Final Paper

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BSSE Section A

Submitted To:
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Paper:
Multivariate Calculus

1. Evaluate

$$\int_0^5 \int_0^x x\ (x+3x)dydx$$

Answer:

$$\int_0^5 \int_0^x x(x+3x)\,dy\,dx$$

$$\int_0^5 \int_{y=0}^{y=x} (x^2+3x^2)\,dy\,dx$$

$$\int_0^5 (x^2+3x^2)\,y \Big|_{0=y}^{x=y} dx$$

$$\int_0^5 (x^2+3x^2)(x-0)\,dx$$

$$\int_0^5 (x^3+3x^3)\,dx$$

$$\frac{x^4}{4}+\frac{3x^4}{4}\Big|_0^5$$

$$\frac{1}{4}\left((5)^4-(0)^4\right)+\frac{3}{4}\left((5)^4-(0)^5\right)$$

$$\frac{1}{4}(625)+\frac{3}{4}(625)$$

$$=156.25+468.75$$

$$=\boxed{625}\ \text{Ans.}$$

2. Evaluate

$$\int_1^4 \int_0^3 (\ xy + x^3y^3)dydx$$

Answer:

$$\int_1^4 \int_0^3 (xy + x^3y^3)\,dy\,dx$$

$$\int_1^4 \left[\int_0^3 xy\,dy + \int_0^3 x^3y^3\,dy\right] dx.$$

$$\int_1^4 \left[\frac{xy^2}{2}\Big|_0^3 + \frac{x^3y^4}{4}\Big|_0^3\right] dx$$

$$\int_1^4 \left[x\left(\frac{3^2}{2}\right) - (0) + x^3\frac{(3)^4}{4} - (0)\right] dx$$

$$\int_1^4 \left[\frac{9}{2}x + \frac{81x^3}{4}\right] dx$$

$$\frac{9}{2}\int_1^4 x\,dx + \frac{81}{4}\int_1^4 x^3\,dx$$

$$\left(\frac{9}{2}\right)\frac{x^2}{2}\Big|_1^4 + \left(\frac{81}{4}\right)\frac{x^4}{4}\Big|_1^4$$

