Name : Asad ullah

ID : 7697

Sec : A

Subject : intro to structural dynamics and earthquake engg

Submitted to :

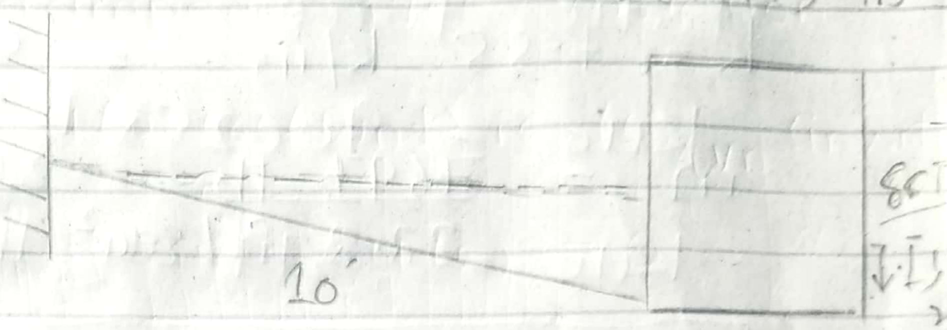
Engr Yaseen Mahmood.

Date : 29/06/020

Q No 1:-

Ans:-

ID(7697) lb



Sol:- The general E.O.M for SDOF System is

$$Ku + C\dot{u} + m\ddot{u} = p(t)$$

In our case system is undamped ($c=0$) undergoing the vibration ($p(t)=0$)

Hence general EOM become

$$Ku + m\ddot{u} = 0$$

$$K = 3EI/L^3$$

$$K = \frac{3 \times 29000\ K/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$\boxed{K = 7.55\ k/in}$$

In order to eliminate the chances of mistake during calculation it is more appropriate to use foundation unit like lb, ft, sec or Kg

(2)

$m = sec$

$k = 7.55 \ k/in = 90625 \ lb/ft$

$$m = \frac{7697 \ lb}{32.2 \ ft/sec^2}$$

$$\boxed{m = 239.0687 \ slug}$$

$$w_n = \sqrt{\frac{k}{m}}$$

$$w_n = \sqrt{\frac{90625}{239.0687}}$$

$w_n = 19.47$ rad/sec

$T_n = 2\pi / w_n = 2\pi / 19.47$

$$\boxed{T_n = 0.322 \ sec}$$

Substituting the corresponding value in eq- I

$$90625 u + 238.088 \ddot{u} = 0$$

where "k" is in lb/ft and 'm' is in $lb \ sec/ft^2$

(3)

General solution to the EOM for undamped free vibration is.

$$u(t) = u(0)\cos(\omega_n t) \text{ is } \dot{u}(0)/\omega_n \sin\omega_n t$$

$u(0) = 1/24$" ft and $\dot{u}(0) = 0$

$$u_t = ((1/24) \times \cos(19.46t) + 0)$$

$$u_t = \frac{1}{24} \times \cos(19.46t)$$

Equivalent static force at any time "t" is

$$f_s(t) = K \cdot u(t)$$

$$f_s(t) = \frac{90625 \times \cos(19.46t)}{24}$$

$$f_s(t) = 3776.04 \cos(19.46t)$$

Amplitude of dynomic displacement u_0 for undamped free vibration.

