M. Saqib

I.D 11381

Engr Anwar SHAH MIM

Math II

B. Tech Civil

29/09/2020

Q2/A

The Sum of two number is k. find the minimum value of the Sum of their cubes.

Solution :-

Let

x & y = The number

z Sum of Their Cubes

$k = x + y$

$y = k - x$

$z = x^3 + y^3$

$z = x^3 + (k-x)^3$

$\frac{dz}{dx} = 3x^2 + 3(k-x)^2(-1) = 0$

#08

$$x^2 - (k^2 - 2kx + x^2) = 0$$

$$x = \frac{1}{2} k$$

$$y = k - 1/2\, k$$

$$y = \frac{1}{2} k$$

$$z = \left(\frac{1}{2} k\right)^3 + \left(\frac{1}{2} k\right)^3$$

$$z = \frac{1}{4} k^3$$

Ans

#03

Q 1/B The Sum of two positive number is 2. find the smallest value possible for the sum of the cubes of one number & square of the other.

Sol.:

Let x and y = The number

$x + y = 2 \rightarrow$ Equation(1)

$1 + y = 0 \quad y = -1$

$z = x^3 + y^2 \rightarrow$ Equation (1)

$z = x^3 + y^2 \rightarrow$ Equation (2)

$\frac{dz}{dx} = 3x^2 + 2yy' = 0$

$3x^2 + 2y(-1) = 0$

#04

$y = \frac{3}{2} x^2$

from Equation ①

$x + \frac{3}{2} x^2 = 2$

$2x \pm 3x^2 = 4$

$3x^2 + 2x - 4 = 0$

$x = 0.86855 \;\&\; -1.5352$

use

$x = 0.86855$

$y = 32(0.86852)$

$y = 1.1315$

$z = 0.86853 + 1.13152$

$\boxed{z = 1.9354}$

Ans

#05

Q2/A Let $f(x)$ be a differentiable function such that $f(3) = 12$, $f'(3) = -2$ Estimate the value of (3.5) using the local approximation at $a = 3$.

Sol :- The linear approximation is given by the equation

$f(x) = L(x)$

$= f(a) + f'(a)(x-a)$

we just need to plug in the new value calculate the value of $f(3.5)$

#06

$L(x) = f(3) + f'(3)$
$(x-3)$
$= 12 - 2(x-3) \Rightarrow 18 - 2x$
$f(3.5) \neq 18 - 2 \cdot 3.5 = 11$ Ans

Q2/B Estimate $\sqrt[3]{9}$ using a linear approximate at
$a = 8$

Sol :- $f(x) \simeq L(x)$
$= f(8) + f(8)(x-8)$
find the derivative
$f'(x) = (\sqrt[3]{x})' = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

Compute the value of the derivative at $a = 8$

#07

$$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

Substituting This, we get function L(x) in the form.

$$f(x) \approx L(x) = 2 + \frac{1}{12}(x-8)$$

$$= \frac{x}{12} + \frac{4}{13}$$

Hence

$$\sqrt[3]{9} \approx L(9) = \frac{9}{12} + \frac{4}{3} = \frac{9+16}{12} = \frac{25}{12}$$

#08

Q3/ Solve the following differential Equation:

$$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$$

Sol: $(2y + x^2 + 1)\,dy/dx + (2xy - 9x^2) = 0$

we can write this as

$$(2y + x^2 + 1)\,dy + (2xy - 9x^2)\,dx = 0$$

Check for exactness:

let $M = 2xy - 9x^2$

let $N = 2y + x^2 + 1$

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$$d = (2xy - 9x^2)/dy = d(2y + x^2 + 1)/dx$$

$$2x = 2x$$

Thus it is indeed exact to further solve

#09

To get g(y), we differentiate partially with respect to y.

$$\frac{df}{dx} = M = 2xy - 9x^2$$

$$\int df = \int (2xy - 9x^2)\, dx$$

$$f = (x^2)y - 3x^3 + g(y)$$

To get g(y), we differentiate it partially with respect to y:

$$\frac{df}{dy} = \frac{d}{dy}\left((x^2 y - 3x^3 + g(y) \right) = N$$

$$\frac{df}{dy} = x^2 + g'(y) = 2y + x^2 + 1$$

$$g'(y) = 2y + 1$$

#10

Integrating

$g(y) = y^2 e + y + C$

Therefore

$P = (x^2)y - 3x^3 + y^2 + y + C$

Ans