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SUBJECT: DIFFERENTIAL EQUATIONS

SEMESTER: 3RD

PROGRAMME: BS (SOFTWARE ENGINEERING)

Q1) $x^2y'' - 4xy' + 6y = 0$ $\quad y(1) = 0.4$, $y'(1) = 0$

Sol:-

~~Pet~~ Put $y = x^n$

$y' = m \cancel{x}^{m-1}$

$y = m(m-1)x^{m-2}$

$x^2 m.(m-1)x^{m-2}\ 4x m^{m-1} + 6x^n = 0$

Common factor

$m(m-1) - 4m + 6 = 0$

$m^2 - 5m + 6 = 0$

Now finding roots

$m^2 - 5m + 6 = 0$

$m\ 1/2 = \frac{5 + \sqrt{(-5)^2 + 4}}{2}$

$m^1/2 = \frac{5+1}{2}$

$m = 3$ & $m_2 = 2$

$y = x^m = x^3$ & $x^2 = x^{m_2} = x^2$

General Solution is

$y = C_1 y + C_2 y,$

$= C \cdot x + C = x^2$

$y' = 3Cx^2 + 2C4$

Now to determine C_1 and C_2

$$\begin{cases} 0.4 = y(1) = C_1 \cdot 1^3 + C \cdot 1^2 \\ 0 = y'(1) = 3C \cdot 1 + 1C + 1 \end{cases}$$

$$\begin{cases} 0.4 = C + C_2 \\ 0.2 \quad 3C_1 + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 - C_1 \\ 0 = 3(0.4 - C_1) + 2C_1 \end{cases}$$

$$\begin{cases} 0.4 - 1.2 = C \\ 1.2 = C_2 \end{cases}$$

$$\begin{cases} 0.8 = C_1 \\ 1.2 = C_1 \end{cases}$$

The particular solution

$y = (-0.8x^3) + 1.2x^2$ Ans

Q2)

$$x^2y'' + 3xy' + 0.75y = 0$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-1}$$

Putting in the given DE

$$x^2 m(m-1)x^{m-2} + 3x\, mx^{m-1} + 0.75x^m = 0$$

Common factor

$$m(m-1) + 3m + 0.75 = 0$$

$$m^2 + 2m + 0.75 = 0$$

$$m^2 + 2m + 0.75 = 0$$

$$m_{1/2} = \frac{-2 \pm \sqrt{1^2 - (4)(0.75)}}{2}$$

$$m_{1/2} = \frac{-2 \pm 1}{2}$$

$$m_1 = -1/2 \quad \wedge \quad m_2 = -3/2$$

$y_1 = x^{m_1} = x^{-1/2} = x^{-0}$ $\lambda = x^{m_2} = x^{-3/2}$

$= C_1 x^{0.5} + C_2 x^{1.5}$

$y' = -0.5 C_1 x^{-1.5} - 1.5 C_2 x^{-2.5}$

To determine C_1 and C_2

$$\begin{cases} 1 = C_1 + C_2 \\ 1.5 = 0.5C - 1.5C_2 \end{cases}$$

$$\begin{cases} 1 = C_2 + C_1 \\ 3 = C_1 + 3C_2 \end{cases}$$

$$\begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$

$y = x^{-1.5}$ Ans

5

Q3) $x^2y'' + xy' + 9y = 0$

$x^2 m(m-1)x^{m-2} + mx^{m} + 9x^m = 0$

$\cancel{x^2} m(m-1)x^{m-\cancel{2}} + mx^m + 9x^m = 0$

$m(m-1) + m + 9 = 0$

$m^2 - \cancel{m} + \cancel{m} + 9 = 0$

finding roots

$\Rightarrow \quad m_1 = 3i \quad \Lambda \quad m_2 = 3i$

$e^{m_2} = x^3 = (e^{\ln x})^{3i} = e^{3i\ln x}$

$e^{m_2} = x^{-3i} = e^{\ln x} = e^{3i\ln x}$

$e^{3i\ln x} = e^{0}(\cos(3\ln x) \cdot i\sin(3\ln x) = \cos(3\ln$

$e^{m_2} = \cos(3\ln x) = i\sin(3\ln x)$

$$\frac{x^{m} + x^{m_2}}{2} = \cos(3\ln x)$$

$$\frac{x^{m_1} - x^{m_2}}{2i} = \sin(3\ln x)$$

6

$y = \cos(3\ln x)$ & $y = \sin(3\ln x)$

$y = c_1 y_1 + c_2 y_2$

$c_1 \cos(3\ln x) + c_2 \sin(3\ln x)$

$y' = -c_1 \sin(3\ln x)(3\ln x)' + c_2 \cos(3\ln x)$

$$\begin{cases} 0 = y(1) = c_1 \cos(3\ln x) + c_2 (\sin)(3\ln x) \\ 2.5 = y'(1)2 - 3c_1 \sin(3\ln x) + 3c_2 \cos(3\ln x) \end{cases}$$

