



Iqra National University, Peshawar
Department of Electrical Engineering



Assignment
Date: 20/4/2020

Course Code: MTH 102 Course Title: Calculus and analytic geometry
Prerequisite: _____ Instructor: HIMAYATULLAH
Module: 3 Program: BEE Total Marks: 30

Q1.	(a)	. Identify $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$	Marks 5
			CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$	Marks 5
			CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a height of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10
			CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where ?	Marks 10
			CLO1 C1

NAME: MUHAMMAD IDREES KHAN

ID: 16431

SEMESTER: 2ND

DEPTT. BEE

INSTRUCTOR: SIR HIMAYAT ULLAH

Q NO. 1(a)

Solution:-

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \quad (0/0)$$

So

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{2+h - 2}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

= ~~limit~~ putting limit
 $\lim_{h \rightarrow 0}$

$$= \frac{1}{\sqrt{2+0} + \sqrt{2}} = \frac{1}{\sqrt{2} + \sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

QNO. 1 (b)

Solution:-

$$y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$$

$$y = (x + x^{-1}) (x - x^{-1} + 1)$$

Taking derivation

$$\frac{dy}{dx} = (x + x^{-1}) \frac{d}{dx} (x - x^{-1} + 1) + (x - x^{-1} + 1) \frac{d}{dx} (x + x^{-1})$$

$$\frac{dy}{dx} = (x + x^{-1}) (1 + x^{-2}) + (x - x^{-1} + 1) (1 - x^{-2})$$

$$y' = \left(x + \frac{1}{x}\right) \left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x} + 1\right) \left(1 - \frac{1}{x^2}\right)$$

$$y' = \left(\frac{x^2 + 1}{x}\right) \left(\frac{x^2 + 1}{x^2}\right) + \left(\frac{x^2 - 1 + x}{x}\right) \left(\frac{x^2 - 1}{x^2}\right)$$

$$y' = \frac{(x^2 + 1)(x^2 + 1)}{x^3} + \frac{(x^2 - 1 + x)(x^2 - 1)}{x^3}$$

$$y' = \frac{x^4 + x^2 + x^2 + 1}{x^3} + \frac{x^4 - x^2 - x^2 + 1 + x^3 - x}{x^3}$$

$$y' = \frac{x^4 + 2x^2 + 1 + x^4 - 2x^2 + 1 + x^3 - x}{x^3}$$

$$y' = \frac{2x^4 + x^3 - x + 2}{x^3}$$

Ans :-

QNO. 2 (a)

Solution:

$$\text{Height} = s = 160t - 16t^2$$

$$\text{Velocity} = v = 160 \text{ m s}^{-1}$$

(a) How high does the sack go.

(b) $v = ?$, when height is 256 ft

(c) $a = ?$, when $t = 5 \text{ sec}$

Let we take $t = 1 \text{ sec}$

$$v = \frac{s}{t}$$

$$160 = \frac{160t - 16t^2}{t}$$

$$160 = \frac{160(1) - 16(1)^2}{1}$$

$$160 = \frac{160 - 16}{1}$$

$$160 = \frac{144}{1}$$

$$t = \frac{144}{160}$$

$$t = 0.9 \text{ sec}$$

$$(a) \quad v = \frac{s}{t}$$

$$160 = \frac{s}{0.9}$$

$$160 \times 0.9 = s$$

$$s = 144 \text{ ft}$$

(b) for finding velocity $v = ?$

$$v = \frac{s}{t}$$

$$s = 256 \text{ ft}$$

$$v = \frac{256}{0.9}$$

$$v = 284.4 \text{ ms}^{-1}$$

(c) for finding acceleration $a = ?$

$$t = 5 \text{ sec}$$

$$a = \frac{v}{t}$$

$$a = \frac{160}{5}$$

$$a = 32 \text{ ms}^{-2}$$

Ans.

QNO. 3

Solution:-

$$y = x^4 - 2x^2 + 2$$

Taking derivative

$$\frac{dy}{dx} = 4x^3 - 4x$$

$$\text{Let put } \frac{dy}{dx} = 0$$

$$0 = 4x^3 - 4x$$

$$4x = 4x^3$$

$$1 = x^2$$

$$x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$\boxed{x = \pm 1}$$

This is point of Tangent on curve.

$$\tan \theta = m$$

$$\boxed{\text{Slope} = m = \frac{dy}{dx}}$$

$$\theta = \tan^{-1}(m)$$

$$\theta = \tan^{-1}(0)$$

$$\boxed{\theta = 0}$$

(So it has horizontal tangent.)

