NAME: OWAIS USMAN

ID: 7897

SECTION: A

SUBJECT: DIFFENTIAL

TEACHER: MAM SHUMAILA

ASSIGNMENT# 2

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Question #1 (1)

$$x^3y''' + 2x^2y' + 2y = 10x + \frac{10}{x}$$

Sol:

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3D^3y + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3D^3 + 2x^2D + 2)y = 10x + 10x^{-1} \quad \text{—} \quad (1)$$

let $x = e^t \Rightarrow t = \ln x$

$$xD = D$$

$$x^2D^2 = D(D-1) = D^2 - D$$

$$x^3D^3 = D(D-1)(D-2)$$

Substituting into eq (i)

$$(D - 3D^2 + 2D + 2(D^2 - D) + 2)y = 10^4 + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{et}$$

using synthetic division

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	$\boxed{0}$

$$D^2 - 2D + 2 = 0$$

Now using Quadratic formula

$a = 1, \quad b = -2, \quad c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

$$D = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{+4} \times \sqrt{-1}}{2}$$

$$D = \frac{2 + 2i}{2}$$

$$D = \cancel{2}\left(\frac{1 \pm i}{\cancel{2}}\right)$$

$$D = 1 \pm i$$

Since root are complex

$$y_c = e^{-x}\left(c_1 \cos t + c_2 \sin t\right).$$

Now Particular integration

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^t$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{\cancel{10}^{5} e^t}{2} + \frac{\cancel{10}^{5} e^{-t}}{\cancel{2}}$$

$$= 5e^t + 5e^{-t}$$

$$y_p = 5e^t + 5e^{-t}$$

General solution:

$$y = y_c + y_p$$

$$y = e^u (c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}$$

Put $e^t = u$ and $t = \ln u$

$$\boxed{y = e^{-u} (c_1 \ln u + c_2 \sin \ln u) + 5e^u + 5e^{-u}}$$

Ans

Question #2

$$x^3 \frac{d^3y}{du} + 4u^2 \frac{d^3y}{du} - \frac{5u\,dy}{du} - 15y = u^4$$

Sol:

Let $\frac{d}{du} = D$

$$u^3 D^3 y + 4u^2 D^2 y - 5u Dy - 15y = u^4$$

$(u^3D^3 + 4u^2D^2 - 5uD - 15)y = u^4$

Let

$u = e^t \Rightarrow t = \ln u$

$uD = D$

$x^2D^2 = D(D-1) = D^2 - D$

$u^3D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$

Now subtituting

$(x^3D^3 + 4u^2D^2 - 5xD - 15)y = u^4$

$(D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$

$(D^3 + D^2 - 7D - 15)y = e^{4t}$

Synthetic Division

5	1	-1	-7	-15
		5	12	15
	1	4	5	0

$D^2 + 4D + 5 = 0$

Quadratic Equation

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$D = \frac{2(-2 \pm i)}{2}$$

$$y_c = e^{sn}(c_1 \cos t + c_2 \sin t)$$

For $y_p = ?$

$$y_p = \frac{1}{D^3 + D^2 - 7D + 15} e^{4t}$$

$$= \frac{1}{(4)^3 + (4^2) - 7(4) - 15} e^{4t}$$

$$= \frac{1}{64 + 16 - 28 - 15} e^{4t}$$

$$= \frac{1}{80 - 43} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}$$

Hence

$$y = y_c + y_p$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

again put $t = \ln u$ and $u = \ln u$

$$\boxed{y = e^{3u}(c_1 \cos \ln u + c_2 \sin \log u) + \frac{1}{37} e^{4u} \text{ Ans}}$$

Question # 3 ⑥

$u^2 y'' + 2uy' - 6y = 10u^2$

Sol:

$y(1) = 1$ and $y'(1) = -6$

$$u^2 \frac{d^2y}{du^2} + 2u \frac{dy}{du} - 6y = 10u^2$$

$$\Rightarrow \left(u^2 \frac{d^2}{du^2} + 2u \frac{d}{du} - 6\right) y = 10u^2$$