$$\begin{cases} 0 = c_1 \\ 5/2 = 3c_1 \quad / : 3 \end{cases}$$

$$\begin{cases} 0 = c_1 \\ 5/6 = c_2 \end{cases}$$

$y = 5/6 \sin(3\ln x)$ Ans

Q4) $x^2 y'' + 3xy' + y = 0$

put $y = x^m$, $y'' = m(m-1)x^{m-2}$

$x^2 m(m-1) x^m \cdot x^{-2} + 3m x^m + x^m = 0$

Common factor

$m(m-1) + 3m + 1 = 0$

$m^2 + 1m + 1 = 0$

$m^2 + 2m + 1 = 0$, $(m+1)^2 = 0$

$y_1 = x^m = x^{-1} = 1/x$

$y'' + 3/x \, ; \, y' + \frac{1}{x^2} y = 0$

$P(x) = \frac{3 \cdot 1}{x} \Rightarrow \int P dx = 3 (\ln x)$

$y_2 = vy$

$e^{-\int P dx} \quad e^{-3\ln(x)} \quad \ln|x|^{-3}$

$C = C = (C) = x^{-3}$

$x = \int \frac{dx}{x} = \ln|x|$

$y = uy = y \cdot \ln x = \frac{1}{x} \ln x$

$y = c_1 y_1 + c_2 y$

$\frac{1}{x} c_1 + c_2 \quad \ln x$

$= -x^2 (c_1 + c_2 \ln x) + \frac{1}{x} \ln \cdot \frac{1}{x}$

$= \frac{1}{x^2} (-c_1 - c_2 \ln x + c_2)$

$\begin{cases} 3.6 = y(1) = \frac{1}{1} (c_1 + c_2 \ln 1) \\ 8.4 = y'(1) = \frac{1}{2} (-c_1 - c_2 \ln 1 + c_2) \end{cases}$

$\begin{cases} 3.6 = c_1 \\ 0.4 = -3.6 + c_2 \end{cases}$

$\begin{cases} 3.6 = c_1 \\ 4.0 = c_2 \end{cases}$

$y = (3.6 + 4.0 \cdot 0.\ln x)^{1/x}$

Ans

Q5 $(x^2 D^2 - 3xD + 4) = 0 \quad y(1) = f(1) = 2\pi$

$x^2 y'' - 3xy' + 4y = 0$

$x^2 m(m-1) x^{m-2} - 3x m x^{m-1} + 4x^m = 0$

$\cancel{x^2} m(m-1) x^m \cancel{x} - 3\cancel{x} m e^m \cancel{x} + 4x^m = 0$

$m(m-1) - 3m + 4 = 0, \quad m^2 - 4m + 4 = 0$

hence $y = x^1$ is a solution

$m^2 - 4m + 4 = 0, \quad (m-1)^2 = 0$

$y = x^m = x^2$

$y'' = \frac{-3}{x} y' + \frac{4}{x^2} y = 0$

$P(x) = -3\frac{1}{x} \Rightarrow \int p(x = -3\ln(x)$

$y_2 = uy_1$

$u = x^3 \cdot \frac{1}{(x)^{2^2}} = x^{3-4} = x^1 = 1/x$

$u = \int \frac{dx}{x} = \ln(x)$

$y_2 = uy_1 = y_1 \ln(x) = u \ln(x)$

$y_1 = y_2 \in R$

General solution

$y = c_1 y_1 + c_2 y_2$

$u^2 (c_1 + c_2 \ln x)$

$y' = (x^2)' (c_1 + c_2 \ln x) + x^2 (c_1 + c_2 \ln x)'$

$= 2c_1 x + 2c_2 \ln x + c_2 u$

$= 2c_1 u + c_2 x (2 \ln x + 1)$

$\begin{cases} -\pi = y(1) = 1^2 (x_1 + c_2 \ln 1) \\ 2\pi = y'(1) = 2c_1 + c_2 (2 \ln x \end{cases}$

