed^2}{K}$$

$$Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$Q = \frac{40 \times 900}{9 \times 10^9}$$

$$Q = 4 \times 10^{-6} C$$

$$Q = 4 \mu C$$

Q1 (h)

A charge of 2×10^{-7} is acted upon by a force of 0.1N. Determine the distance to the other charge of $4.5 \times 10^{-7} C$, both the charges are in vacuum.

Ans: Given data

$q_1 = 2 \times 10^{-7} C$ $\quad F = 0.1N$

$q_2 = 4.5 \times 10^{-7} C$ $\quad K = 9 \times 10^9$

Required

$d = ?$

By using formula

$$F = K \frac{q_1 q_2}{d^2}$$

$$d^2 = K \frac{q_1 q_2}{F}$$

Now Putting values

$$d^2 = \frac{9\times10^9 (2\times10^{-7}) (4.5\times10^{-7})}{0.1}$$

$$d^2 = 8.1\times10^{-3}$$

As

$$d^2 = 0.0081$$

Now taking underroot on bothside

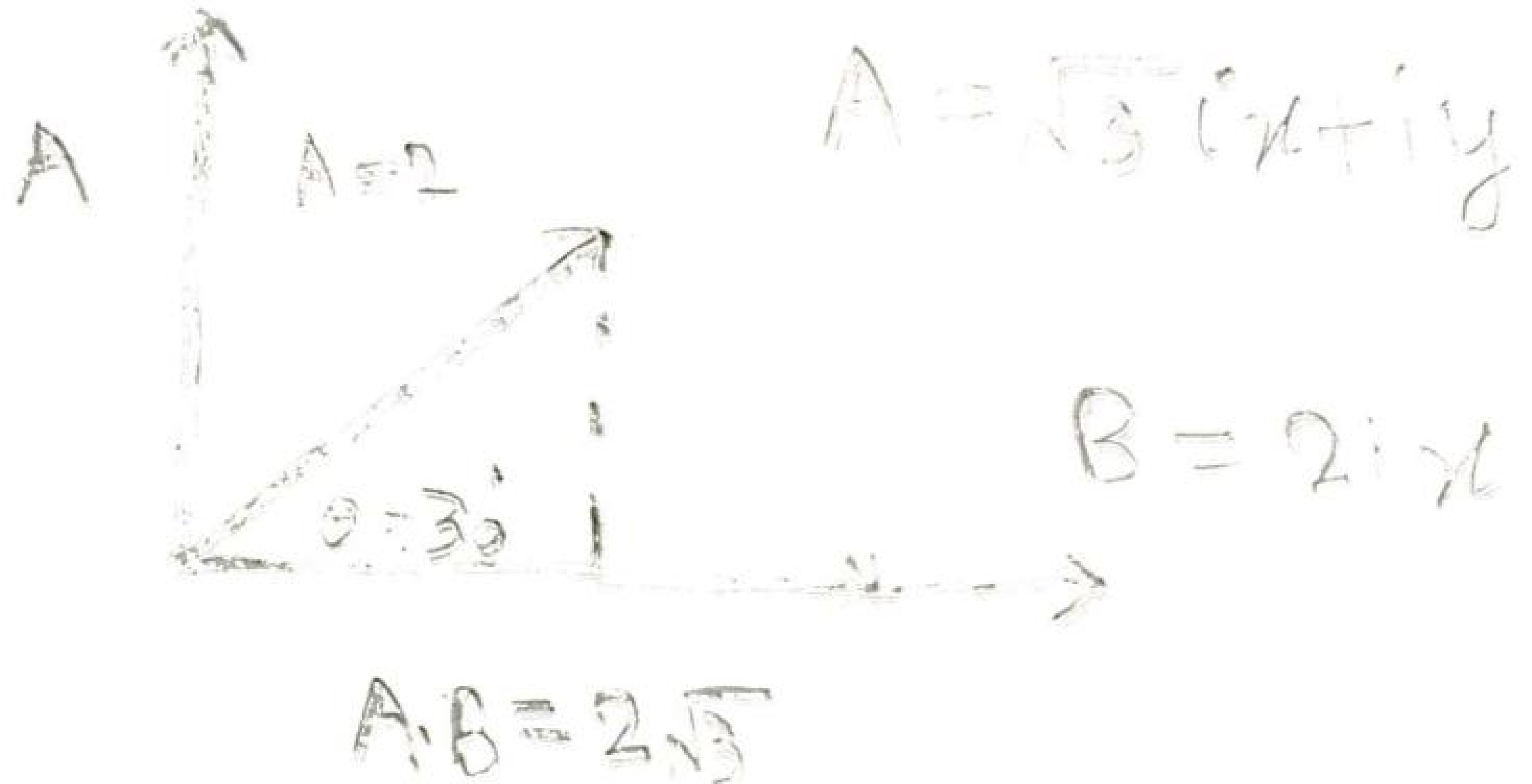
$$\sqrt{d^2} = \sqrt{0.0081}$$

$$d = 0.09\ m$$

$$d = 9\times10^{-2} m$$

Q2 (a)

Find the angle between the vectors shown in fig



Ans:

$$A.B = |A||B|\cos\theta$$

$$A.B = 2\sqrt{3}$$

$|A| = \sqrt{2^2}$ $\qquad$ $|B| = \sqrt{2^2}$

$|A| = 2$ $\qquad$ $|B| = 2$

Put these values in $A.B = |A||B|\cos\theta$

$$2\sqrt{3} = 2 \times 2\cos\theta$$

$$2\sqrt{3} = 4\cos\theta$$

$$\frac{\cancel{2}\sqrt{3}}{\cancel{4}} = \cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \cos^{-1}\left(\frac{1.73}{2}\right)$$

$$\theta = \cos^{-1}(0.866)$$

$$\theta = 30^\circ$$

Q2 (b)

Find the gradient of each of the following function where a and b are constant

i) $f = ax^2 + by^3z$

