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Section (B)

I:D 7985

Paper differetional Equation

Submitted to

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Question # 1

$w = \ln(2x+2ct)$

$$\frac{\partial w}{\partial x} = \frac{2}{(2x+2ct)} = 2(2x+2ct$$

$$\frac{\partial w^2}{\partial x^2} = 2(-1)[2x+2ct])^{-2} + 2$$

$$= \frac{-4}{2x+2ct^2}$$

$$\frac{\partial w}{\partial x} = \frac{1+2c}{2x+2ct} = 2c(2x+2ct)^{-1}$$

$$\frac{\partial w^2}{\partial t^2} = 2c * (2x+2ct)^{-2} * -1+2c$$

$$= \frac{-4c^2}{(2x+2ct)^2}$$

$$= \frac{-4}{(2x+2ct)^2} * c^2$$

$$\frac{\partial^2 w}{\partial x^2} * c^2$$

parts

$w = \tan(2x + ct)$

Sol: differential w.r.t to (t

$\partial w^2 = c\, 2\sec(2x + ct) \frac{\partial}{\partial t} \sec$

$=$

$\Rightarrow 2c^2 \sec(2x + ct) \sec(2x+ct) \tan(2$

$\frac{\partial w^2}{\partial t^2} = 2c^2 \sec^2(2x + ct) \tan(2x+ct$

differentiate w.r.t x

$\frac{\partial x}{\partial x} = 2\sec^2(2x+ct)$

$\frac{\partial^2 w}{\partial x^2} = 2(2\sec(2x+ct) \cdot \sec(2x+$

$(\tan(2x+ct)$

$= 8\sec^2(2x+ct)\tan(2x+ct$

$= 2c^2 \sec^2(2x+ct)\tan(2x+ct \neq$

$c^2\, 8\sec^2(2x+ct)\ldots\ldots$

$\tan(2x+ct)$ so it is Not possible.

Question No (2)

Given function is

$$f(x) \begin{cases} x; & -\pi < x \leq 0 \\ 2x; & 0 \leq x \leq \pi \end{cases}$$

We have to find the Fourier's Co-efficients a_0, a_n & b_n

Now:-

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{\cancel{2}}{\pi}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \pi/2} \rightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\Rightarrow \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right)-\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n+2(-1)^n-2}{n^2}\right] = \frac{(-1)^n-1}{\pi n^2}$$

So

$$a_n = \begin{cases} \frac{-2}{\pi n^2}; & \text{if } n \text{ is odd} \\ 0; & \text{if } n \text{ is even} \end{cases} \rightarrow (2)$$

$$b_x = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$\Rightarrow \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n} + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]\right] =$$

$$\frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So The required Fourier Serise is :.

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$\Rightarrow \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)x}{2n-1} + 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

Question = 3

$$y'' - 4y' + 13y = 8\sin 3x \rightarrow (1)$$

$$y(0) = 1 \quad \text{and} \quad y(0) = 2$$

Sol:. Associated Homogenious Eq of (1) is

$$y'' - 4y' + 13y = 0 \rightarrow (2)$$

Change (2) into Auxiliary equation:

put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

used Quadratic Formula

$a = 1 \quad b = -4 \quad c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= \frac{4 \pm \sqrt{36\, i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

2

$= 2 \pm 3i$

$m_1 = 2 \pm 3i$

$m_2 = 2 - 3i$

$y_c = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) \rightarrow A$

$y_p = A \cos 3x + B \sin 3x \longrightarrow x$

Diff: w.r.to x

$y_p' = -3A \sin 3x + 3B \cos 3x$

Agian Diff: w.r.t 'x'

$y_p'' = -9A \cos 3x - 9B \sin 3x$

Put in eq (1)

$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos^3 x) + 13(A\cos 3x + B\sin 3x) = 8\sin 3x$

$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

Comparing Co-efficents

$\sin 3x \Rightarrow \quad 4B + 12A = 8 \longrightarrow (a)$

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow 4A = 12B$

$\boxed{A = 3B} \longrightarrow (b)$

put (b) in (a)

$$4B + 12(3B) = 8$$

$$4B + 36B = 8$$

$$\frac{\cancel{40}^{5} B}{\cancel{8}} = \frac{\cancel{8}}{\cancel{8}}$$

$$\boxed{B = \frac{1}{5}} \rightarrow c$$

put (c) in (b)

$$\Rightarrow A = 3/5 \rightarrow (d)$$

put (c) & (d) in (x)

$$y_p = \frac{B}{5} \cos 3x + \frac{1}{5} \sin 3x \rightarrow B$$

The G. Sol is

$$y = y_c + y_p$$

$$y = e^{-2x}(C_1 \cos 3x + C_2 \sin 3x) + \frac{3}{5} \cos 3x$$

$$+ 1/5 \sin 3x \rightarrow (C)$$

$$\cancel{1 = e^{x(2)}(C_1 \cos 3(0) + C_2 \sin 3(0))}$$

Now we need to Find The Valuse of c_1 & c_2 For This put $x = 0$ & $y = 1$ (C)

$$1 = e^{x(0)}(C_1 \cos 3(0) + C_2 \sin 3(0)) + 3/5 \cos 3(0)$$

$$+ 1/5 \sin 3(0)$$

$1 = C_1(1) + C_2(0) + 3/5\,(1) + \frac{1}{5}(0)$

$1 = C_1 + 3/5$

$C_1 = 1 - \frac{3}{5}$

$\boxed{C_1 = 2/5} \longrightarrow xx$

Diff (C) w.r.t 'x'

$y' = C_1(2e^{2x}\cos 3(x) - 3e^{2x}\sin 3x + C_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x$

$- \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \rightarrow D$

put $y' = 2$ $x = 0$ in D

$y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x + C_2(2e^{2x}\sin 3x +$
$3e^{2x}\cos 3x - \frac{6}{5}\sin 3x + 3/5\cos 3x$

put $y = 2$ $x = 0$

$2 = C_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0) + C_2(2e^{2(0)}$
$\sin 3(0) + 3e^{2(0)}\cos 3(0)) - \frac{6}{5}\sin 3(0)$
$+ 3/5\cos 3(0)$

$2 = C_1(2) + C_2(3) - 0 + \frac{3}{5}$

$2 = 2C_1 + 3C_2 + 3/5$

put $C_1 = 2/5$

$2 = \frac{4}{5} + 3C_2 + 3/5$

$2 = \frac{7}{5} + 3C_2$

$$3c_2 = 2 - \frac{7}{5}$$

$$3c_2 = \frac{3}{5}$$

$$\boxed{c_2 = 3/15} \longrightarrow xxx$$

put (xx) & (xxx) in (c)

$$y = e^{3x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + 3/5\cos 3x + \frac{1}{5}\sin 3x}$$

→ Requried Genrall Solution.

Q:- No (4)

$$(D^2 - DD')z = \cos x \cos 2y$$

The given PDE can be rewrite as:-

$$D(D-D')u = \cos x \cos 2y$$

In CF is give by:-

$$CF = \phi_1(y) + \phi_2(y+x)$$

while its PI is give by:

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)} \cos(x+2y) + \frac{1}{(-1-2)} \cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Hance The complete Solution of given PDE is given by

$$u = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$