Course Title :- Electrical Network Analysis

Module :- 4th

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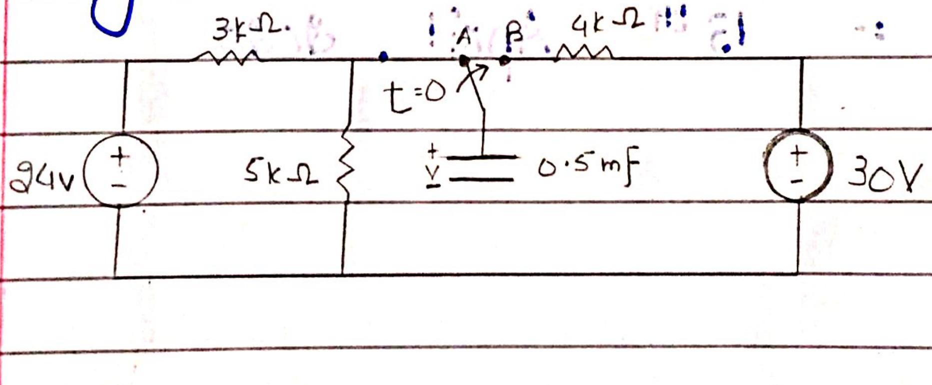
Question No :- 1 :

The switch in figure has been in position A for a long time. At t = 0 switch moves to B. Determine v(t) for t > 0 & calculate its value at t = 1s and 8s.

figure :-



Solution :-

for t < 0 :

The switch is at position A. The capacitor acts like an open Circuit to dc, but v is the same as the voltage across 5 Ω resistor. Hence the across the capacitor just before t = 0 is obtained by voltage division as,

$$v(0) = \frac{5}{5+3}(24) = 15\,V$$

As the capacitor cannot change instantaneously.

$$v(0) = v(0) = 15\,V.$$

For $t > 0$:

The switch is in position B. The $R_{in} = 4k\,\Omega$.

Time constant is,

$\tau = R_{in} C = 4 \times 10^{3} \times 0.5 \times 10^{-3}$

$\tau = 2s$

Since the capacitor acts like an open circuit to dc at steady state,

$V(\infty) = 30V$

Thus

$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/2}$

$= 30 + (15 - 30) e^{-t/2}$.

$= (30 - 15 e^{-0.5t})$ V.

At $t = 2$:

$V(2) = 30 - 15 e^{-2/2}$

$= 30 - 15 e^{-1}$

$V(2) = 24.48$ V

At $t = 8$

$V(8) = 30 - 15 e^{-8/2}$

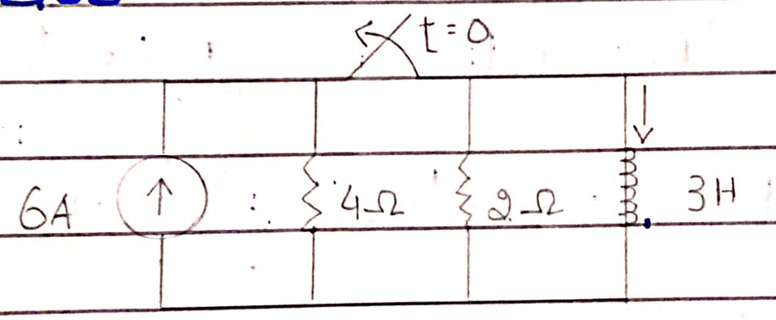
$= 30.15 e^{-4}$

$V(8) = 29.72$

Question No :- 2

Determine the inductor current for both $t > 0$ and $t < 0$ for the circuit.

Figure :-



Solution :-

For $t < 0$:

The switch is closed and inductor acts as short circuit

Therefore inductor current

$i = 6A$

For $t > 0$:

The switch is opened and time constant

$\tau = L/R$

$\tau = 3/2$

Now the inductor current $i(t) = e^{-t/\tau}$

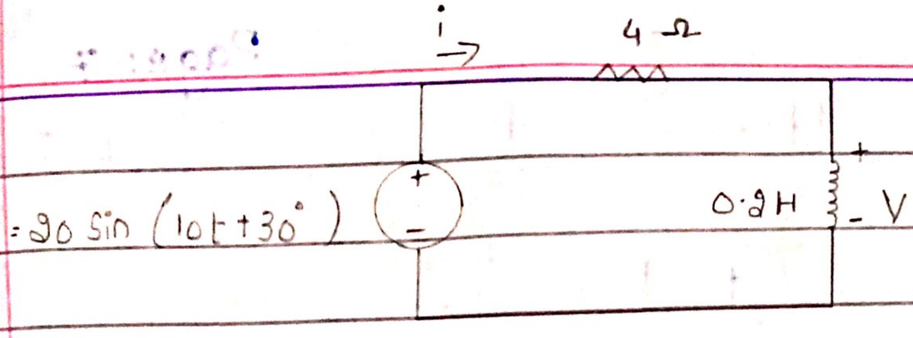
$i(t) = 6e^{-t/3/2}$

$i(t) = 6e^{-2t/3} u(t) A$.

Question No : 5

Find $v(t)$ and $i(t)$ in the circuit in figure.

Figure :-



Solution :-

For $i(t)$

From the voltage source

$V_s = 20 \sin(10t + 30°)$ V

$V_s = 20 \cos(10t + 30° - 90°)$ V

$V_s = 20 \cos(10t - 60°)$ V

$V_s = 20 \angle -60°$ V

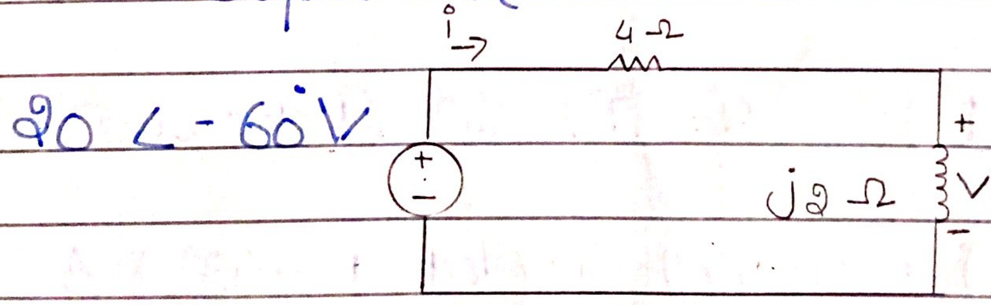
$\omega = 10$ rad/sec

$X_L = j\omega L$

$0.2\,H = j \times 10 \times 0.2$

$0.2\,H = j2\,\Omega$

Given circuit can be represented as :



From the above Circuit,

$Z = 4 + j2 \ \Omega$

Hence the Current i is,

$$i = \frac{20 \angle -60^\circ}{4 + j2}$$

$$i = \frac{20 \angle -60^\circ}{\sqrt{4^2 + 2^2} \angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$i = \frac{20 \angle -60^\circ}{4.472 \angle 26.57^\circ}$$

$$i = 4.472 \angle -86.57^\circ.$$

Converting this into domain,

$$i(t) = 4.472 \cos(10t - 86.57^\circ)$$

$$i(t) = 4.472 \sin(10t - 86.57 + 90^\circ)$$

$$i(t) = 4.472 \sin(10t + 3.43^\circ) \ A$$

For v(t):

From the circuit voltage across the inductor is,

$V = j2 \times i$

$V = j2 \times (4.472 \angle 86.57°)$

Converting polar form to rectangular form we get;

$V = j2 \times (0.26756 - j4.464)$

$V = 8.928 + j0.53512$

Converting rectangular form to polar form

$V = (\sqrt{(8.926)^2 + (0.53512)^2}) \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$

$V = 8.944 \angle 3.4\ V$

Converting this into time domain

$v(t) = 8.944 \cos(10t + 3.4°)$

$v(t) = 8.944 \sin(10t + 3.4° + 90°)$

$v(t) = 8.944 \sin(10t + 93.4°)$ V

Question No 3 :

A Series RLC circuit is described by

$$L\frac{d^2t}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

find the response when L = 0.5 H, R = 4 Ω and C = 0.2 F.