$$u_0 = \sqrt{(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$$

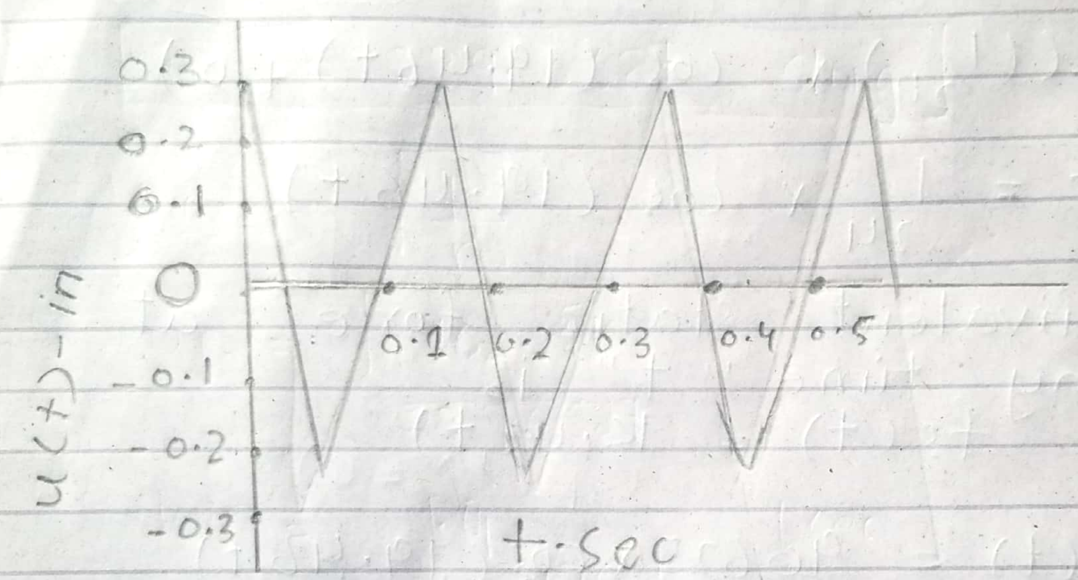
$$u_0 = \sqrt{(1/24)^2 + 0}$$

$$u_0 = 1/24 \text{ ft}$$

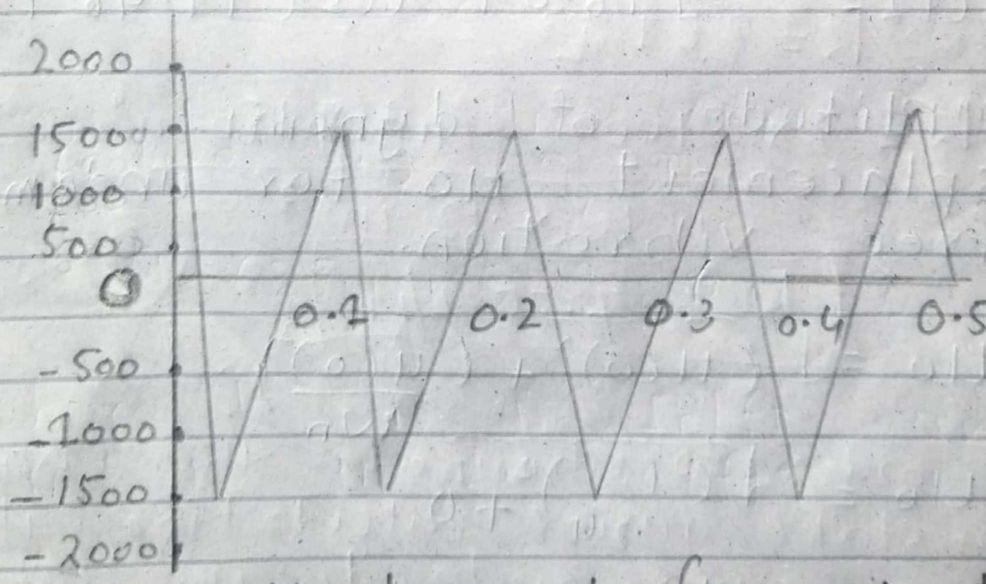
Amplitude of equivalent static force:

$$Ku_0 = 90625 \times \frac{1}{24}$$

$$Ku_0 = 377.04 \text{ lb}$$



Undamped force vibration

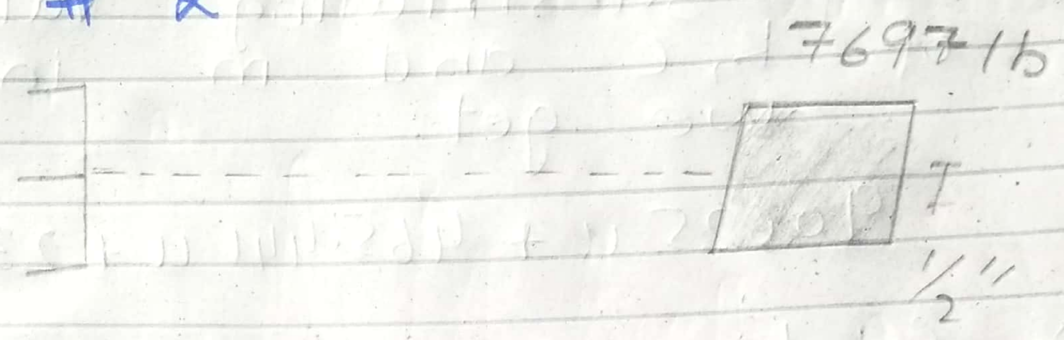


Undamped free vibration

(1)

Q NO # 2

Sol:-



* E.O.M for damped free vibration is;