$\begin{cases} -\pi = c_1 \\ 4\pi = c_2 \end{cases}$

$y = x^2 (-\pi + 4\pi \ln x)$

Ans

Q6) $(x^2 D^2 + xD + I)y = 0$, $y(1) = 1$, $y' =$

$x^2 D^2 y + x D y + I y = x^2 D(D y) + x D y + y$

$= x^2 y'' + xy' + y$

Now

$x^2 y'' + x y' + y = 0$

$y = x^m$, $y' = m e^{n}$, $y'' = m(m-1)$

$x^2 m(m-1) \frac{m}{x} \cancel{x} \quad \cancel{x} m x^m x^{-1} + x^m = 0$

$m(m-1) + m + 1 = 0 \Rightarrow m^2 - \cancel{m} + \cancel{m} + 1 = 0$

$= m^2 + 1 = 0$

$m_1 = i \wedge m = -i$

$x = e^{\ln x}$

$x^m = x^i = (e^{\ln x})^i = e^{i \ln x}$

$x^n = x^{m} = (e^{\ln x}) = e^{\ln x}$

$e = e^{\ln x} = e^{m}(\cos 1 + i \sin b)$, $2 \in C$

$e^{\ln x} = (c(\cos \ln x) + i \sin(\ln x))$

$= \cos(\ln x) + i\sin(\ln x)$

$x^{m_2} = \cos(\ln x) - i\sin(\ln x)$

$x^{m_1} + x^{m_2} = \cos(\ln x) + i\sin(\ln x) + \cos(\ln x) - i\sin(\ln x)$

$= 2\cos(\ln x)$

$\frac{x^{m_1} - x^{m_2}}{2i} = \sin(\ln x)$

$y_1 = \cos(\ln x)$ & $y_2 = \sin(\ln x)$

$= -\frac{C_1}{x}\sin(\ln x) + \frac{C_2}{x}\cos(\ln x)$

$\begin{cases} 1 = C_1\cos(0) + C_2\sin(0) \\ 1 = -C_1\sin(0) + C_2\cos(0) \end{cases}$

$\begin{cases} 1 = C_1 \\ 1 = C_2 \end{cases}$

$y = \sin(\ln x) + \cos(\ln x)$

Ans

(Q7) $(9x^2D^2 + 3xD + I)y = 0$ $\quad y(1) = 1$, $y'(1) = 0$

$9x^2D^2y + 3xDy + Iy = 9x^2 D(Dy) + 3xD$

$9x^2y'' + 3xy' + y = 0$

$9x^2m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0$

$9m(m-1) + 3m + 1 = 0$, $9m^2 - 9m + 3m + 1 = 0$

$= 9m^2 - 6x + 1 = 0$

Finding the roots

$m^2 - 4m + 4 = 0$, $(m-2)^2 = 0$

$9m^2 - 6m + 1 = 0$ $m_{1/2} = \frac{6 \pm \sqrt{6^2 - 40}}{18}$

$m_{1/2} = 6/18$

$m_{1/2} = 1/3$

$y_1 = x^m = x^{1/3}$

$y_2 = uy_1$

$u = \int U dx$ / $U = \frac{1}{y_2} e^{\int P dx}$

$$v = x^{-1/3} \frac{1}{(x^{1/2})} = x^{-1/3 - 2/3}\, x^{-1} = \frac{1}{x}$$

$$y = vy = y(x \ln x) = x^{1/3} \ln x$$

$$y = c y_1 + c_2 y_2$$

$$y' = (x^{1/3})'(c_1 + c_2 \ln x) + x^{1/3}(c_1 + c_2 \ln x)'$$

$$= \tfrac{1}{3} x^{-2/3}(c_1 + c_2 \ln x) + x^{1/3} c_2$$

$$\left\{ \begin{array}{l} 1 \cdot y(1) = 1^{1/3}(c_1 + c_2 \ln 1) \\ 0 = y'(1) = \tfrac{1}{3} \cdot 1^{-2/3}(c_1 + c_2 \ln x) + 1^{1/3} c_2 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 1 = c_1 \\ -1/3 = c_2 \end{array} \right\}$$