Put $uD = D \Rightarrow u^2D^2 = \Delta(D-1)\ D^2 - D$

$u = e^t$ and $\log u = t$

$(D^2 - D + 2D - 6)y = 10e^{2t}$

$(\Delta^2 + \Delta - 6)y = 10e^{2t}$

The characteristic equation

$D^2 - D - 6 = 0$

$D^2 + 3D - 2D - 6 = 0$

$\Rightarrow D(D+3) - 2(D+3) = 0$

$\Rightarrow (D+3)(D-2) = 0$

$D + 3 = 0 \quad , \quad D - 2 = 0$

$D = -3 \quad , \quad D = 2$

Since roots are real and distinct

For $y_c = ?$

$$y_c = c_1 e^{-3t} + c_2 e^{2t}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - D - 6} 10 e^{2t}$$

$$= \frac{10}{D^2 - D - 6} e^{2t}$$

$$= 10 \frac{1}{0} e^{2t} \quad \text{fails}$$

Now

$$10 \frac{1}{d/dD (D^2 + D - 6)} e^{2t}$$

$$\Rightarrow 10 \frac{t}{2D + 1} e^{2t}$$

$$= 10 \frac{t}{4 + 1} e^{2t}$$

$$y_p = 2 t e^{2t}$$

General solution

$$y = y_c + y_p$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2 t e^{2t}$$

$$y = c_1 u^{-3} + c_2 u^2 + 2 (\log u) u^2 \quad \text{—(B)}$$

Put $y(1) = 1$ i.e. $u = 1, y = 1$ in eq (B)

$$1 = c_1 (1)^{-3} + c_2 (1)^2 + 2 \log (1)$$

$$1 = c_1 + c_2 \longrightarrow (C)$$

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Now differentiate eq (B) w.r.t x

$$y_1 = -3c_1 x^{-4} + 2(2x + \frac{2}{x}(x^2)) + 4x\log x$$

Now put $y'(1) = -6$ i.e $y' = -6$ and $x = -6$

$$-6 = -3c_1 + 2c_2 + 2 + 0$$

$$\Rightarrow -6 = -3c_1 + 2c_2 + 2$$

$$\Rightarrow -6 - 2 = -3c_1 + 2c_2 + 2$$

$$-8 = -3c_1 + 2c_2 \rightarrow D$$

Xing eq(i) with (2) and ÷ing from D

$$2c_1 + 2c_2 = 2$$
$$3c_1 \pm \cancel{2c_2} = -8$$

$$5c_1 = 10$$

$$c_1 = \frac{10}{5} \quad \boxed{c_1 = 2}$$

$$-8 = -3(2) + 2c_2$$

$$-8 = -6 - 2c_2$$

$$2c_2 = -2$$

$$c_2 = \frac{-2}{2}$$

$$\boxed{c_2 = -1}$$

Now put the value of c_1 and c_2 in eq B

$$\boxed{y = \frac{2}{x^3} - x^2 + 2x^2 \log x} \text{ Ans}$$

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Question # 4 (a)

$x^2y'' + 7xy' + 5y = x^5$

$y(0) = 2$ and $(y')(1) = 2$

Sol:-

$$\frac{x^2 dy^2}{dx^2} + \frac{7x\,dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 7x\frac{d}{dx}\right) + 5y = x^5 \quad (A)$$

Put $xD = \Delta \Rightarrow x^2 D^2 = D(D-1) = D^2 - D$

$x = e^t \Rightarrow \log x = t$ in eq (A)

$$\Rightarrow (D^2 - D + 7D + 5)y = e^{5t}$$

$$\Rightarrow (D^2 + 6D + 5)y = e^{5t}$$

By Quadratic formula

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 + \sqrt{36 - 20}}{2}$$

$$= \frac{-6 \pm 2}{2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$$D = -3 + 2$$