Sol: $f = ax^2 + by^3z$

$$\frac{df}{dx} = \frac{d}{dx}(ax^2 + by^3z)$$

$$\frac{df}{dx} = 2ax$$

$$\frac{df}{dy} = \frac{d}{dy}(ax^2 + by^3z)$$

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$$\frac{df}{dy} = 3by^2z$$

$$\frac{df}{dz} = \frac{d}{dz}(ax^2+by^3z)$$

$$\frac{df}{dz} = by^3$$

$$\nabla f(x,y,z) = (2ax+3by^2z, by^3)$$

ii) $f = ar^2\sin\phi + brz\cos 2\phi$

Sol: gradient for spherical

$$\nabla f = \frac{df}{dr}\hat{r} + \frac{1}{r}\frac{df}{d\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{df}{d\phi}\hat{\phi}.$$

So

$$\nabla f = \frac{d}{dr}(ar^2\sin\phi + brz\cos 2\phi)\hat{r} + \frac{1}{r}\frac{d}{d\theta}$$

$$(ar^2\sin\phi + brz\cos 2\phi)\hat{\theta} + \frac{1}{r\sin\theta}\frac{d}{d\theta}(ar^2\sin\phi +$$

$$brz\cos 2\phi)\hat{\phi}.$$

Now take Partial derivatives

$$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r}(0)$$
$$+ \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}.$$

So

$$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} +$$
$$\frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}$$

Now gradient for cylindrical

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla f = \frac{\partial}{\partial \rho}(ar^2\sin\phi + brz\cos 2\phi)\hat{\rho} +$$
$$\frac{1}{\rho}\frac{\partial}{\partial \phi}(ar^2\sin\phi + brz\cos 2\phi)\hat{\phi} +$$
$$\frac{\partial}{\partial z}(ar^2\sin\phi + brz\cos 2\phi)\hat{z}$$

Now take Partial derivatives:

The first term becomes zero

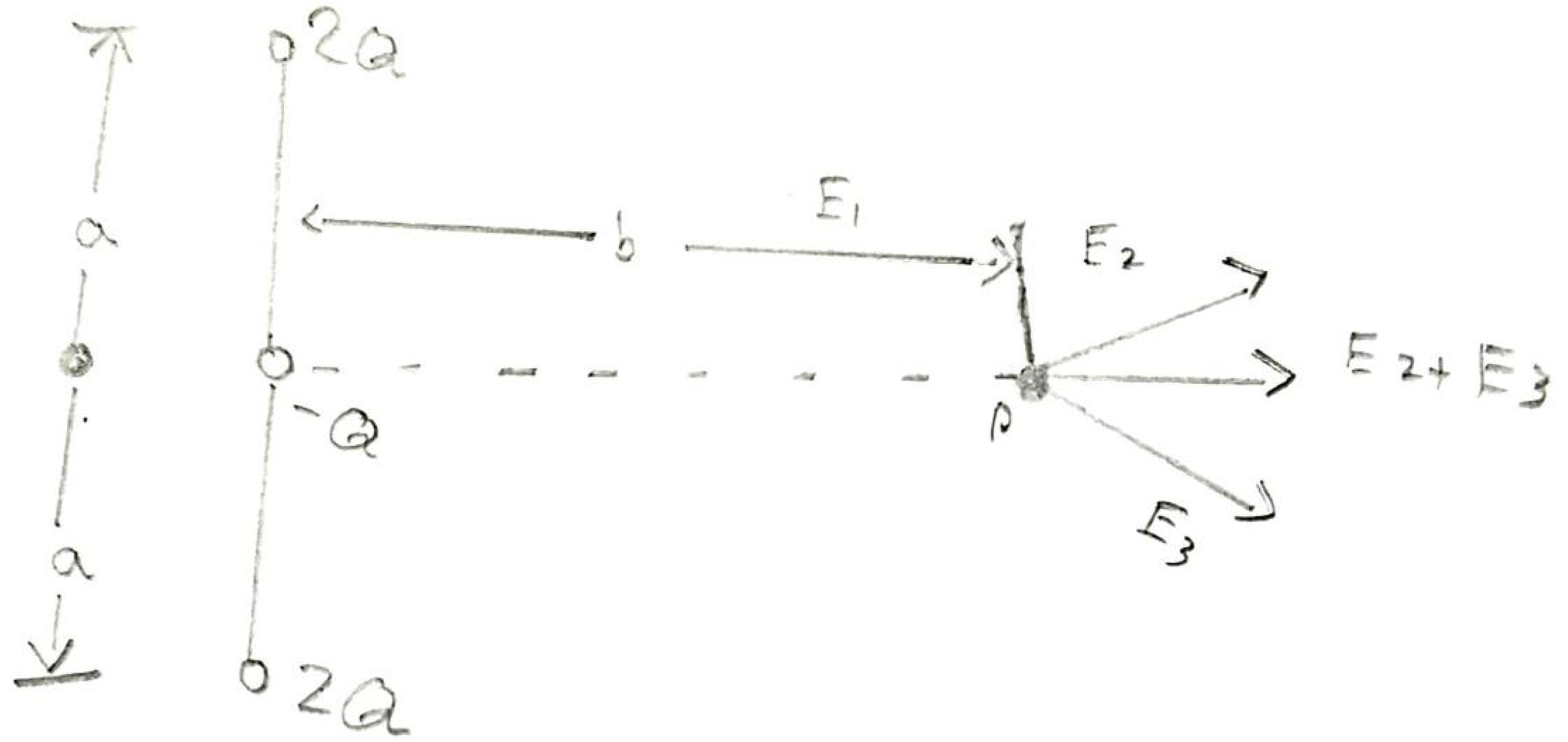
$$\nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\hat{\phi})\hat{\phi} +$$
$$(br\cos 2\phi)\hat{z}.$$

So

$$\nabla f = \frac{1}{\rho}\left(ar^2\cos\phi - 2brz\sin 2\phi\right)\hat{\phi} + \left(br\cos 2\phi\right)\hat{z}$$

Q3

Three Pointer Charges are Placed on the y axis as shown. Find the electric field at Point P on the x axis