$$y = x^{1/3}\left(1 - \tfrac{1}{3} \ln x\right)$$

Ans

Q2-1-a) $x' = \sqrt{x}$

$$\frac{dx}{dt} = x$$

$$\frac{dx}{\sqrt{x}} = 1\,dt$$

$$\frac{1}{\sqrt{x}}\,dx = dt$$

$$\int \frac{1}{\sqrt{x}}\,dx = \int dt$$

$$2\sqrt{x} + C_1 = t + C_2$$

$$2\sqrt{x} = t + C$$

$$4(x) = (t + c)^2$$

$$x = \frac{(t+c)^2}{4}$$

Ans

Q2-1b) $x' = e^{-2x}$

$$= \frac{dx}{dt} = e^{-2x} \Rightarrow \frac{dx}{e^{-2x}} = dt$$

$$= \frac{e^{2x}}{2} = t + C$$

$$e^{2x} = 2(t+C)$$

$$x = \ln \frac{2(t+C)}{2} \quad \text{Ans}$$

Q2-1-c) $\quad y' = 1 + y^2$

Sol:-

$$\frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{dy}{1+y^2} = dt$$

$$y(1) = \tan(t+C) \quad \text{Ans}$$

Q2-1-d) $\quad u' = \frac{1}{5 - 2u}$

Sol:-

$$\frac{dy}{dt} = \frac{1}{5-2u} \Rightarrow \frac{dy}{5-2u} = dt \ (1)$$

$$-(u-5)\, u + C = t + C_2$$

$$-(u-5)u = t + C$$

$$= \frac{t+C}{-(u-5)} \quad \text{Ans}$$

Q2)-1-j)

$$Q' = \frac{Q}{4+Q^2}$$

$$\frac{dQ}{dj} = \frac{Q}{4+Q} \Rightarrow \frac{Q^2 dQ}{Q} = dj$$

$$= \int \frac{Q^2}{Q} dQ = \int dj$$

$$3\ln(Q) + \frac{Q^2}{2} + C \quad \text{Ans}$$

Q2-1 g)

$$x' = e^{x^2}$$

$$\frac{dx}{dt} = e^{x^2}$$

$$\frac{dx}{e^{x^2}} = dt$$

$$x = \frac{1\ t+e}{\sqrt{\pi}\ j}$$

$$x(1) = \frac{2j + C}{\sqrt{\pi}} \quad \text{Ans}$$

Q2-1-h

$$y' = r(a-y)$$

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)}\,dy = \int dt$$

$$\frac{(a-y)}{r}\ u = t + c$$

$$x(t) = \frac{r(t+c)}{a-y}$$ Ans

Q2-3(a)

$(1a)7 \quad , \quad x(0) = 1$

$x(t) = \frac{(t + c)^2}{4}$

$u(0) = 1 , \; u(t) = u(0) = 1$

$1 = \frac{(0 + c)^2}{4}$

$4 = c^2$

$c = 2$ Ans

(Q2-4a)

$u' = \frac{2x}{t+1}$

$\frac{dx}{dt} = \frac{2x}{t+1}$

$\frac{dx}{2x} = \frac{1}{t+1} dt$

$\int \frac{dx}{2x} = \int \frac{1}{t+1} dt$

$\frac{u^2}{4} = \ln(t+1) + C$

$= C = \frac{x^2}{4\ln(t+1)}$ Ans

Q2-4-b)

$\theta' = t\sqrt{t^2+1}\ \sec\theta$

$d\theta = t\,t\sqrt{t^2+1}\ \sec\theta$

$\frac{d\theta}{\sec\theta} = t\sqrt{t+1}\ dt$

$= \sin\theta = \frac{(t^2+1)^{3/2}}{3} + C$

$= C = \frac{3\sin\theta}{(t^2+1)^2}$ Ans

Q2-4 e) $(2v+1)v' - (t+1) = 0$

Sol:-

$(2v+1)\,dv = (t+1)\,dt$

$\int (2v+1)\,dv = \int (t+1)\,dt$

$\frac{2(v^2+v)}{t^2} - 1 = e$

$$C = 2\left(\frac{v^2 + v}{t^2}\right) - 7$$

Q 2-4 d)

$$R' = (t+1)(R^2+1)$$

$$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\int \frac{dR}{R^2+1} = \int (t+1)\,dt$$

$$C = \frac{2\,(\cot(R))}{t^2} - 6 \quad \text{Ans}$$