Let $i(0) = 1A$, $\frac{di(0)}{dt} = 0$

Solution :-

The step response of the branch voltage of the given RLC circuit is described by;

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divide by L;

$$\frac{d^2i}{dt^2} + \frac{R\,di}{L\,dt} + \frac{i}{LC} = \frac{10}{L}$$

xing by $\frac{C}{C}$

$$\frac{d^2i}{dt^2} + \frac{R\,di}{L\,dt} + \frac{i}{LC} = \frac{10C}{LC}$$

As $C = 0.2F$, thus;

$$\frac{d^2i}{dt^2} + \frac{R\,di}{L\,dt} + \frac{i}{LC} = \frac{2}{LC}$$

Substitute;

$$\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 10i = 20 \quad \ldots (i)$$

The general form for source-free RLC is

given by,

$$\frac{d^2 i}{dt^2} + \frac{R di}{L dt} + \frac{1}{LC} i = \frac{I_s}{LC} \quad \cdots (2)$$

Comparing 1 and 2 we get,

$$\frac{R}{L} = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 10 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow (5)$$

From (3), α is given by;

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/sec} \rightarrow (6)$$

The natural Frequency ω_0 is given by;

Substitude $t = 0$;

$$\frac{di(0)}{dt} = A_1 s_1 + A_2 S_2$$

Substitude the values;

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0 \rightarrow (11)$$

Solving (10) and (11) Simultaneously

$$A_1 = -1.316$$
$$A_2 = 0.316$$

Substitute in (9)

$$i(t) = 2 - 1.316 e^{(-4+\sqrt{6})t} + 0.316 e^{(-4-\sqrt{6})t} A$$

Question No 4 :-

A Series RLC Circuit has

$R = 100\ \Omega$
$L = 240\ H$
$C = 10\ mF$

If the input voltage is $v(t) = 10 \cos 2t$, find the current flowing through the circuit.

Solution :-

The input voltage is,

$$v(t) = 10 \cos 2t \ V$$

Amplitude = V_m = 10 V

Angular frequency = 2 rad/s

Phase angle, $\phi = 0°$

So

Phasor for the voltage $v(t)$:

$$v(t) = 10 \angle 0° \ V$$

Now For inductive reactance

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From (4)

$$\omega_0 = \sqrt{10} \text{ rad/sec} \rightarrow (7)$$

From (6) and (7):

$\because \alpha > \omega_0$

$\therefore$ The circuit is overdamped.

The root of the characteristic equation are given by;

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 + \sqrt{6} \text{ rad/sec}$$

And,

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -4 - \sqrt{4^2 - \sqrt{10}^2}$$

$= -4 - \sqrt{6}$ rad/sec

From (5), the steady current is given by

$I_s = 20 \times LC = 20 \times 0.5 \times 0.2 = 2A \rightarrow (8)$

The current for overdamped is given by ;

$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \rightarrow (9)$

Substitude $t = 0$:

$i(0) = I_s + A_1 + A_2$

Substitude :

$1 = 2 + A_1 + A_2$

Thus,

$A_1 + A_2 = -1 \rightarrow (10)$

From 9, find $\frac{di(t)}{dt}$,

$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$

$X_L = \omega L$

So $\omega = 2\ rad/s$, $L = 240\ H$

$X_L = \omega L$

$X_L = (2)(240)$

$X_L = 480\ \Omega$

Now for capacitive reactance,

$$X_C = \frac{1}{\omega C}$$

$\omega = 2\ rad/s$, $C = 10\ mF$

$$\frac{1}{2(10 \times 10^{-3})}$$

$$\frac{1}{2 \times 10^{-2}}$$

$$\frac{1 \times 10^{2}}{2}$$

$X_C = 50\ \Omega$

Now for impedence ;

$Z = R + jXL - jXC$

$R = 100\,\Omega$, $XL = 480\,\Omega$, $XC = 50\,\Omega$

Putting in equation

$Z = (100 + 480 - 50)\,\Omega$

$Z = (100 + j430)\,\Omega$

Represent "Z" in Phasor form.

$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$

$= \sqrt{10{,}000 + 184{,}900} \angle \tan^{-1}(4.3)$

$= \sqrt{194.900} \angle \tan^{-1}(4.3)$

$Z = 441.47 \angle 76.90^\circ\,\Omega$

Now for Current flowing in the circuit ;

$i = \frac{v(t)}{Z}$

$v(t) = 10\angle 0^\circ$, $Z = 441.47\angle 76.90^\circ\Omega$.

Putting in equation

$$i = \frac{10\angle 0^\circ V}{441.47\angle 76.90^\circ\Omega}$$

$$i = \frac{10}{441.47}\angle[0 - 76.90^\circ]\,A$$

$$= 22.6 \times 10^{-3}\angle -76.90^\circ A$$

$$= 22.6\angle -76.90^\circ mA$$

So, the general expression for "i"

$$i = 22.6\cos(2t - 76.90^\circ)\,mA.$$