$$\left(\frac{9}{4}\right)(4)^2-(1)^2 + \frac{81}{16}(4)^4-(1)^4$$

$$\left(\frac{9}{4}\right)(16-1) + \frac{81}{16}(256-1)$$

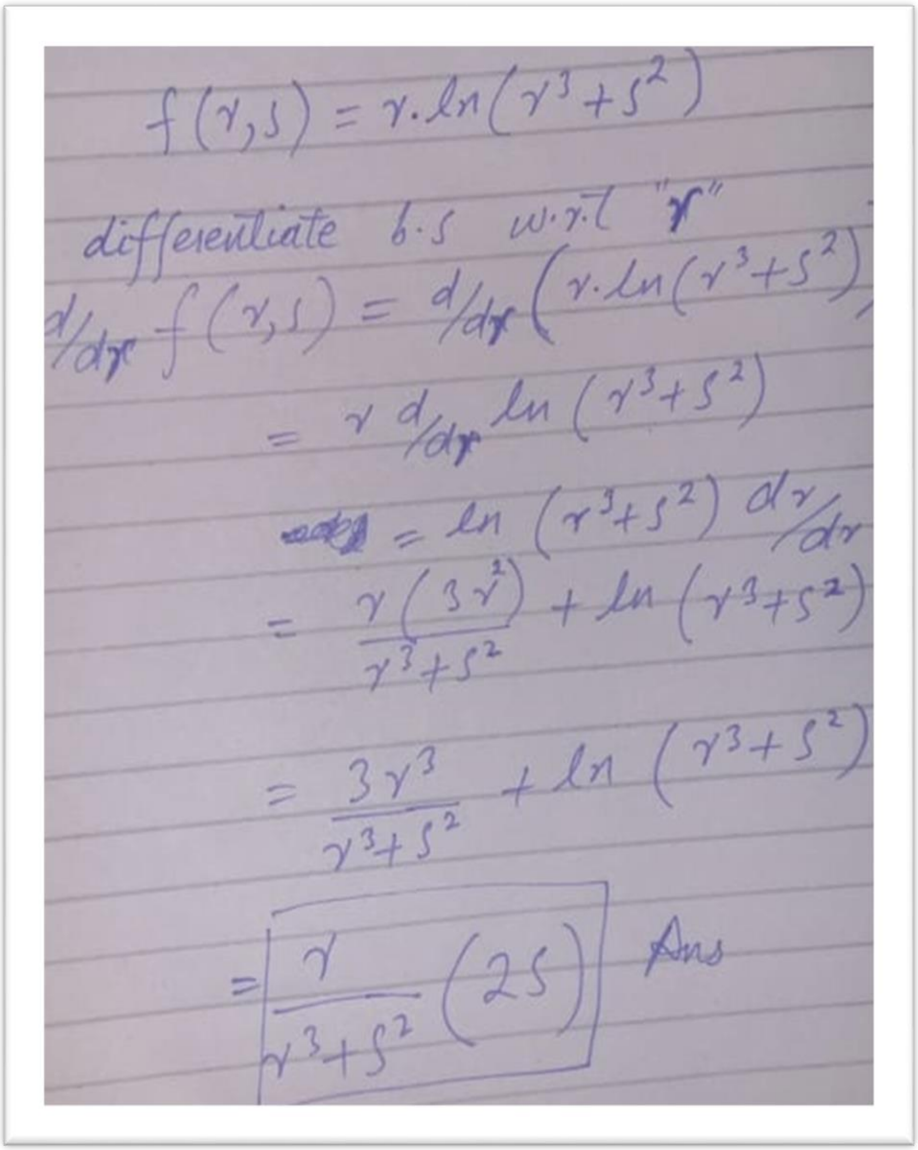
$$\left(\frac{9}{4}\right)(15) + \frac{81}{16}(255)$$

$$= \boxed{1319.125}$$ Ans.

3. Find partial derivatives w.r.t r and s

f (r,s) = r.ln(r^3 +s^2)

Answer:



$f(r,s) = r.\ln(r^3+s^2)$

differentiate b.s w.r.t "r"

$$\frac{d}{dr} f(r,s) = \frac{d}{dr}\left(r.\ln(r^3+s^2)\right)$$

$$= r \frac{d}{dr} \ln(r^3+s^2)$$

$$= \ln(r^3+s^2) \frac{dr}{dr}$$

$$= \frac{r(3r^2)}{r^3+s^2} + \ln(r^3+s^2)$$

$$= \frac{3r^3}{r^3+s^2} + \ln(r^3+s^2)$$

$$= \boxed{\frac{r}{r^3+s^2}(2s)}$$ Ans

4. Finding partial derivatives w.r.t "x"

   $F(x,y,z) = xy^2z^4 + 3yz^2$

Answer:

$f(x,y,z) = xy^2z^4 + 3yz^2$

taking derivatives on b.s w.r.t "x"

$\frac{d}{dx} f(x,y,z) = \frac{d}{dx}(xy^2z^4 + 3yz^2)$

$= y^2z^4 + 0$

$= \boxed{y^2z^4}$ Ans.

5. Find the value of x and y

8x – y = -1, 7x – y = -2

Answer:

$8x - y = -1$ ——— (1)

$7x - y = -2$ ——— (2)

∴ subtracting equation (1) & (2)

$$\begin{array}{rcl} 8x - \cancel{y} & = & -1 \\ \overset{}{\underset{-}{+}}7x \underset{+}{-}\cancel{y} & = & \underset{+}{-}2 \\ \hline x & = & 1 \end{array}$$

So, $\boxed{x = 1}$

∴ putting $x = 1$ in equation (1)

$8(1) - y = -1$

$8 - y = -1$

$-y = -1 - 8 \Rightarrow \cancel{-}y = \cancel{-}9$

So, $\boxed{y = 9}$