Ans: The distance between Charge 2Q and Point "P" is

So $r^2 = b^2 + a^2$

$$r = \sqrt{b^2 + a^2}$$

let assume that charge $2Q$ make angle (α) and $(-\alpha)$ with x-axis

magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{Kq}{r^2}$

$$= \frac{K(2Q)}{r^2}$$

$$= \frac{K(2Q)}{b^2 + a^2}$$

So Resultant of $\vec{E_1}$ and $\vec{E_2}$ is

$$\vec{E}_{1+2} = \vec{E_1} + \vec{E_2} = E_{1x} + E_{2x}$$

(y-component will be cancel)

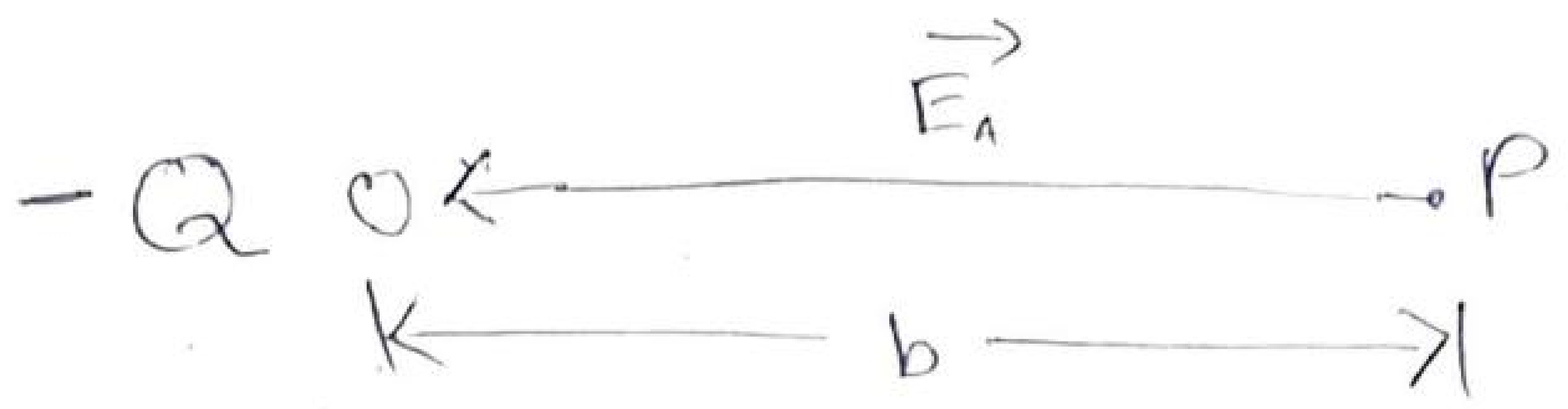
$$= \frac{K(2Q)}{b^2 + a^2} (\cos(\alpha) + \cos(-\alpha))$$

$$= \frac{K(2Q)}{b^2 + a^2} (2\cos(\alpha)) \quad \because \cos(\alpha) = \cos(-\alpha)$$

$$\vec{E}_{1+2} = \frac{4KQ\cos(\alpha)}{b^2 + a^2}$$

- Now electric field at Point 'p' due to charge "$-Q$"

- As charge is negative electric field at point will be directed towards charge "$-Q$".



$$\vec{E_A} = \frac{-K(Q)}{b^2}$$

Net electric field at point P will be

$$\vec{E}_{net} = \vec{E_A} + (\vec{E_1} + \vec{E_2})$$

$$= \frac{-K(Q)}{b^2} + \frac{4KQ\cos\alpha}{b^2+a^2}$$

$$= \frac{-KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $K = 9\times10^9\ Nm^2/C^2$

$$\vec{E}_{net} = \frac{9\times10^{9}\,Q}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

Now

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

So

$$\vec{E}_{net} = \frac{9\times10^{9}\,Q}{b^2(a^2+b^2)}\left[4b^2\cos\left[\tan^{-1}\left(\frac{a}{b}\right) - (a^2+b^2)\right]\right]$$