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{------} (i)$$

* it is known from (ques-1)

$k = 90625$ lb/ft and $m = 239.068$ slug

$w_n = 19.48$ $m = 239.068$ sec^2/ft

$\Rightarrow C = \xi * 2mw_n = 2 * 239.068 * 19.48 * \xi$

($\xi = 0.03 - 0.05$ with considerable cracking The damping ratio)

$\Rightarrow C = 2 * 239.068 * 19.48 * 0.05$

$$\boxed{C = 465.441}$$

(2)

* by substituting values of k, c and m in eq(i) we get.

$$90625\, u + 465.441\, u + 239.068\, u = 0$$

* Solution to the E.O.M for damped free vibration is;

$$u(t) = e^{-\xi w_n t}\left(u(0)\cos(w_D t) + \frac{1}{w_D}(u(0))t\right.$$
$$\left. u(0)\,\xi\, w_n \cdot \sin(w_D T)\right)$$

$$w_D = 19.469 \text{ rad/sec}$$

$$u(t)\, e^{-0.05 * 19.46}\left(\frac{1}{24} * \cos(19.469) + \frac{1}{19.469}\right.$$
$$\left. * \left(0 + \frac{1}{24} * 0.05 * 19.469 * \sin(19.469t)\right)\right.$$

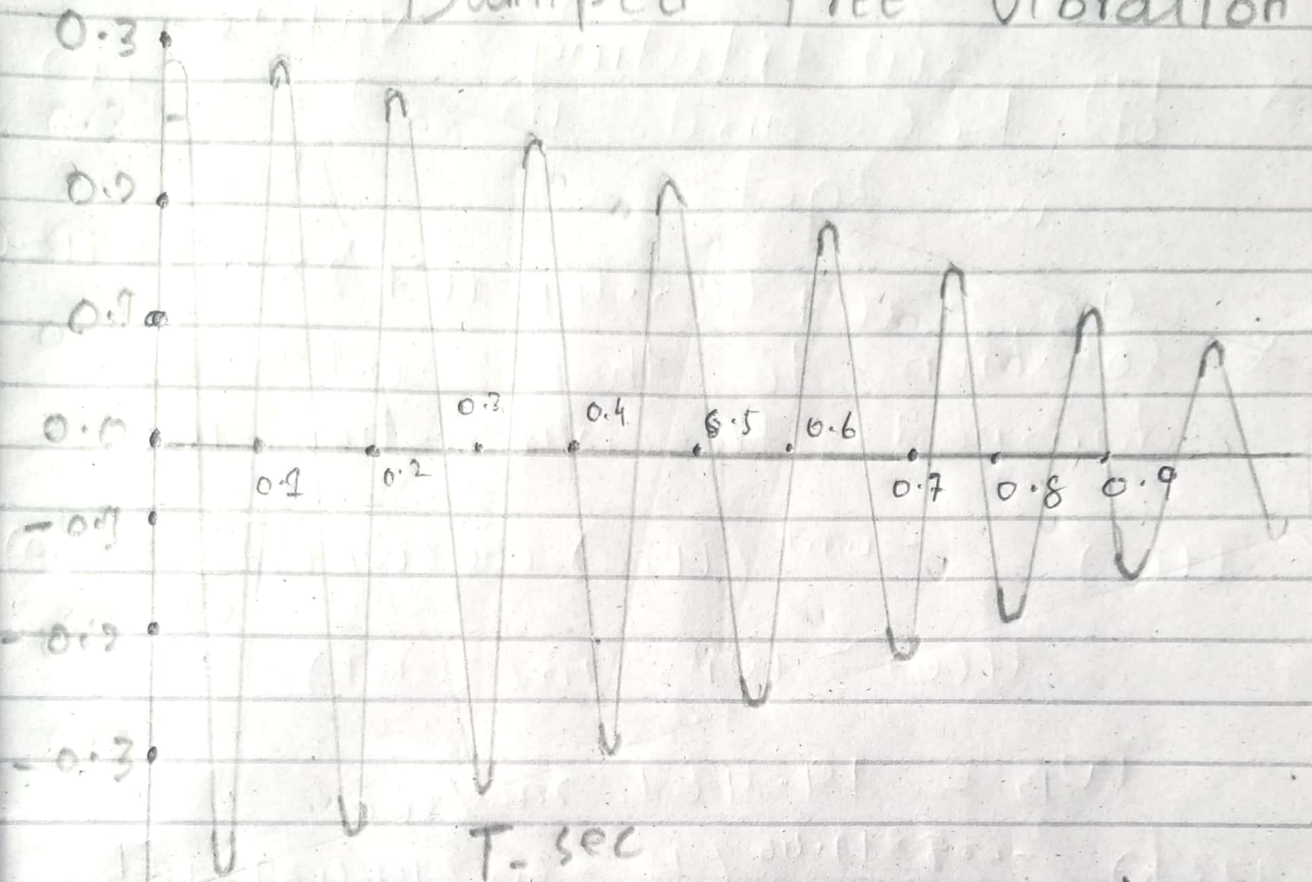
$$u(t) = e^{-0.973t}\left(0.0417 * \cos(19.469t)\right.$$
$$\left. + 0.051 \times 0.0417 \times \sin(19.469)\right.$$
$$t$$

