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 INSTRUCTOR : APPLIED MATHS II  
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Q1

$$(a) \int x^2 e^x dx$$

Sol:-

Using integration by parts:

$$\Rightarrow x^2 \int e^x dx - \int [ \int e^x dx \cdot \frac{d}{dx} x^2 ] dx$$

$$\Rightarrow x^2 e^x - \int e^x \cdot (2x) dx$$

$$\Rightarrow x^2 e^x - 2 \int e^x x dx$$

again using I.B.P

$$\Rightarrow x^2 e^x - 2 \left\{ x \int e^x dx - \int [ e^x dx \cdot \frac{d}{dx} x ] dx \right\}$$

$$\Rightarrow x^2 e^x - 2 \left\{ x e^x - \int e^x (1) dx \right\}$$

$$\Rightarrow x^2 e^x - 2 \left\{ x e^x - e^x \right\} + C$$

$$\Rightarrow x^2 e^x - 2x e^x + 2e^x + C$$

$$\Rightarrow \underline{e^x [ x^2 - 2x + 2 ] + C} \quad \text{Ans:}$$

Q1 Part B

(b)  $\int (1 + 3t) t^3 dt$

SOL:-

$$\Rightarrow \int (t^3 + 3 \cdot t \cdot t^3) dt$$

$$\Rightarrow \int (t^3 dt + 3) t^4 dt$$

$$\Rightarrow \frac{t^{3+1}}{3+1} + 3 \frac{t^{4+1}}{4+1} + C$$

$$\Rightarrow \frac{t^4}{4} + 3 \frac{t^5}{5} + C$$

$$\Rightarrow \boxed{\frac{1}{4} t^4 + \frac{3}{5} t^5 + C} \quad \text{Ans.}$$

Q1 Part C

c)  $\int (e^x - e^3) dx$

$$\Rightarrow \int e^x dx - \int e^3 dx$$

$$\Rightarrow e^x - \frac{e^3}{3} + C$$

$$\Rightarrow \boxed{e^x - \frac{1}{3} e^3 + C} \quad \text{Ans.}$$

Question - 2-

$$f(x) = e^{-6x} \quad \text{at } x = -4$$

Solution :-

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0)$$

$$+ \dots + \frac{(x-x_0)^n}{n!} f^{(n)}(x_0) \rightarrow \textcircled{1}$$

$$f(x) = e^{-6x}$$

$$f(-x) = e^{-6x-4} = e^{24}$$

$$f'(x) = e^{-6x}(-6) = -6e^{-6x}$$

$$= -6e^{-6x-4} = -6e^{24}$$

$$f''(x) = -6e^{-6x} - 6 = 36e^{-6x}$$

$$f''(-4) = 36e^{-6x(-4)} = 36e^{24}$$

put these values in eq  $\textcircled{1}$ .

$$e^{-6x} = e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2!} (36e^{24})$$

+ .....

$$e^{-6x} = e^{24} - 6xe^{24} - 24e^{24} + \frac{(x+4)^2}{2}$$

$$36e^{24} + \dots$$

Answer.

Q2)  $f(x) = e^{-6x}$  at  $x = -4$

We know that

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0) + \dots +$$

$$\frac{(x-x_0)^n}{n!} f^n(x_0) \rightarrow \textcircled{1}$$

$$f(x) = e^{-6x}$$

$$f(-4) = e^{-6 \times -4} = e^{24}$$

$$f'(x) = e^{-6x} (-6) = -6e^{-6x} = -6e^{-6 \times -4} = -6e^{24}$$

$$f''(x) = -6e^{-6x} \times -6 = 36e^{-6x}$$

$$f''(-4) = 36e^{-6 \times (-4)} = 36e^{24}$$

Put these values in equation ①

$$e^{-6x} = e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2!} (36)e^{24} + \dots$$

$$e^{-6x} = e^{24} - 6xe^{24} - 24e^{24} + \frac{(x+4)^2}{2} 36e^{24} + \dots$$

Ans

Q3 Part "A":

$$F(y) = x \sin x$$

Sol.:

$$\frac{d}{dx} (x \sin x)$$

Using products rule

$$\Rightarrow x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x$$

$$\Rightarrow x \cos x + \sin x (1)$$

$$\Rightarrow \underline{x \cos x + \sin x} \quad \text{Ans}$$

PART "B"

$$F(y) = x^2 \cos x$$

Sol.:-

$$\frac{d}{dx} (x^2 \cos x)$$

Using product rule.

$$\Rightarrow x^2 \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x^2$$

$$\Rightarrow x^2 (-\sin x) + \cos x (2x)$$

$$\Rightarrow -x^2 \sin x + 2 \cos x (x)$$

$$\Rightarrow -x^2 \sin x + 2x \cos x$$

$$\Rightarrow \boxed{-x(x \sin x - 2 \cos x)} \text{ Ans.}$$

Part "C"

$$F(z) = z \cdot (2z - 2)^2$$

Sol:-

$$\frac{d}{dz} [z \cdot (2z - 2)^2]$$

Using product rule

$$\Rightarrow z \frac{d}{dz} (2z - 2)^2 + (2z - 2)^2 \cdot \frac{d}{dz} (z)$$

$$\Rightarrow z \{ 2(2z - 2)(2) \} + (2z - 2)^2 \cdot (1)$$

$$\Rightarrow z \{ 4(2z - 2) \} + (2z - 2)^2$$

$$\Rightarrow 4z(2z - 2) + (2z - 2)^2$$

$$\Rightarrow (2z - 2) \{ 4z + 2z - 2 \}$$

$$\Rightarrow \boxed{(2z - 2) \{ 6z - 2 \}} \text{ Ans.}$$