$y_c = c_1 e^{-5t} + c_2 e^{-t}$

For $y_p = ?$

$y_p = \frac{1}{D^2+6D+5} e^{5t}$

$= \frac{1}{(5)^2+6(5)+5} e^{5t}$

$= \frac{1}{60} e^{5t}$

Now General solution is

$y = y_c + y_p$

$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{1}{60} e^{5t}$

$y = c_1 u^{-5} + c_2 u^{-1} + \frac{1}{60} u^{5} \rightarrow$ B

$u=0$ Put in this ~~equilibrio~~ equation   No eq (B $e^0 = 1$

Put $y(0) = 2$ 1-eq = 2 & $u = 2$

$2 = c_1(2)^{-5} + c_2(2)^{-1} + \frac{1}{60}(2)^5$

$2 = -32c_1 - 2c_2 + \frac{1}{\cancel{60}\,15}(\cancel{32})\,8$

$2 = -32c_1 - 2c_2 + \frac{8}{15}$

$2 - \frac{8}{15} = -32c_1 - 2c_2$

$\frac{22}{15} = -32c_1 - 2c_2 \rightarrow$ C

Now differentiate eq (B) w.r.t (u)

$$y' = -5a\,u^{-6} - C_2 u^{-2} + \frac{1}{12} u^4 \rightarrow$$

Put $y'(1) = 2$ i.e $y' = 2$ and $u = 2$ in above equation

$$2 = -5C_1 C_2^{-} - C_2(2)^{-2} + \frac{1}{12}(2)^4$$

$$2 = -5C_1(-64) - C_2(4) + \frac{1}{12}(16)$$

$$2 = 320C_1 + 4C_2 + 4/3$$

$$\Rightarrow 2 - \frac{4}{3} = 320C_1 + 4C_2$$

$$\Rightarrow 2/3 = 320C_1 + 4C_2 \rightarrow (D)$$

Xing eq (C) with 2 and then sing (D) from (B)

$$\frac{-44}{15} = 64C_1 + 4C_2$$

$$\frac{-44}{15} = 64C_1 + 4C_1$$

$$\frac{+2}{3} = +320C_1 + 4C_2$$

$$\frac{34}{15} = -256C_1$$

$$C_1 = \frac{34}{15} \times 256$$

$$\boxed{C_1 = 580}$$

Put the value of C_1 in eq (C)

$$\frac{22}{15} - 32(580) - 2C_2$$

$$\Rightarrow \frac{22}{15} = -18560 - 2C_2$$

$\Rightarrow \frac{22}{15} + 18560 = -2c_2$

$\frac{18561}{-2} = c_2$

$\boxed{c_2 = -9280}$

Now put the value of c_1 and c_2 in eq (B)

$y = 580u^{-5} - 9280u^{-1} + \frac{1}{60}u^5$

$\boxed{y = \frac{580}{u^5} - \frac{9280}{u} + \frac{1}{60}u^5}$ Ans

$(u+1)^2 y'' - 3(u+1)y' + 4y = u^2$ Q#5

Sol:

$(u+1)^2 \frac{d^2y}{du^2} - 3(u+1)\frac{dy}{du} + 4y = u^2$

$\Rightarrow \left((u+1)^2 \frac{d^2}{du^2} - 3(u+1)\frac{d}{du} + 4\right)y = u^2 \rightarrow (A)$

Put $(u+1)D = D \Rightarrow (u+1)^2 D^2 = D(D-1) = D^2 - 1$

$u = e^t$ in eq (A)

$\Rightarrow (D^2 - D - 3D + 4)y = e^{2t}$

$\Rightarrow (D^2 - 4D + 4)y = e^{2t}$

$\Rightarrow (D^2 - 4D + 4)z = e^{2t}$

for y_c we find the roots

$D^2 - 4D + 4 = 0$

$D^2 - 2D - 2D + 4 = 0$

$D(D-2) - 2(D-2) = 0$

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$D-2=0$, $D=2$

$D-2=0$, $D=2$

so the root are real and reaped

The general solution are

$y=(c_1+c_2 u)^{mu}$

$y=(c_1+c_2 u)^{2u}$

for $yp = ?$

$yp = \frac{1}{D^2-4D+4}$ | $(2)^2-4(2)+4 \Rightarrow 0$

$yp = \frac{2}{2D-4} e^{2t}$

if we put 2

$2D-4 \Rightarrow 2(2)-4=0$

we take again derivative

$yp = \frac{\cancel{2}}{\cancel{2}} \cdot e^{2t}$

$$\boxed{y=(c_1+c_2 u)^{2t}+e^{2t}}$$ Ans

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