$$u(t) = e^{-6.973t}\left(0.0417 \times \cos(19.469t)\right. *$$
$$\left. 0.0021 \times \sin(19.469)\right)$$

$$f_s(t) = K \cdot u(t) = 90625 \times u(t)$$

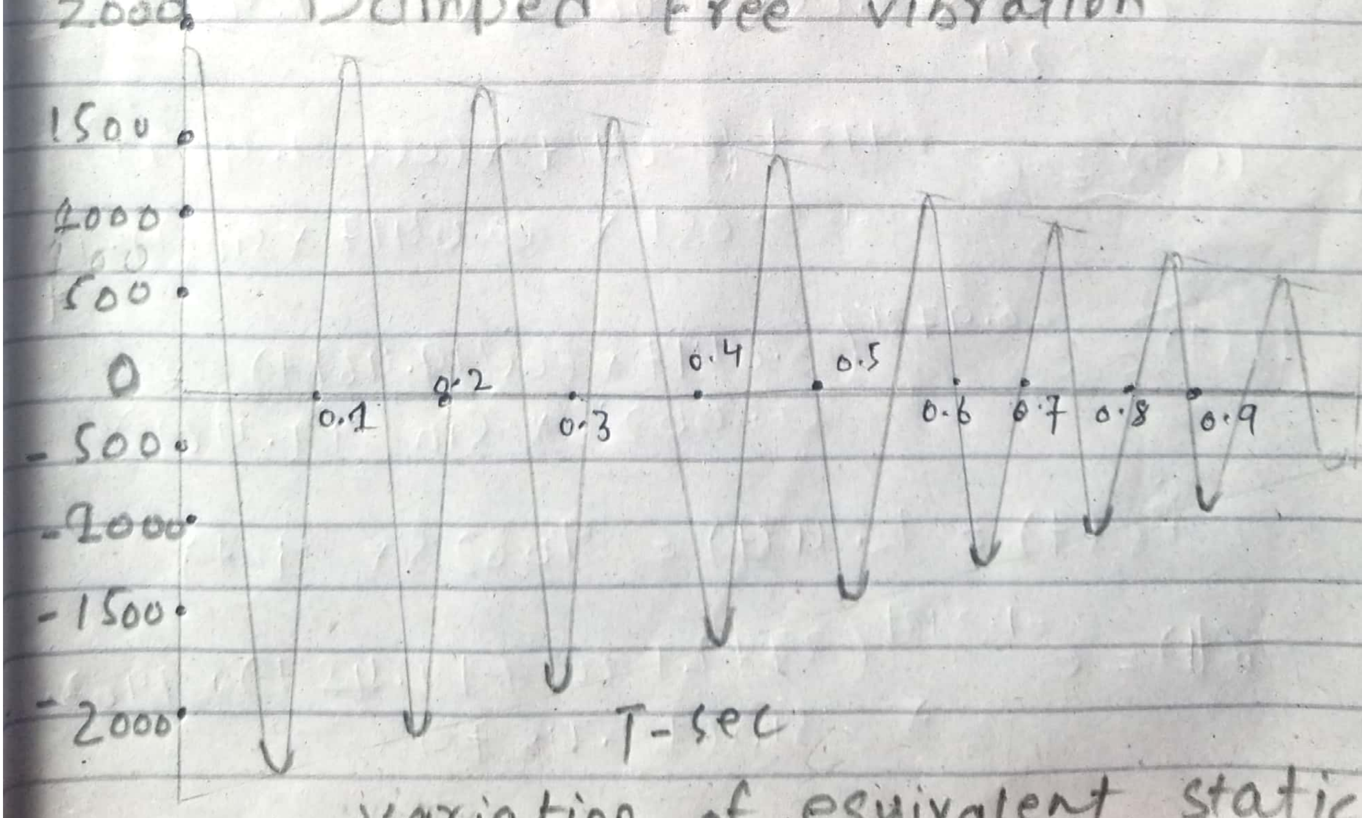
$$f_s(t) = e^{-1.373t}\left(3779.1 \cos(19.48t) + 1903.1\right.$$
$$\left. * \sin(19.48t)\right)$$

