



## **Final Paper**

**Submitted By:  
Yahya Riaz (12280)  
BSSE Section A**

**Submitted To:  
Sir Shakeel**

**Dated:  
24<sup>th</sup> September 2020**

**Paper:  
Multivariate Calculus**

1. Evaluate

$$\int_0^5 \int_0^x x(x+3x) dy dx$$

Answer:

The image shows a handwritten solution on lined paper. It starts with the original double integral:  $\int_0^5 \int_0^x x(x+3x) dy dx$ . The next step is to integrate with respect to y, treating x as a constant, resulting in  $\int_0^5 \int_{y=0}^{y=x} (x^2 + 3x^2) dy dx$ . This is then simplified to  $\int_0^5 (x^2 + 3x^2) y \Big|_{0 \pm y}^{x-y} dx$ . The next line shows the integration of the constant term:  $\int_0^5 (x^2 + 3x^2) (x - 0) dx$ . This is further simplified to  $\int_0^5 (x^3 + 3x^3) dx$ . The next step is to find the antiderivative:  $\frac{x^4}{4} + \frac{3x^4}{4} \Big|_0^5$ . The final calculation is  $\frac{1}{4} ((5)^4 - (0)^4) + \frac{3}{4} (5^4 - (0)^4)$ , which simplifies to  $\frac{1}{4} (625) + \frac{3}{4} (625)$ . This is then calculated as  $= 156.25 + 468.75$ , resulting in  $= \boxed{625}$  Ans.

$$\int_0^5 \int_0^x x(x+3x) dy dx$$
$$\int_0^5 \int_{y=0}^{y=x} (x^2 + 3x^2) dy dx$$
$$\int_0^5 (x^2 + 3x^2) y \Big|_{0 \pm y}^{x-y} dx$$
$$\int_0^5 (x^2 + 3x^2) (x - 0) dx$$
$$\int_0^5 (x^3 + 3x^3) dx$$
$$\frac{x^4}{4} + \frac{3x^4}{4} \Big|_0^5$$
$$\frac{1}{4} ((5)^4 - (0)^4) + \frac{3}{4} (5^4 - (0)^4)$$
$$\frac{1}{4} (625) + \frac{3}{4} (625)$$
$$= 156.25 + 468.75$$
$$= \boxed{625} \text{ Ans.}$$

2. Evaluate

$$\int_1^4 \int_0^3 (xy + x^3y^3) dy dx$$

Answer:

The image shows a handwritten solution for the double integral problem. The steps are as follows:

$$\int_1^4 \int_0^3 (xy + x^3y^3) dy dx$$
$$\int_1^4 \left[ \int_0^3 xy dy + \int_0^3 x^3y^3 dy \right] dx$$
$$\int_1^4 \left[ \frac{xy^2}{2} \Big|_0^3 + \frac{x^3y^4}{4} \Big|_0^3 \right] dx$$
$$\int_1^4 \left[ x \left( \frac{3^2}{2} \right) - (0) + \frac{x^3(3^4)}{4} - (0) \right] dx$$
$$\int_1^4 \left[ \frac{9}{2}x + \frac{81x^3}{4} \right] dx$$
$$\frac{9}{2} \int_1^4 x dx + \frac{81}{4} \int_1^4 x^3 dx$$
$$\left( \frac{9}{2} \right) \frac{x^2}{2} \Big|_1^4 + \left( \frac{81}{4} \right) \frac{x^4}{4} \Big|_1^4$$
$$\left( \frac{9}{4} \right) (4)^2 - (1)^2 + \frac{81}{16} (4)^4 - (1)^4$$
$$\left( \frac{9}{4} \right) (16-1) + \frac{81}{16} (256-1)$$
$$\left( \frac{9}{4} \right) (15) + \frac{81}{16} (255)$$
$$= [1319.125] \text{ Ans.}$$

3. Find partial derivatives w.r.t r and s

$$f(r,s) = r \cdot \ln(r^3 + s^2)$$

Answer:

The image shows a handwritten solution on lined paper. It starts with the function  $f(r,s) = r \cdot \ln(r^3 + s^2)$ . The student then says "differentiate b.s w.r.t 'r'". The next line is  $\frac{d}{dr} f(r,s) = \frac{d}{dr} (r \cdot \ln(r^3 + s^2))$ . This is followed by  $= r \frac{d}{dr} \ln(r^3 + s^2)$ . Then,  $= \ln(r^3 + s^2) \frac{dr}{dr}$ . The next line is  $= \frac{r(3r^2)}{r^3 + s^2} + \ln(r^3 + s^2)$ . This is followed by  $= \frac{3r^3}{r^3 + s^2} + \ln(r^3 + s^2)$ . Finally, the answer is boxed as  $= \frac{r}{r^3 + s^2} (2s)$  with "Ans" written next to it.

$$\begin{aligned} f(r,s) &= r \cdot \ln(r^3 + s^2) \\ \text{differentiate b.s w.r.t "r"} \\ \frac{d}{dr} f(r,s) &= \frac{d}{dr} (r \cdot \ln(r^3 + s^2)) \\ &= r \frac{d}{dr} \ln(r^3 + s^2) \\ &= \ln(r^3 + s^2) \frac{dr}{dr} \\ &= \frac{r(3r^2)}{r^3 + s^2} + \ln(r^3 + s^2) \\ &= \frac{3r^3}{r^3 + s^2} + \ln(r^3 + s^2) \\ &= \frac{r}{r^3 + s^2} (2s) \text{ Ans} \end{aligned}$$

#### 4. Finding partial derivatives w.r.t "x"

$$F(x,y,z) = xy^2z^4 + 3yz^2$$

Answer:

The image shows a handwritten solution on lined paper. It starts with the function  $f(x, y, z) = xy^2z^4 + 3yz^2$ . Below this, it says "taking derivatives on b.s w.r.t 'x'". Then, it shows the derivative calculation:  $\frac{d}{dx} f(x, y, z) = \frac{d}{dx} (xy^2z^4 + 3yz^2)$ . This simplifies to  $= y^2z^4 + 0$ . Finally, the result  $y^2z^4$  is boxed and labeled "Ans.".

$$f(x, y, z) = xy^2z^4 + 3yz^2$$

taking derivatives on b.s w.r.t "x"

$$\frac{d}{dx} f(x, y, z) = \frac{d}{dx} (xy^2z^4 + 3yz^2)$$
$$= y^2z^4 + 0$$
$$= \boxed{y^2z^4} \text{ Ans.}$$

5. Find the value of x and y

$$8x - y = -1, \quad 7x - y = -2$$

**Answer:**

The image shows a handwritten solution on lined paper. It starts with two equations labeled 1 and 2. Equation 1 is  $8x - y = -1$  and equation 2 is  $7x - y = -2$ . The student then subtracts equation 2 from equation 1, as indicated by the text "∴ subtracting equation ① Eq ②". The subtraction is shown as follows:  $8x - y = -1$  minus  $7x - y = -2$ . The result is  $x = 1$ . Next, the student substitutes  $x = 1$  into equation 1, resulting in  $8(1) - y = -1$ , which simplifies to  $8 - y = -1$ . Solving for  $y$ , the student gets  $-y = -1 - 8$ , which simplifies to  $-y = -9$ , and finally  $y = 9$ . The final answers are boxed:  $x = 1$  and  $y = 9$ .

$$8x - y = -1 \quad \text{--- ①}$$
$$7x - y = -2 \quad \text{--- ②}$$

∴ subtracting equation ① Eq ②

$$8x - y = -1$$
$$\begin{array}{r} \oplus 7x \ominus y = \ominus 2 \\ - \\ \hline x = 1 \end{array}$$

So,  $x = 1$

∴ putting  $x = 1$  in equation ①

$$8(1) - y = -1$$
$$8 - y = -1$$
$$-y = -1 - 8 \Rightarrow -y = -9$$

So,  $y = 9$