Damped free vibration



variation of displacment with Time

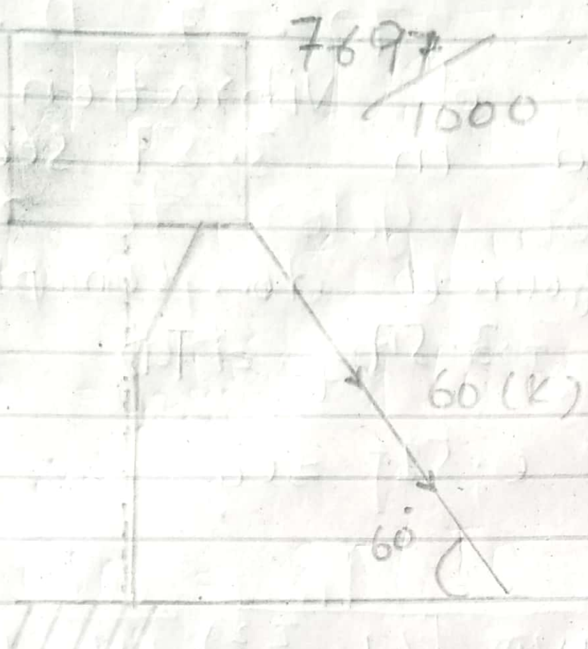
Damped Free vibration



variation of equivalent static force with Time

(1)

Question # No 3



Solution:-

(7.692" or 7.7")

$$u_1 = \frac{7697}{1000} = 7.7''$$

After $j = 7$, $u_{j+1} = u_b = 2.286$ cm

$= 0.9''$

(a) ξ = Damping ratio = ?

$$j = \frac{1}{2\pi\xi} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$7 = \frac{1}{2\pi\xi} \ln(7.7/0.9)$$

$$\xi = 0.049 = \boxed{4.9\ \%}$$

Scanned with CamScanner

b) $T_n = ?$

7 cycles of vibration are completed in 3.57 sec

Time required to complete one cycle $= 3.57/7 = T_D$

$T_D = 0.51$ sec

Now

$\omega_D = \omega_n \sqrt{(1-\xi^2)}$

$2\pi/\omega_D = 2\pi/(\omega_n \sqrt{1-\xi^2})$

$\Rightarrow T_D = \dfrac{T_n}{\sqrt{1-\xi^2}}$

$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$

$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.049)^2}$

$\boxed{\Rightarrow T_n = 0.5094 = 0.51 \text{ sec}}$

c) $K = ?$

$K = \dfrac{60 \times \cos 60^\circ}{7.7} = 3.9 \text{ k/in}$

$\boxed{K = 46800 \text{ lb/ft}}$

d) weight of the tank, $w = ?$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{(w/g)}} = \sqrt{\frac{k * g}{w}}$$

$$\Rightarrow \omega_n^2 = k * g / w$$

$$\Rightarrow w = k * g / \omega_n^2$$

Also

$$\omega_n = 2\pi / T_n$$

$$W = kg\left(\left(\frac{4\pi^2}{T_n^2}\right)\right) = kg * \frac{T_n^2}{4\pi^2}$$

$$W = \frac{46800 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$$W = 9928.5 \text{ lb}$$

$$\boxed{W = 9.93 \text{ k}}$$

e) $c = ?$

it is know that $\zeta = \frac{c}{2m\omega_n}$

$$\Rightarrow c = \zeta \times 2m\omega_n = \zeta \times 2m \times (2\pi / T_n)$$

$$c = 0.049 \times 2 \times 2\left(\frac{\pi}{0.51}\right)\left(\frac{9928.5}{32.2}\right)$$

$$\Rightarrow \quad C = 372.27 \ \text{lb.sec/ft}$$

f) No of cycles to reduce displacement amplitude from 7.7 in to

0.5" , $J = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow j = \frac{1}{2 * \pi(0.049)} \ln\left(\frac{7.7}{0.5}\right)$$

$$\boxed{J = 86.9 \text{ or } 9 \text{ cycles}}$$